

Low frequency disturbances: offset and flicker noise

- ➔ • The offset strongly affects the accuracy of most sensor interfaces and their detection limit
- ➔ • When the quantity estimation requires integration operations, such as dead reckoning (position from acceleration or speed), the offset results in a drift of the final quantity.
- ➔ • The flicker noise, with its power accumulation at low frequency, makes the problem worse, reducing resolution.
- ➔ • Both offset and flicker noise may affect also high-speed or RF systems (e.g.: offset spurs in fast time-interleaved ADCs or $1/f$ noise upconversion in oscillators)

State of art of amplifier offset

The best amplifiers use BJTs and resistor trimming, to achieve offsets as low as 15 μV with 0.1 $\mu\text{V}/^\circ\text{C}$ drift (typical) and low frequency noise $< 0.5 \mu\text{V}$ pp in the 0.1-10 Hz frequency band.

This option is often not convenient:

- It is obtained with very large area and a non-CMOS technology, resulting non suitable for modern SoCs.
- Trimming of the individual amplifiers is required
- In many examples of sensor interfacing, offsets as low as 1 μV are required.

Calibration

One-time calibration is typically performed at the start-up to the whole DAS, providing a reference input (e.g.: 0 V) and storing the result in a digital memory, then subtracted in the digital domain.



Not effective against offset drift and low-frequency noise.

Calibration can be repeated to track the offset drift, but:

- there is a loss of data during the calibration routine
- rejects the offset of the whole system, not of the individual amplifier
- still not effective against low-frequency noise

Dynamic techniques for the offset and noise flicker reduction

Three main techniques

- Auto-zero (AZ)
- Correlated Double Sampling (CDS)
- Chopper modulation (CHS = Chopper stabilization)

C. C. Enz and G. C. Temes, "Circuit techniques for reducing the effects of op-amp imperfections: autozeroing, correlated double sampling, and chopper stabilization," in *Proceedings of the IEEE*, Nov. 1996, doi: 10.1109/5.542410

We are interested in understanding the principle of operation, the residual noise spectrum and possible limitations of the various techniques.

The three techniques are not limited to pure electronic circuits, but can in general be applied to other physical systems and, in particular, in a DAS, may involve also the sensor.

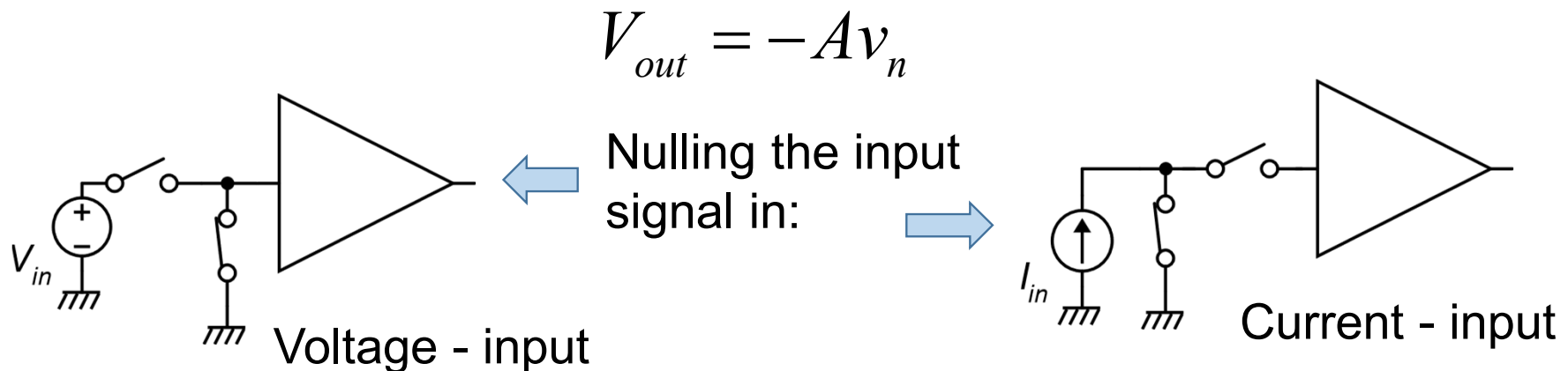
Premise

In all cases, in order to simplify the formalism, we will refer to the case of a voltage amplifier:

$$V_{out} = A(V_{in} - v_n)$$

where v_n includes both the input referred noise and offset voltage.

If we remove the signal from the amplifier input, the output becomes:



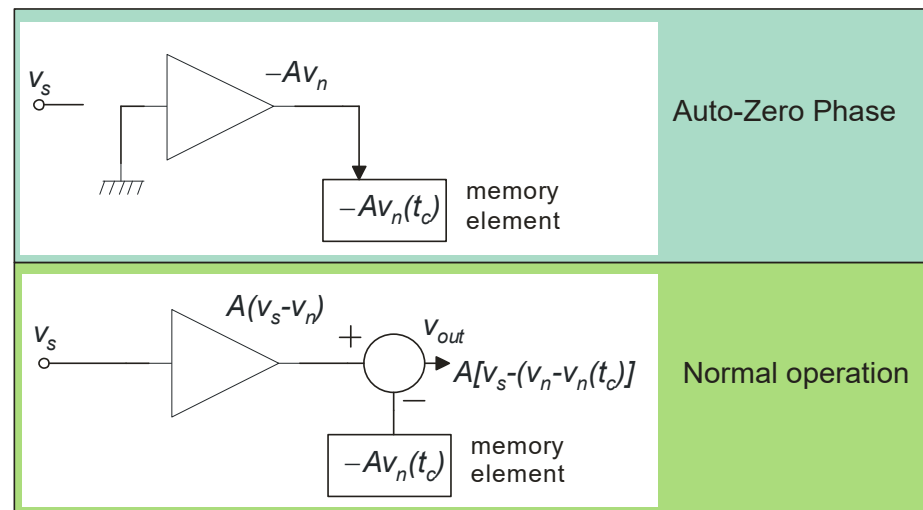
Auto – Zero (AZ)

Two phases: AZ and NO (Normal operation)

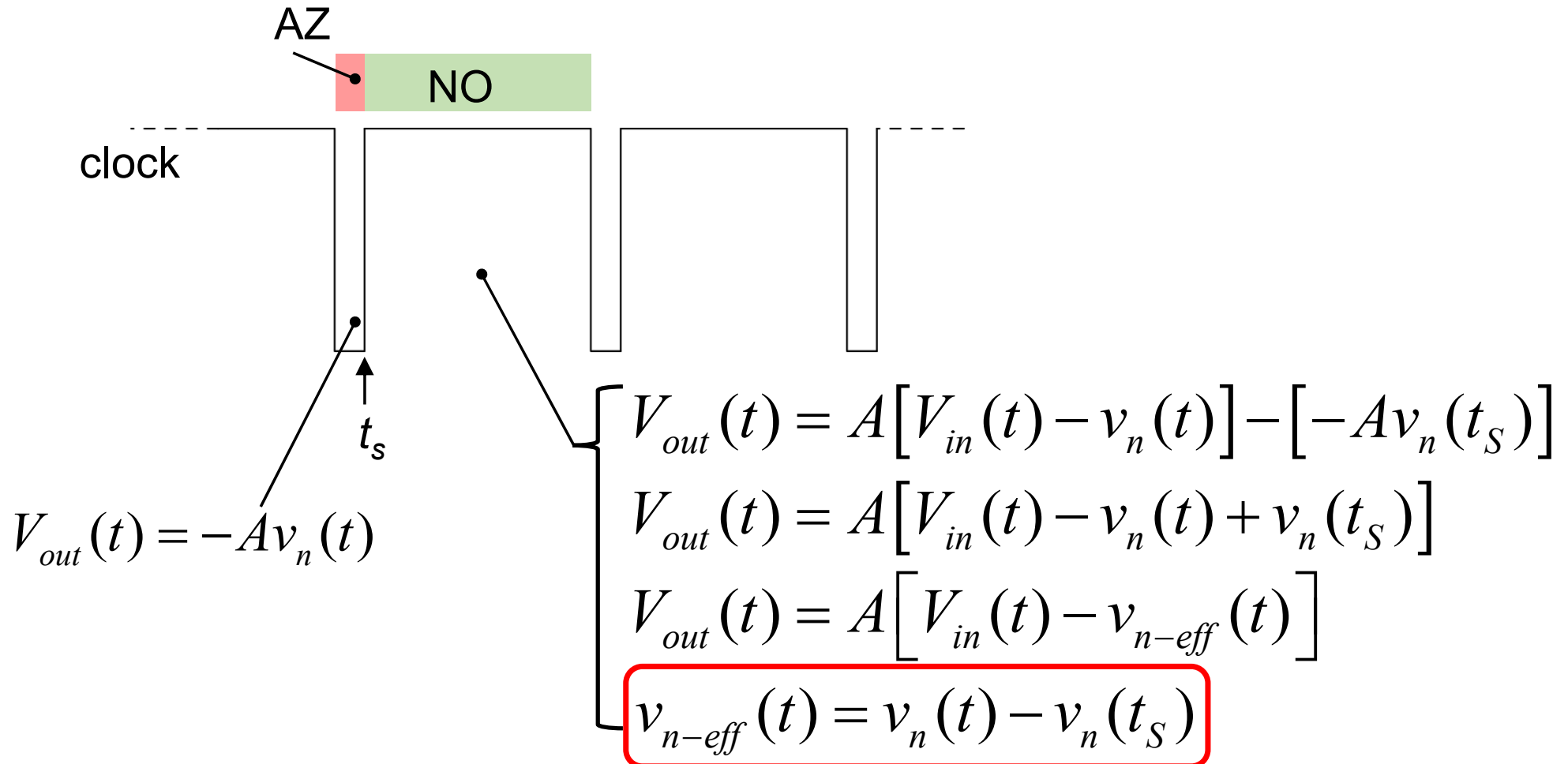
Principle:

1. In the AZ phase the signal is removed and the effect of v_n (noise/offset) is stored in a memory (typically an analog memory, i.e. a capacitor)
2. In the NO phase, the signal is connected and the v_n value stored in previous phase is subtracted.

Example, for an amplifier with voltage input

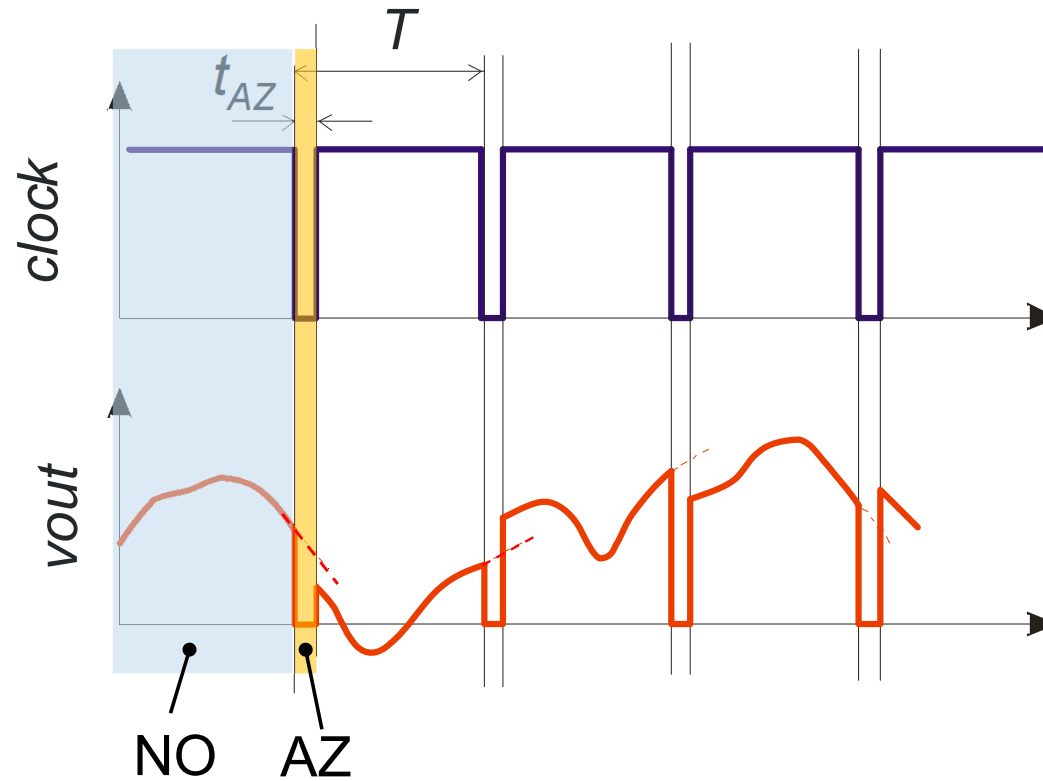


Signals and noise during the AZ cycle



Auto-Zero: phases and signal diagrams

AZ technique is applied to time-continuous systems (AZ phase should be as short as possible)



$$t_{AZ} \ll T$$

Auto-Zero simplified noise model

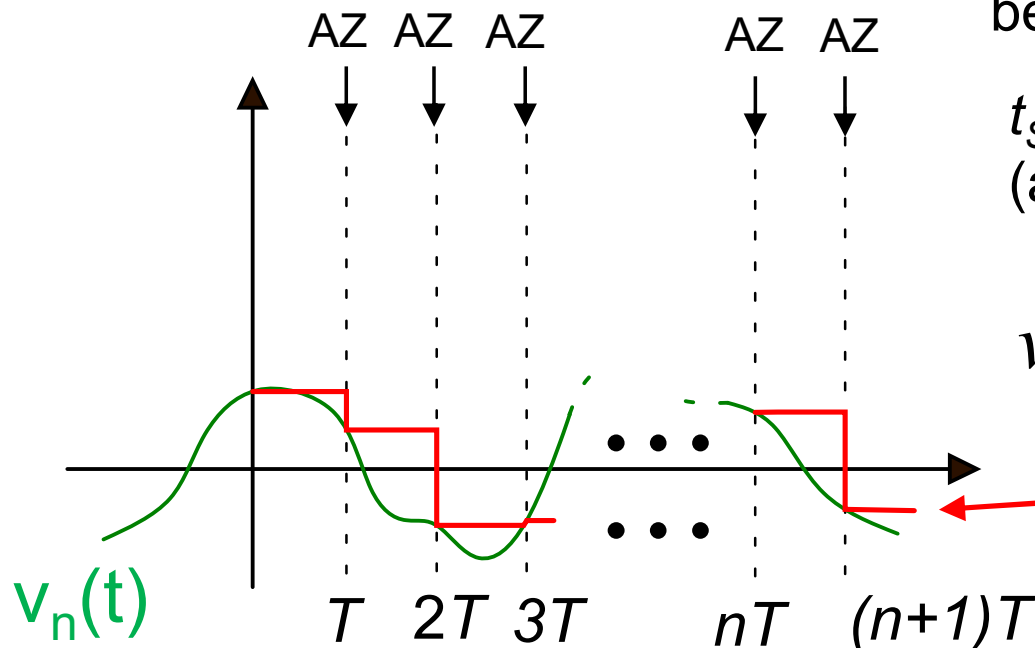
We consider that the duration of the AZ phase is negligible (ideally zero), then the amplifier is in NO phase during the whole period.

$$v_{n-eff}(t) = v_n(t) - v_n(t_S)$$

for each clock cycle, t_S coincides with the beginning of the cycle

t_S instants form a discrete set (a sequence nT)

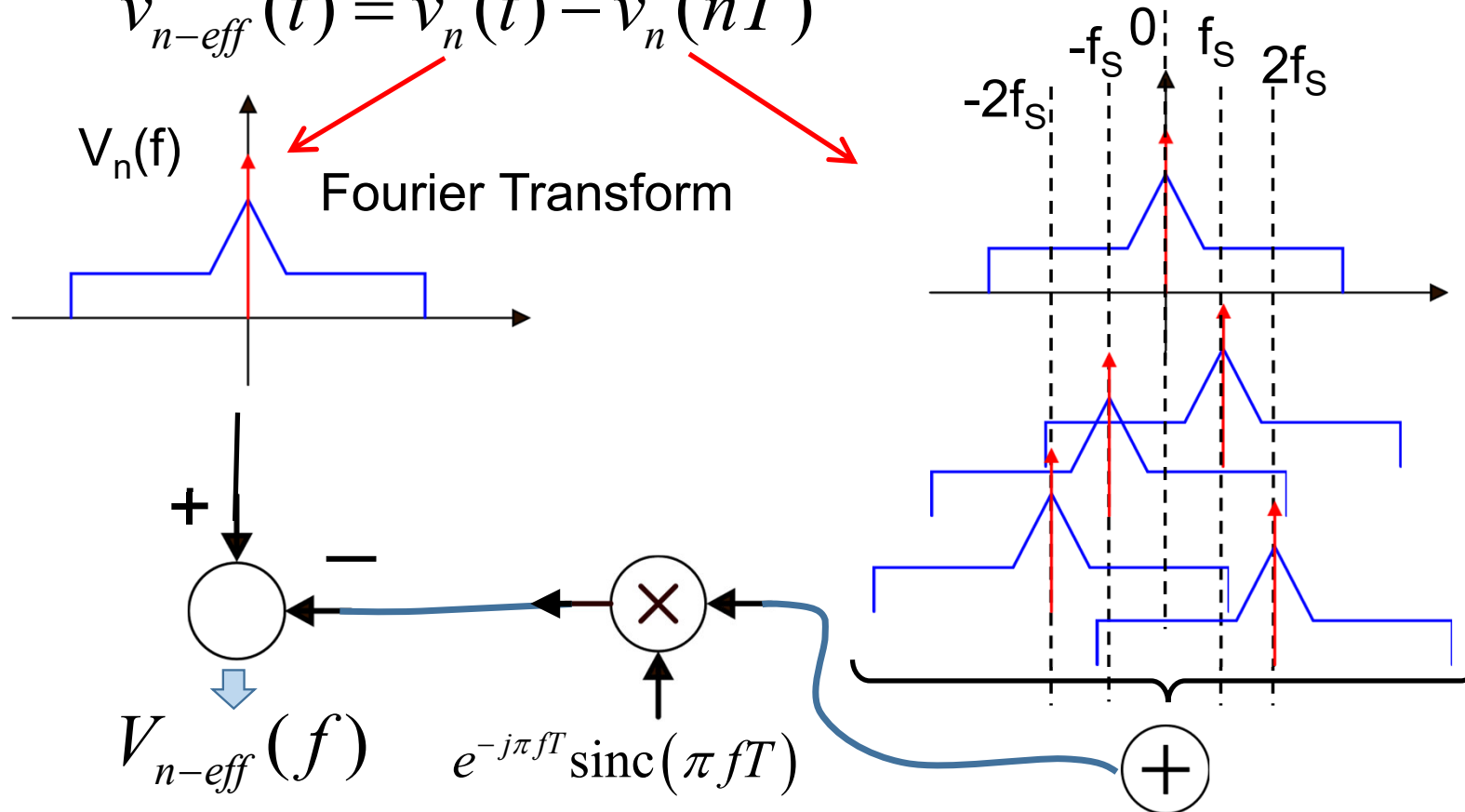
$$v_{n-eff}(t) = v_n(t) - v_n(nT)$$



Calculation of the spectrum of the residual noise v_{n-eff}

We consider a single realization of the noise random process

$$v_{n-eff}(t) = v_n(t) - v_n(nT)$$



Calculation of the spectrum of the residual noise $v_{n\text{-eff}}$

$$V_{n\text{-eff}}(f) = V_n(f) - e^{-j\pi fT} \text{sinc}(\pi fT) \left[\sum_{k=-\infty}^{\infty} V_n(f - kf_{ck}) \right]$$

Zero-order replica extracted from the sum

$$V_{n\text{-eff}}(f) = V_n(f) - e^{-j\pi fT} \text{sinc}(\pi fT) V_n(f) - e^{-j\pi fT} \text{sinc}(\pi fT) \left[\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} V_n(f - kf_{ck}) \right]$$

$$V_{n\text{-eff}}(f) = \underbrace{\left[1 - e^{-j\pi fT} \text{sinc}(\pi fT) \right]}_{H_0(f)} V_n(f) - \underbrace{e^{-j\pi fT} \text{sinc}(\pi fT)}_{H_1(f)} \left[\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} V_n(f - kf_{ck}) \right]$$

Calculation of the spectrum of the residual noise $v_{n\text{-eff}}$

$$V_{n\text{-eff}}(f) = \sum_{k=-\infty}^{\infty} H_k(f) V_n(f - kf_{ck})$$

$$H_k(f) = \begin{cases} H_0(f) = 1 - e^{-j\pi fT} \text{sinc}(\pi fT) & \text{for } k = 0 \\ H_1(f) = -e^{-j\pi fT} \text{sinc}(\pi fT) & \text{for } k \neq 0 \end{cases}$$

Now, remember that this transformation is applied to the random process $v_n(t)$ with spectral density $S_{vn}(f)$:

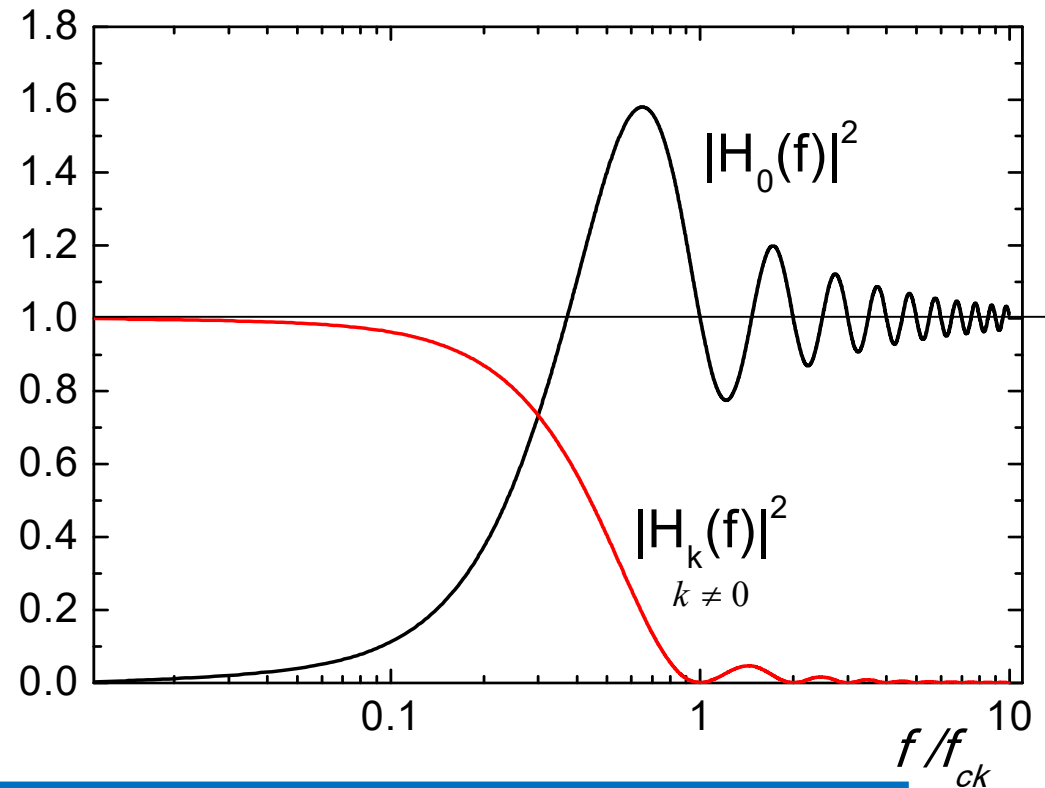
$$S_{vn\text{-eff}}(f) = \sum_{k=-\infty}^{\infty} |H_k(f)|^2 S_{vn}(f - kf_{ck})$$

Calculation of the spectrum of the residual noise $v_{n\text{-eff}}$

$$S_{v_{n\text{-eff}}}(f) = \sum_{k=-\infty}^{\infty} |H_k(f)|^2 S_{v_n}(f - kf_{ck})$$

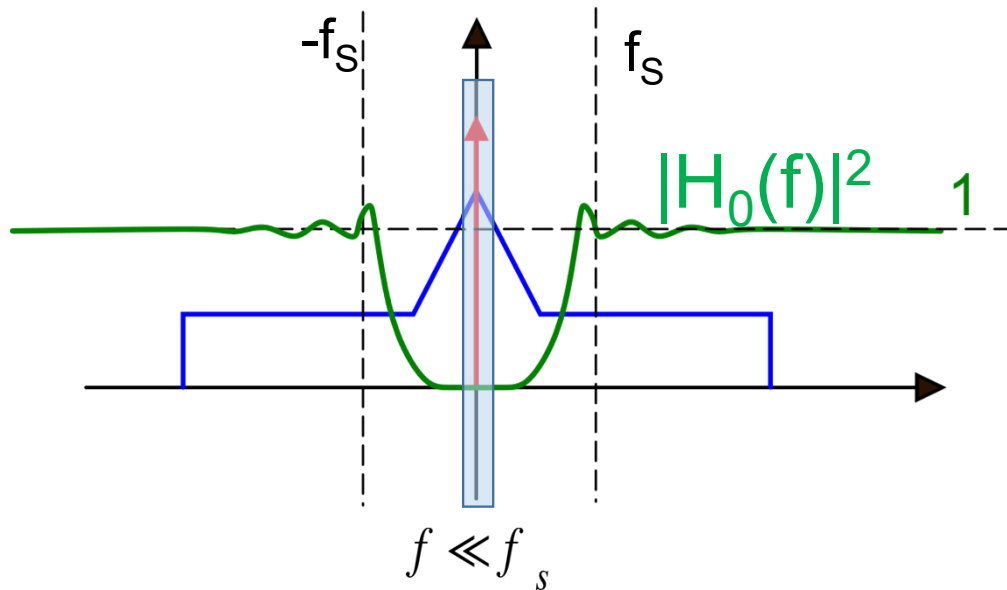
$$H_0(f) = 1 - e^{-j\pi fT} \text{sinc}(\pi fT)$$

$$H_1(f) = -e^{-j\pi fT} \text{sinc}(\pi fT)$$



Contribution of 0-th replica

$$H_0(f) = 1 - e^{-j\pi fT} \text{sinc}(\pi fT)$$



The 0-th replica is weighted by $H_0(f)$:

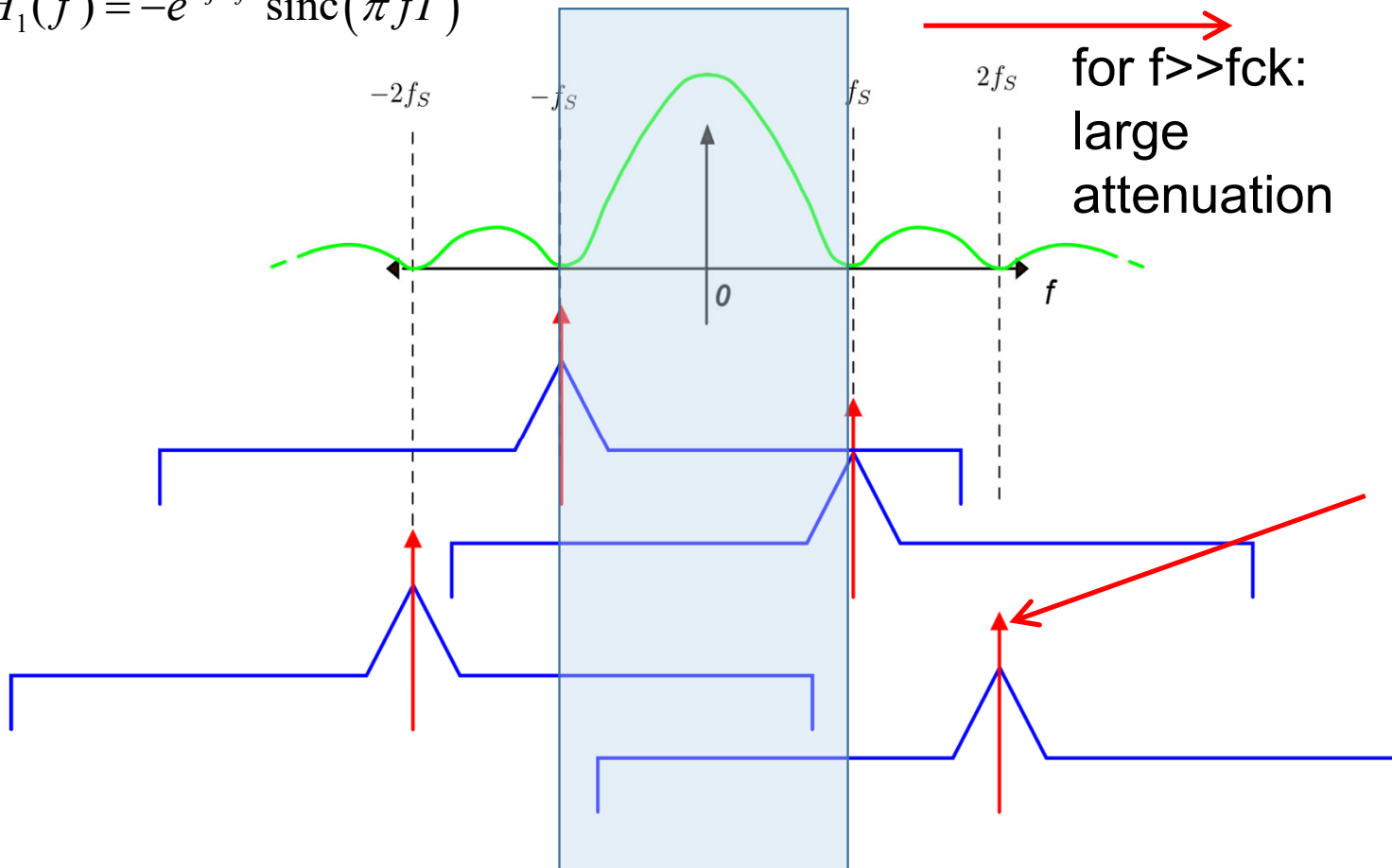
- The offset is cancelled
- The flicker noise is strongly reduced
- For $f \gg f_{ck}$ (f_s) the spectrum is nearly unchanged

$$\left\{ \begin{array}{l} |H_0(f)|^2 \xrightarrow{f \rightarrow 0} f^2 + \dots \\ S_{vn-F}(f) = \frac{k_F}{f^\gamma} \quad \gamma \cong 1 \end{array} \right.$$

$$\Rightarrow S_{vn-F}(f) |H_0(f)|^2 \xrightarrow{f \rightarrow 0} f + \dots$$

Contribution of replicas other than 0-th one

$$H_1(f) = -e^{-j\pi fT} \text{sinc}(\pi fT)$$



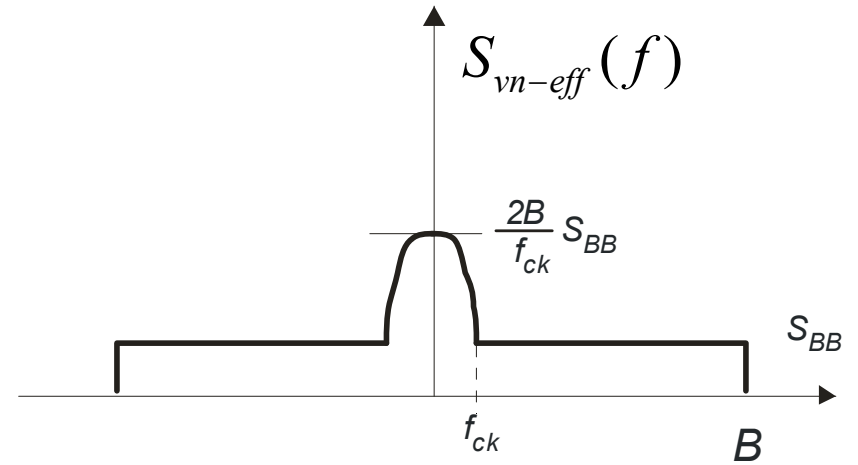
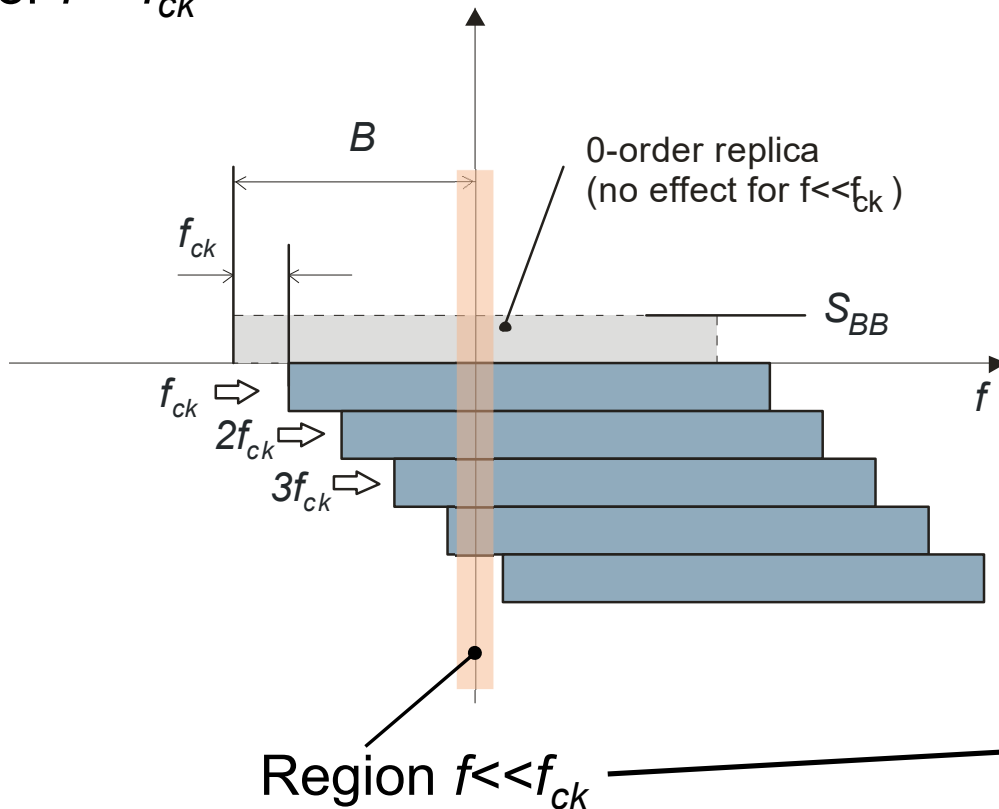
for $f \gg f_{ck}$:
large
attenuation

For $f < f_{ck}$ (f_s), several
contributions equal to S_{VBB}
are added together. For
 $f \ll f_{ck}$ the sinc leaves the
sum unchanged

The offset delta functions
and the flicker regions of
each replica fall over one
of the zeros (at f_{ck}) of the
sinc and are deleted or
strongly reduced

In Summary:

Contribution of replicas different from 0-th
for $f \ll f_{ck}$

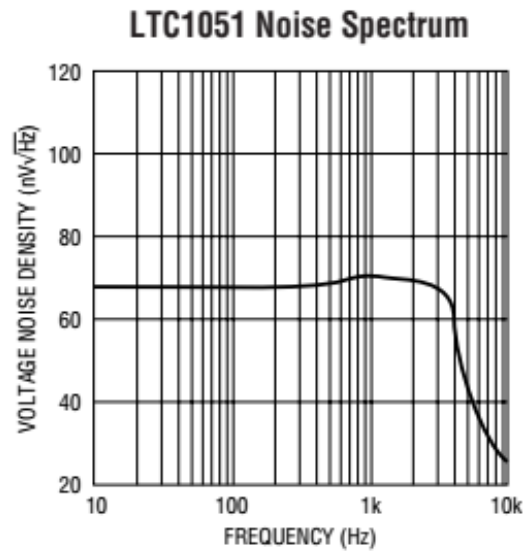


Approximate effective noise
PSD

Noise foldover

$$S_{vn-eff}(f) \cong \frac{2B}{f_{ck}} S_{BB} \quad \text{for } f < f_{ck}$$

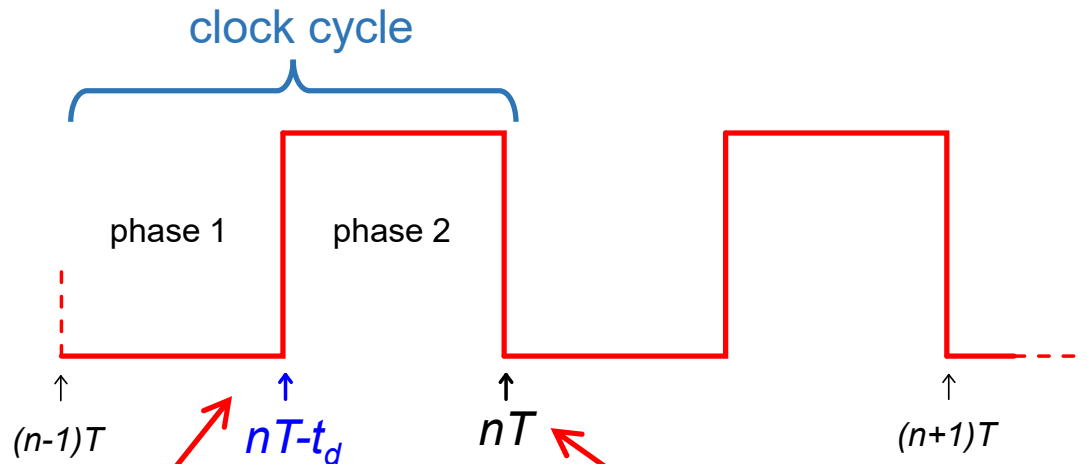
Example: LTC 1051, a commercial auto-zero operational amplifier



Parameter	Conditions	Typical	Max	Min	Max	Unit
Input Offset Voltage		±0.5	±5		±0.5 ±5	μV
Average Input Offset Drift	●	±0.0	±0.05		±0.0 ±0.05	μV/°C
Long Term Offset Drift		50			50	nV/√Mo
Input Bias Current	LTC1051C/LTC1053C ●	±15	±65 ±135		±15 ±50 ±100	pA pA

Correlated Double Sampling (CDS)

- CDS is a sampled data approach. Both the signal and the noise are discrete-time signals.
- It involves two clock phases: phase 1 and phase 2



at $nT-t_D$ the signal is removed
and the noise/offset is sampled

$$s_1 = -A[v_n(nT - t_D)]$$

at nT the signal is sampled
(together with noise/offset)

$$s_2 = A[V_{in}(nT) - v_n(nT)]$$

Correlated Double Sampling

The output voltage at instant nT of the system that adopts the CDS technique is the difference between the two samples:

$$V_{out}(nT) = s_2 - s_1 = A[V_{in}(nT) - v_n(nT)] - \{-A[v_n(nT - t_D)]\}$$

$$V_{out}(nT) = A[V_{in}(nT) - \underline{v_n(nT) + v_n(nT - t_D)}]$$

Differently from the autozero, also the signal is sampled. Then, all limitation coming from the Shannon theorem applies

We have the subtraction of two samples (hence "**Double Sampling**"). If the samples are similar ("**Correlated**") they cancel each other effectively

CDS: effective noise

$$v_{n-eff}(nT) = v_n(nT) - v_n(nT - t_D)$$

Generally, $t_D = \frac{T}{2}$, thus:

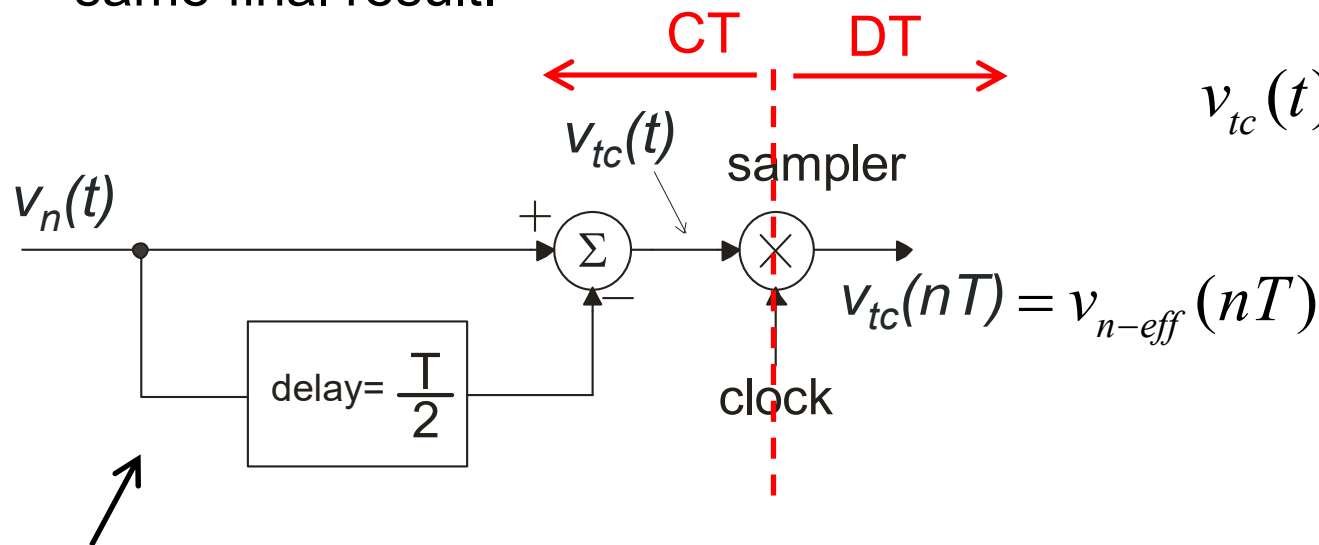
$$v_{n-eff}(nT) = v_n(nT) - v_n(nT - \frac{T}{2})$$

The operation applied to the noise involve sampled data and should be analyzed using the typical approaches of this domain, such as the Z-transform.

Problem: not all samples are sampled at instants that are multiple of T

CDS: effective noise

It is then preferred to use a mixed continuous-time / discrete-time approach, that does not represent the actual operations but that gives the same final result.



$$v_{tc}(t) = v_n(t) - v_n\left(t - \frac{T}{2}\right)$$

\uparrow nT \uparrow nT

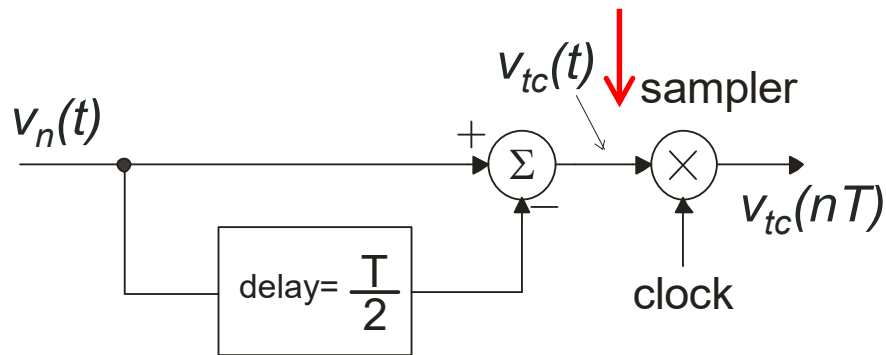
Sampling at intervals nT
we obtain the CDS effective
noise

$$v_{n-eff}(nT) = v_n(nT) - v_n\left(nT - \frac{T}{2}\right)$$

Equivalent model of the operations
applied to the original noise by the
CDS approach.

CDS: effective noise

$$v_{tc}(t) = v_n(t) - v_n\left(t - \frac{T}{2}\right)$$



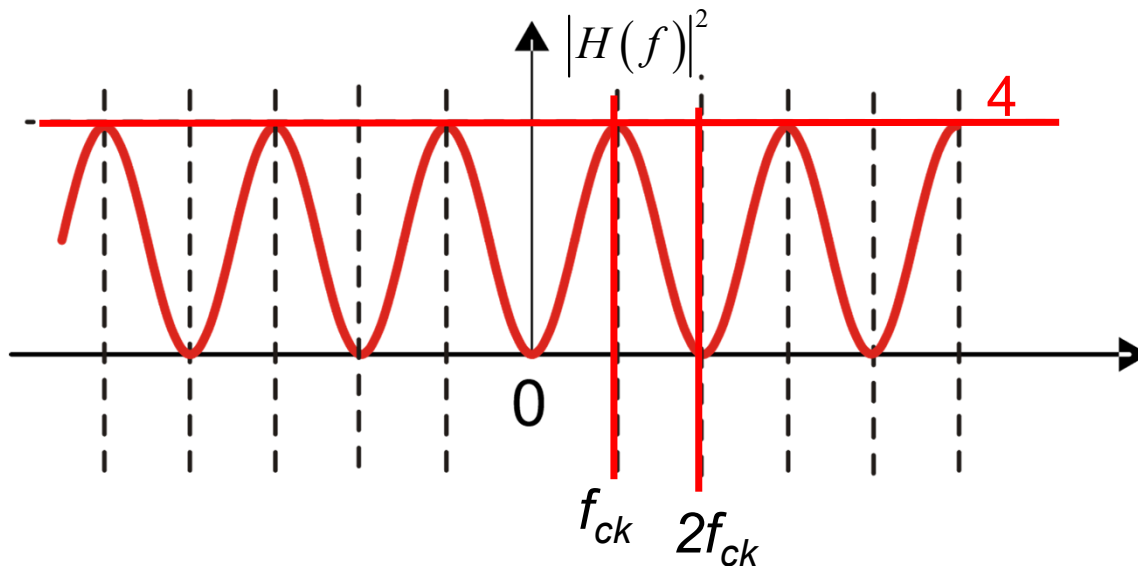
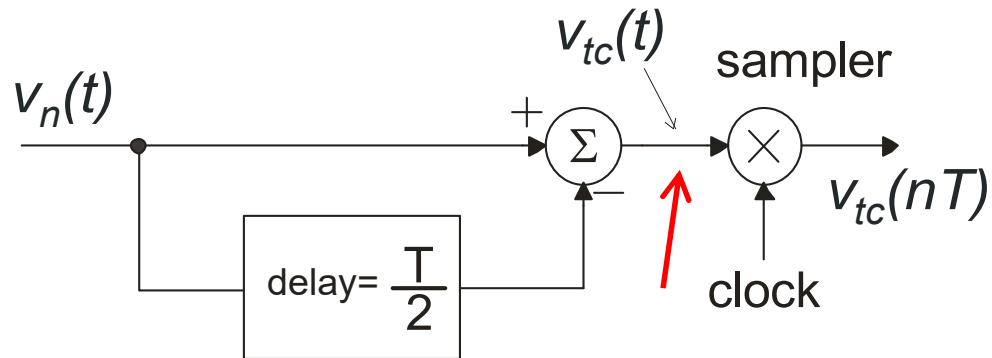
We are interested in the transfer function from the input of the model to the input of the sampler. In the frequency domain:

$$V_{tc}(f) = V_n(f) - V_n(f)e^{-j2\pi f \frac{T}{2}}$$

$$V_{tc}(f) = V_n(f) \underbrace{\left[1 - e^{-j2\pi f \frac{T}{2}} \right]}_{H(f)}$$

$$H(f) = e^{-j\pi f \frac{T}{2}} \left[e^{+j\pi f \frac{T}{2}} - e^{-j\pi f \frac{T}{2}} \right] = e^{-j\pi f \frac{T}{2}} \cdot 2j \sin\left(\pi f \frac{T}{2}\right)$$

CDS: effective noise



Noise spectral density:

$$V_{tc}(f) = V_n(f)H(f)$$

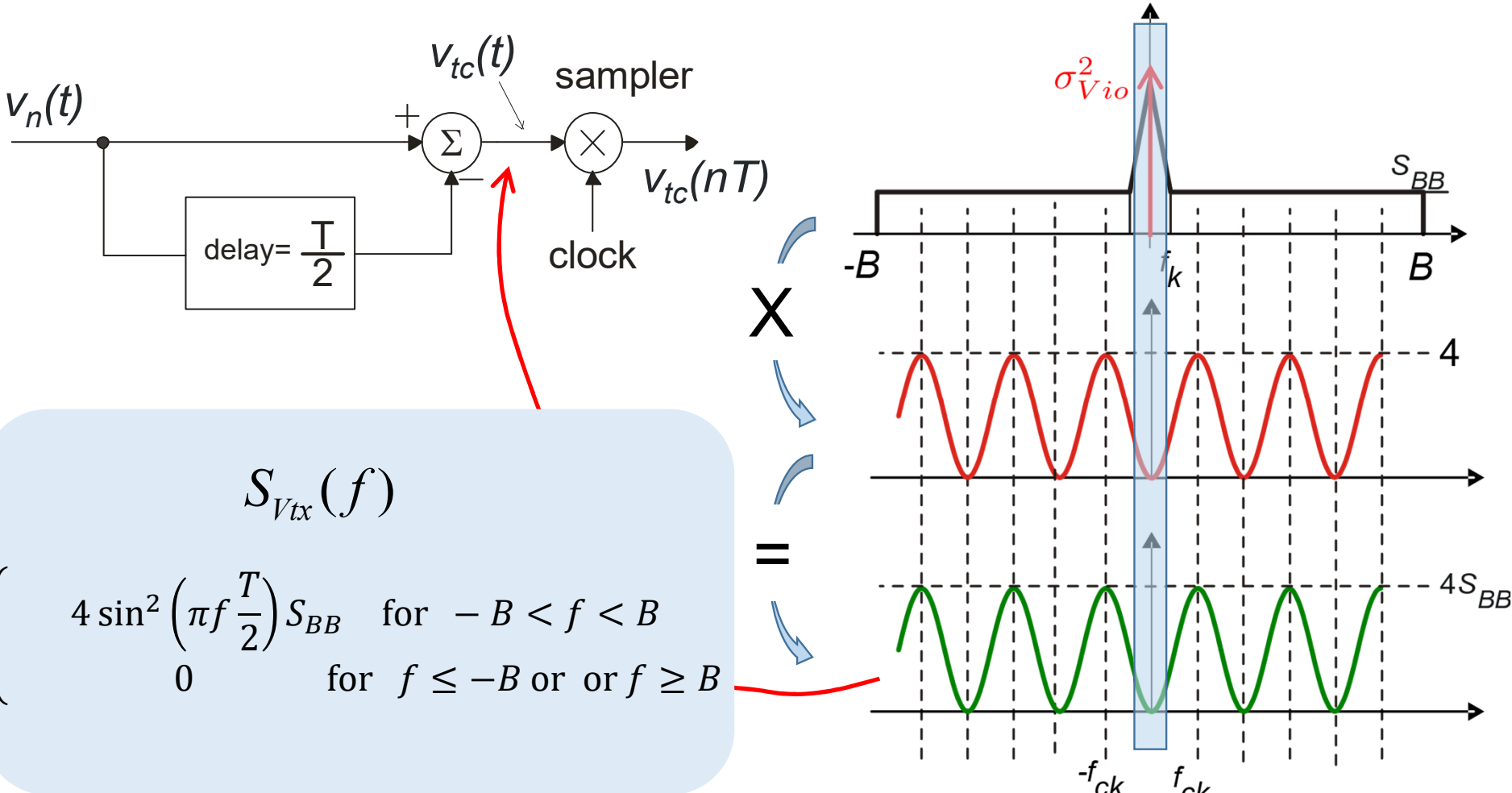
$$H(f) = e^{-j\pi f \frac{T}{2}} \cdot 2j \sin\left(\pi f \frac{T}{2}\right)$$

$$S_{V_{tx}}(f) = S_{V_n}(f)|H(f)|^2$$

$$|H(f)|^2 = 4 \sin^2\left(\pi f \frac{T}{2}\right)$$

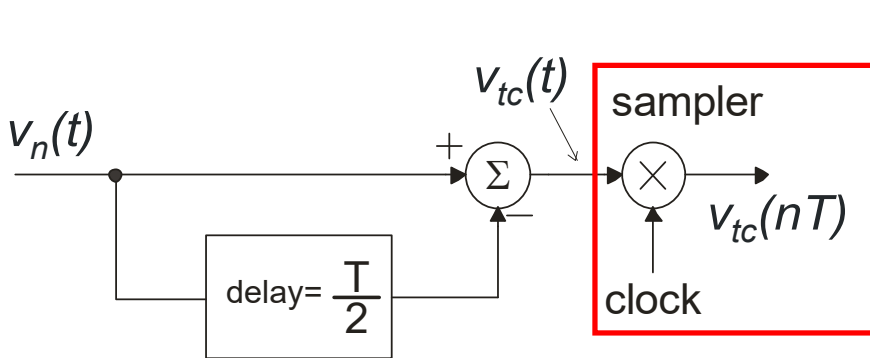
$$T = \frac{1}{f_{ck}} \Rightarrow |H(f)|^2 = 4 \sin^2\left(\frac{\pi}{2} \frac{f}{f_{ck}}\right)$$

CDS: effective noise



$$S_{V_{tx}}(f) = \begin{cases} 4 \sin^2\left(\pi f \frac{T}{2}\right) S_{BB} & \text{for } -B < f < B \\ 0 & \text{for } f \leq -B \text{ or } f \geq B \end{cases}$$

Effect of sampler

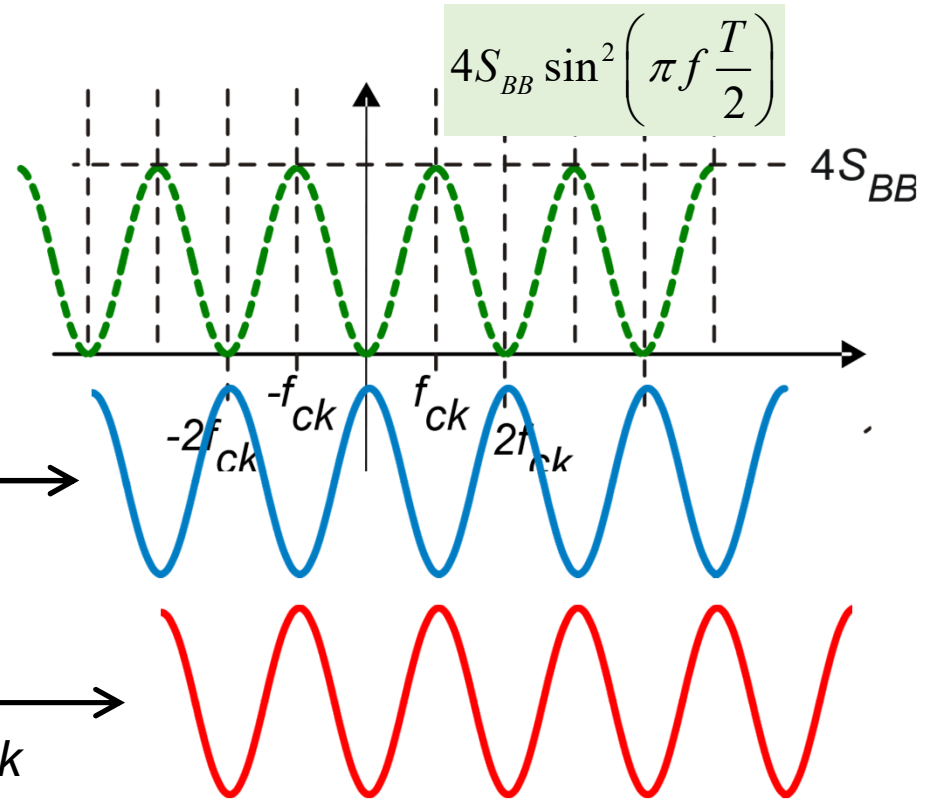


$$\sin^2 \left[\pi (f - f_{ck}) \frac{T}{2} \right] = \sin^2 \left(\pi f \frac{T}{2} - \frac{\pi}{2} \right) = \cos^2 \left(\pi f \frac{T}{2} \right)$$

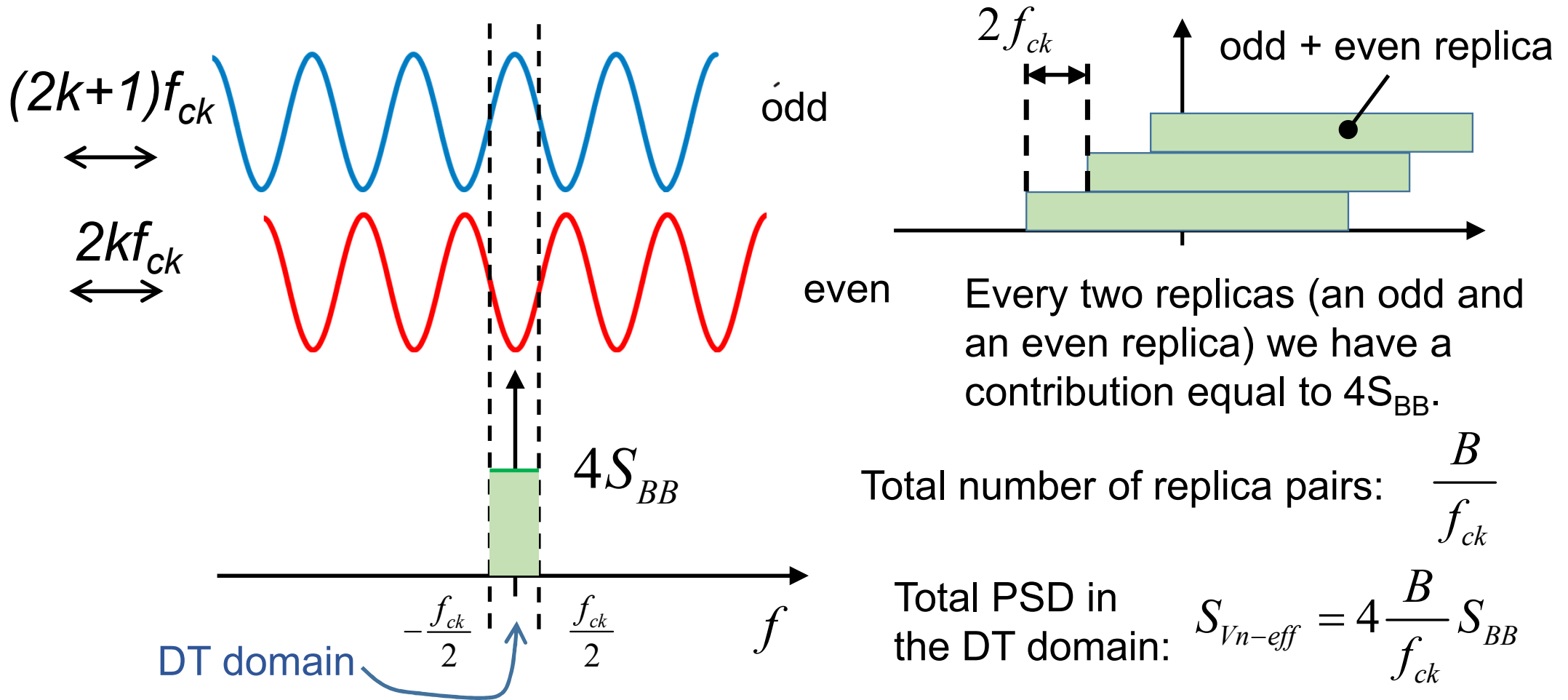
+

$$\sin^2 \left[\pi (f - 2f_{ck}) \frac{T}{2} \right] = \sin^2 \left(\pi f \frac{T}{2} - \pi \right) = \sin^2 \left(\pi f \frac{T}{2} \right)$$

= 1

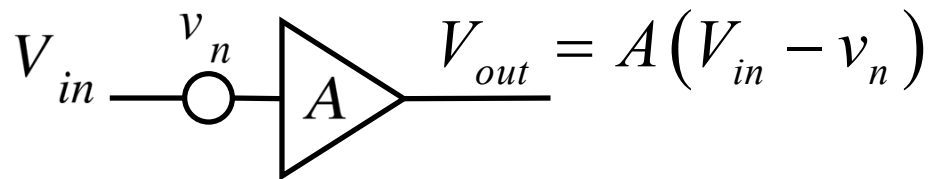


CDS: Residual noise in the DT frequency domain



Chopper modulation: basic principle

Ex.: $A=10$, $v_{io}=10$ mV



$V_{in} = 100$ mV \rightarrow $V_{out} = 900$ mV

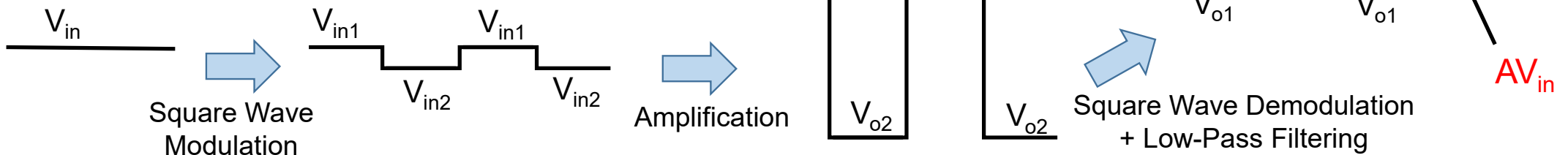
correct value (without offset): 1V

$V_{in1} = V_{in} = 100$ mV $\rightarrow V_{o1} = 900$ mV

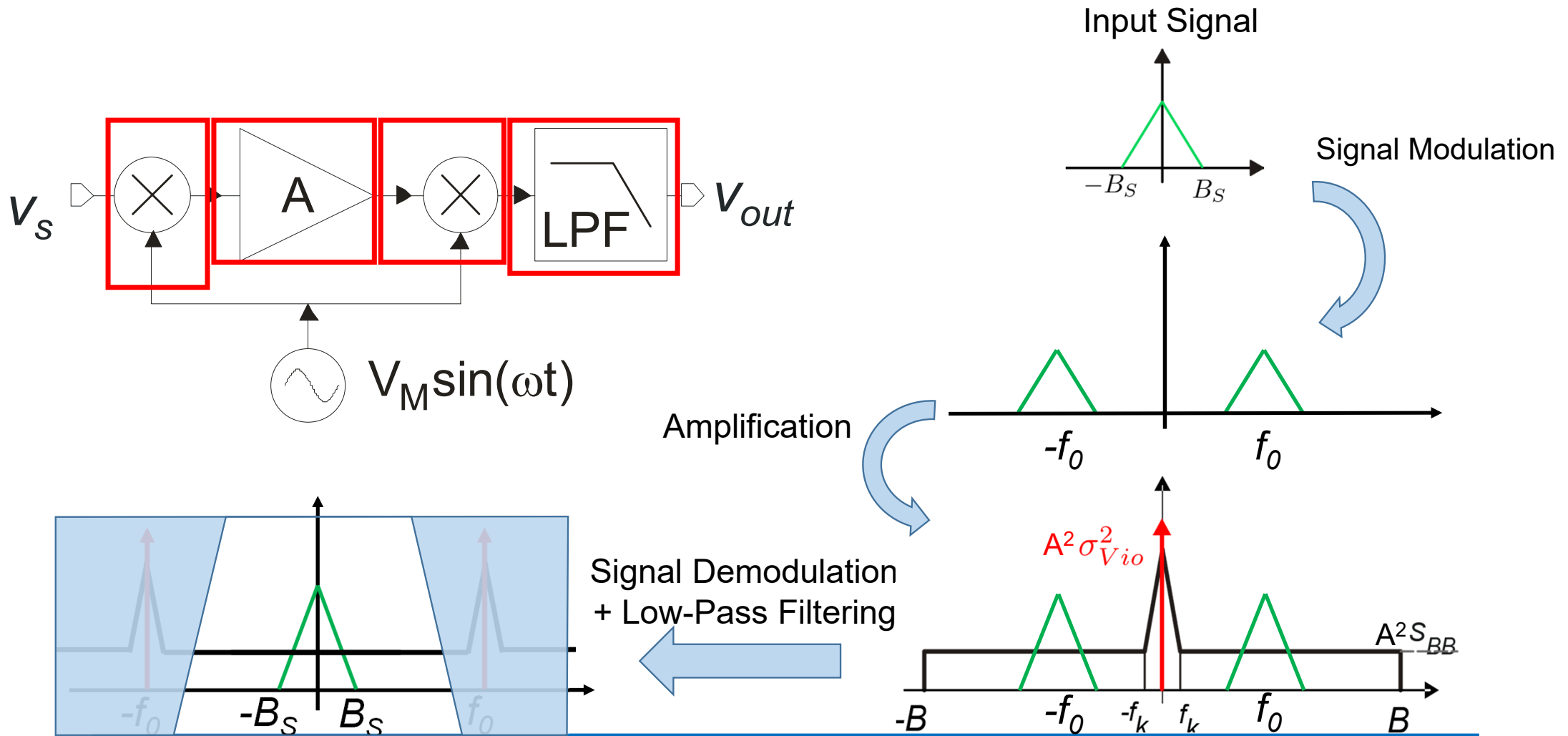
$V_{in2} = -V_{in} = -100$ mV $\rightarrow V_{o2} = -1.1$ V

Average of V_{o1} and $-V_{o2}$

$\frac{V_{o1} - V_{o2}}{2} = 1$ V correct value

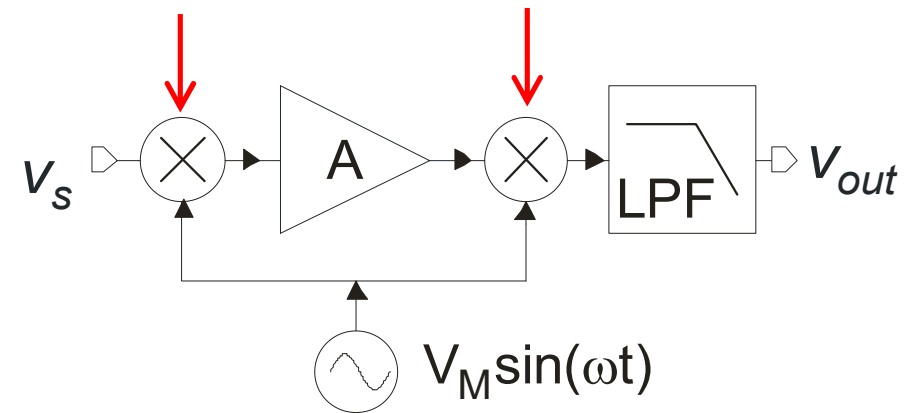


Sinusoidal modulation

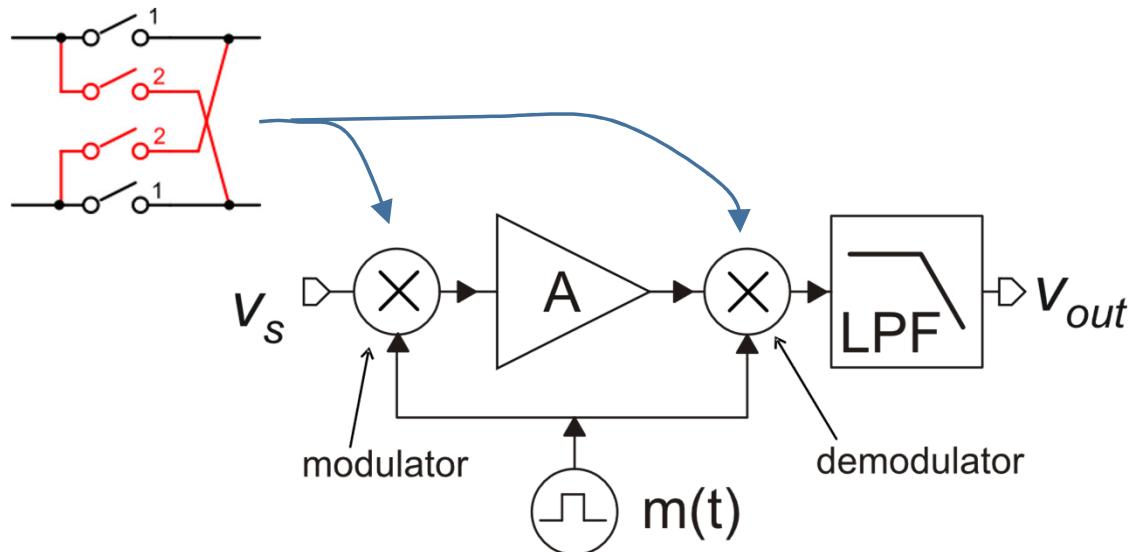


Problems of the sinusoidal modulation

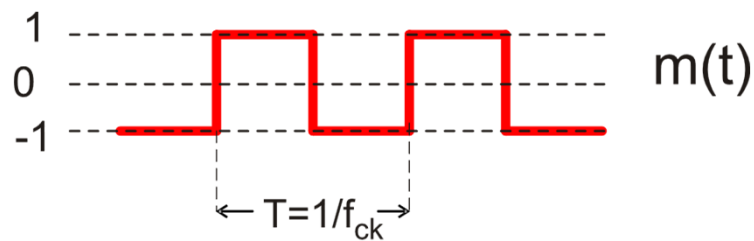
- Sinusoidal modulation requires a real analog multiplier (i.e a Gilbert Cell), that is marked by a very large equivalent input offset and noise.
- Generation of sinusoidal waveform with precise magnitude is not simple using only on-chip components



Chopper modulation



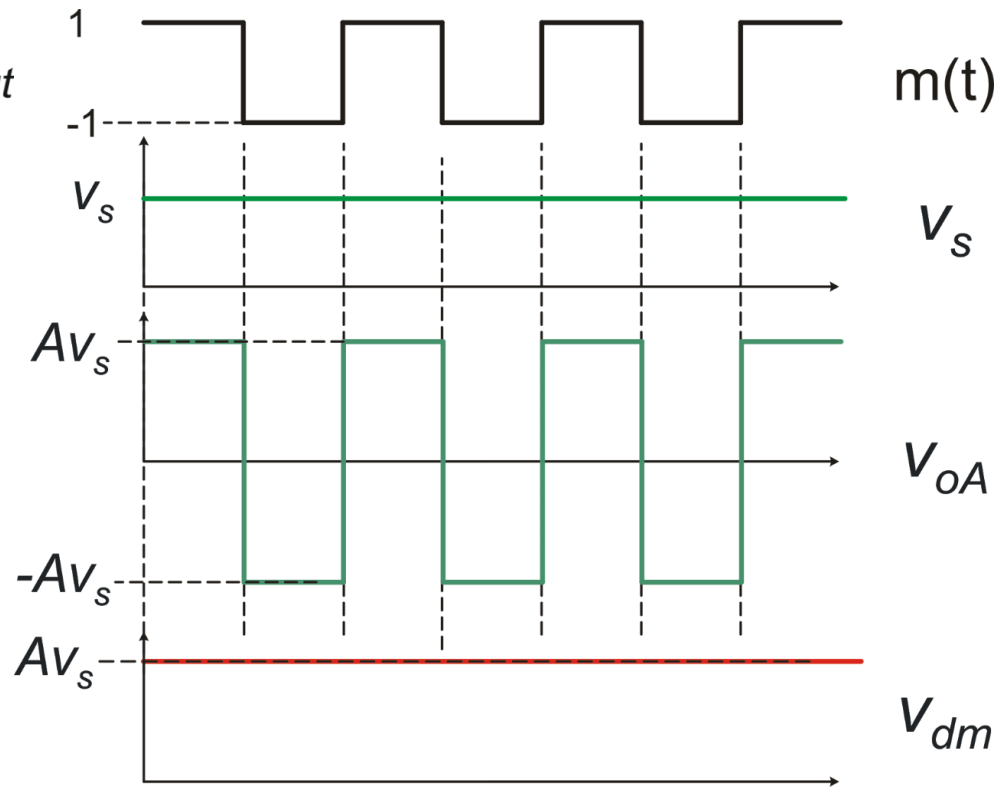
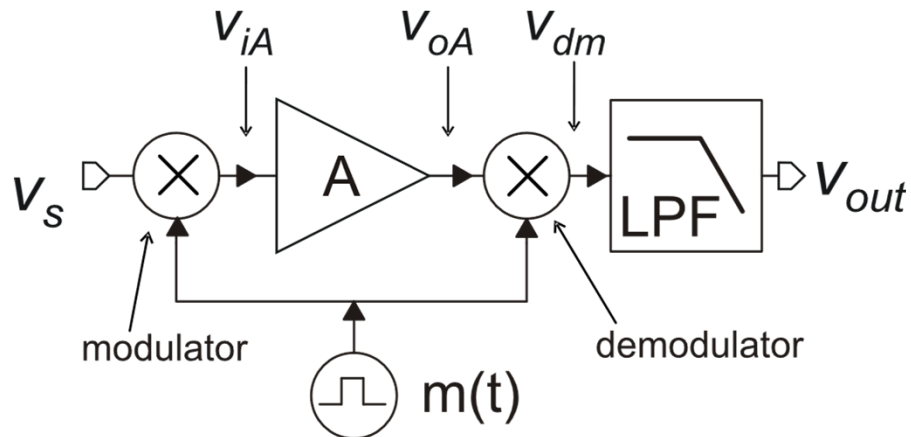
Square Wave Modulation



Modulator and demodulator can be implemented by switch matrices: virtually free from $1/f$ noise and offset

Dimensionless square waveform with unity magnitude and strictly 50 % duty-cycle

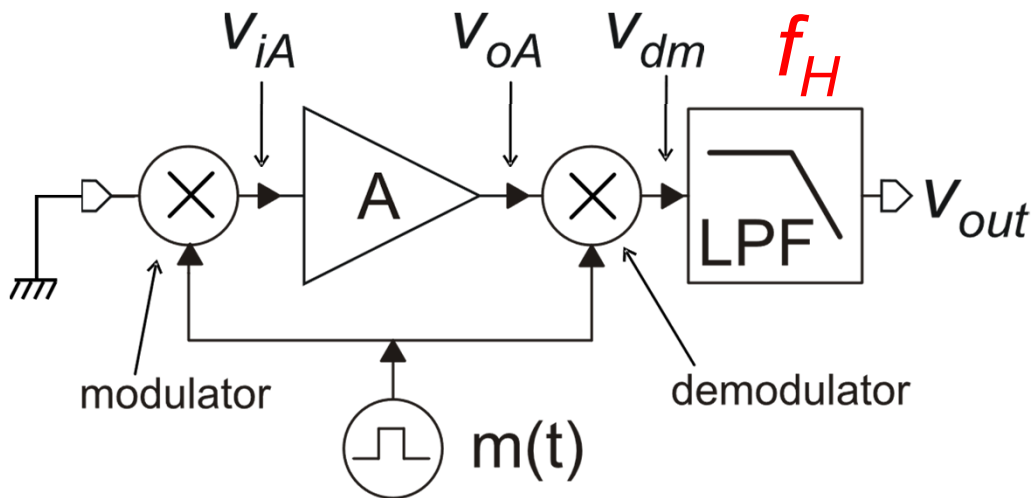
A simplified analysis in the time domain



Hypothesis:

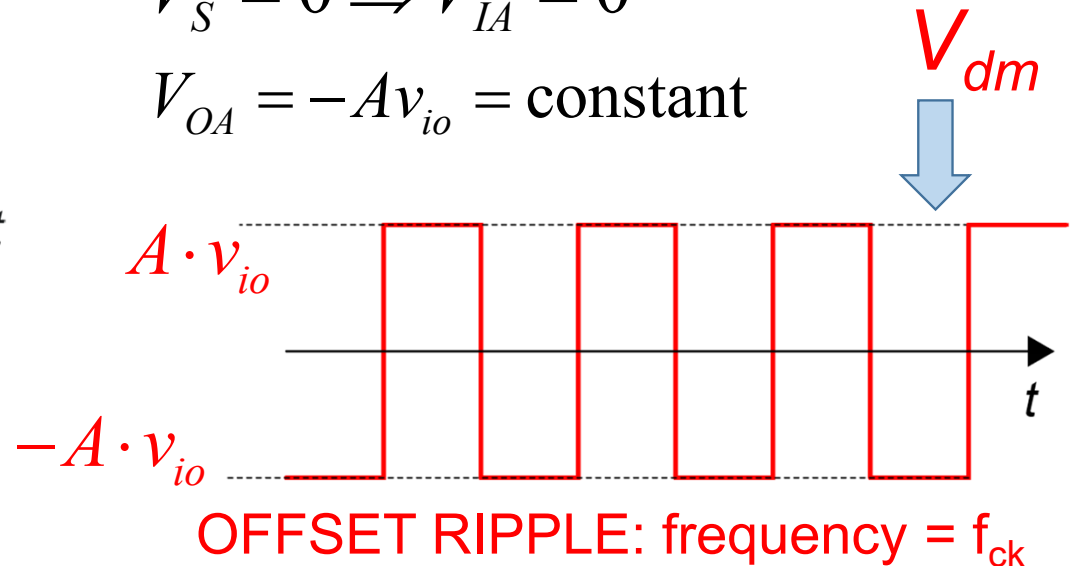
- The amplifier has infinite bandwidth and zero delay
- The input signal (V_s) is constant
- **Zero noise and offset**

Simplified time-domain analysis: how the offset is processed



$$V_S = 0 \Rightarrow V_{IA} = 0$$

$$V_{OA} = -Av_{io} = \text{constant}$$



The offset ripple is completely deleted if:

- LPF: $f_H < f_{ck}$
- $\langle m(t) \rangle = 0$ (requires duty-cycle=50%)

Chopper modulation: analysis in the frequency domain

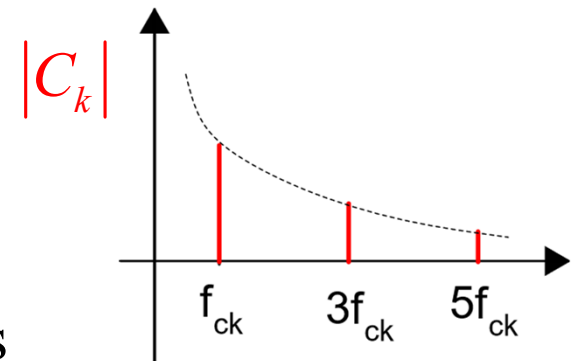
The simplified analysis in the time domain is useful to gain an intuitive understanding of the CHS principle of operation but can give quantitative prediction for non-constant signal and noise components

In order to model the effect on non-constant signal and noise components it is necessary to perform the analysis in the frequency domain.

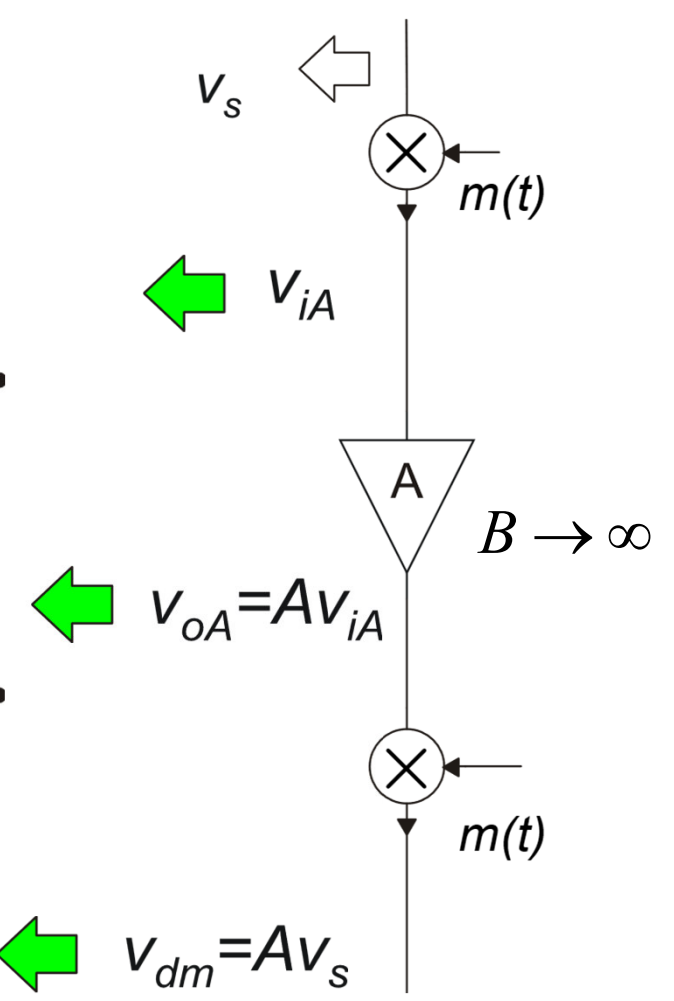
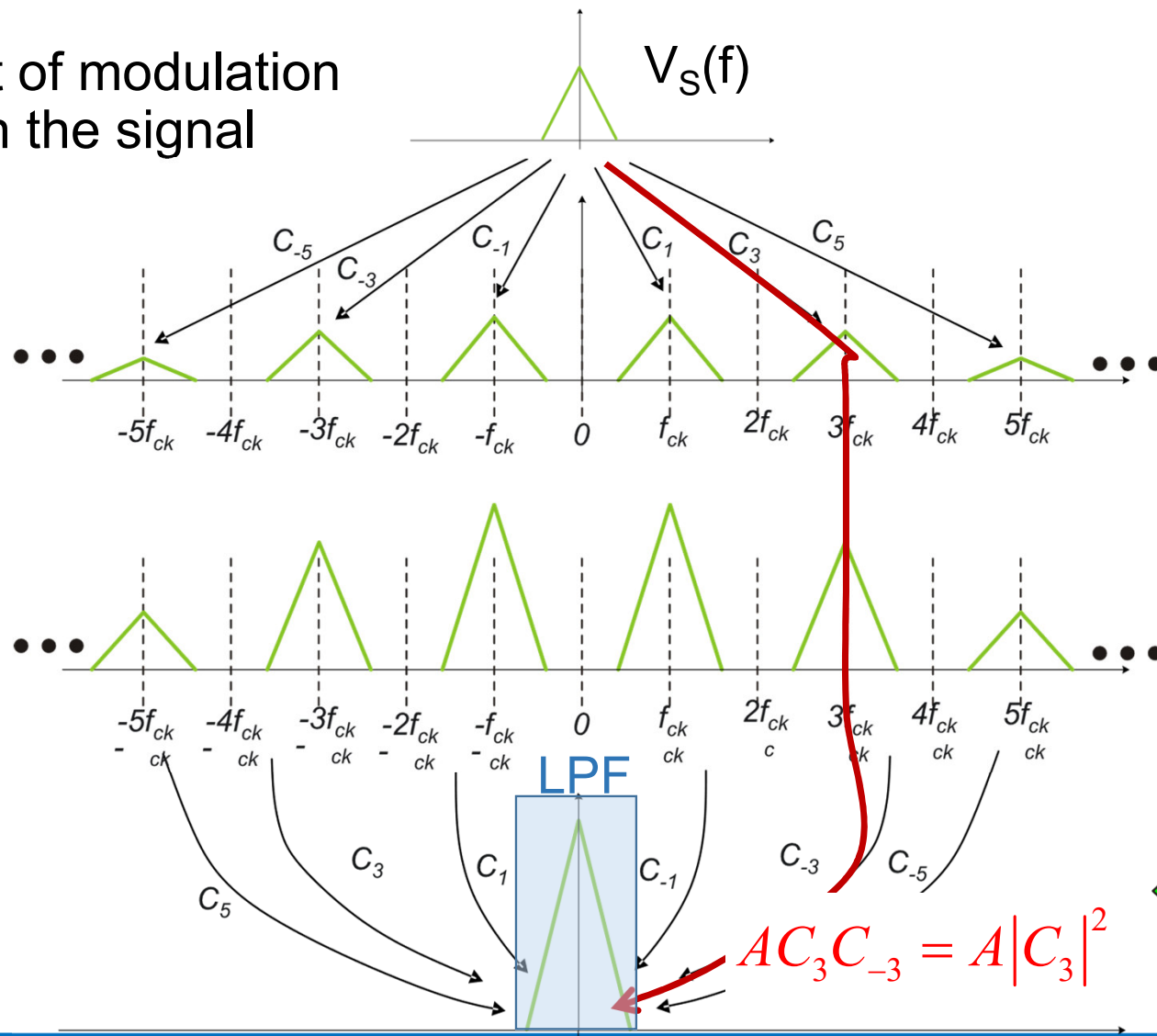
Fourier series of the $m(t)$ waveform

$$m(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k f_{ck} t}$$

with
$$\begin{cases} |C_k| = \frac{2}{\pi k} & \text{for odd } k \text{ values} \\ 0 & \text{for even } k \text{ values} \end{cases}$$

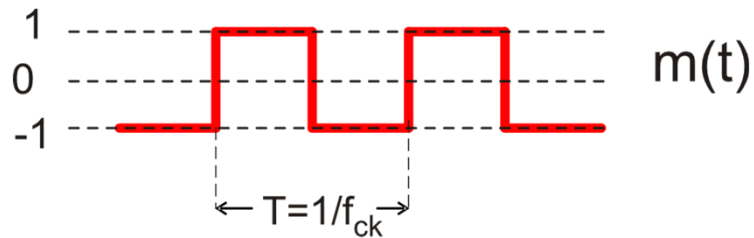


Effect of modulation on the signal



Effect of modulation on the signal

$$V_{out}(f) = \sum_{k=-\infty}^{\infty} A|C_k|^2 V_S(f) = V_S(f) A \underbrace{\left[\sum_{k=-\infty}^{\infty} |C_k|^2 \right]}$$



Power of the modulating waveform m(t)

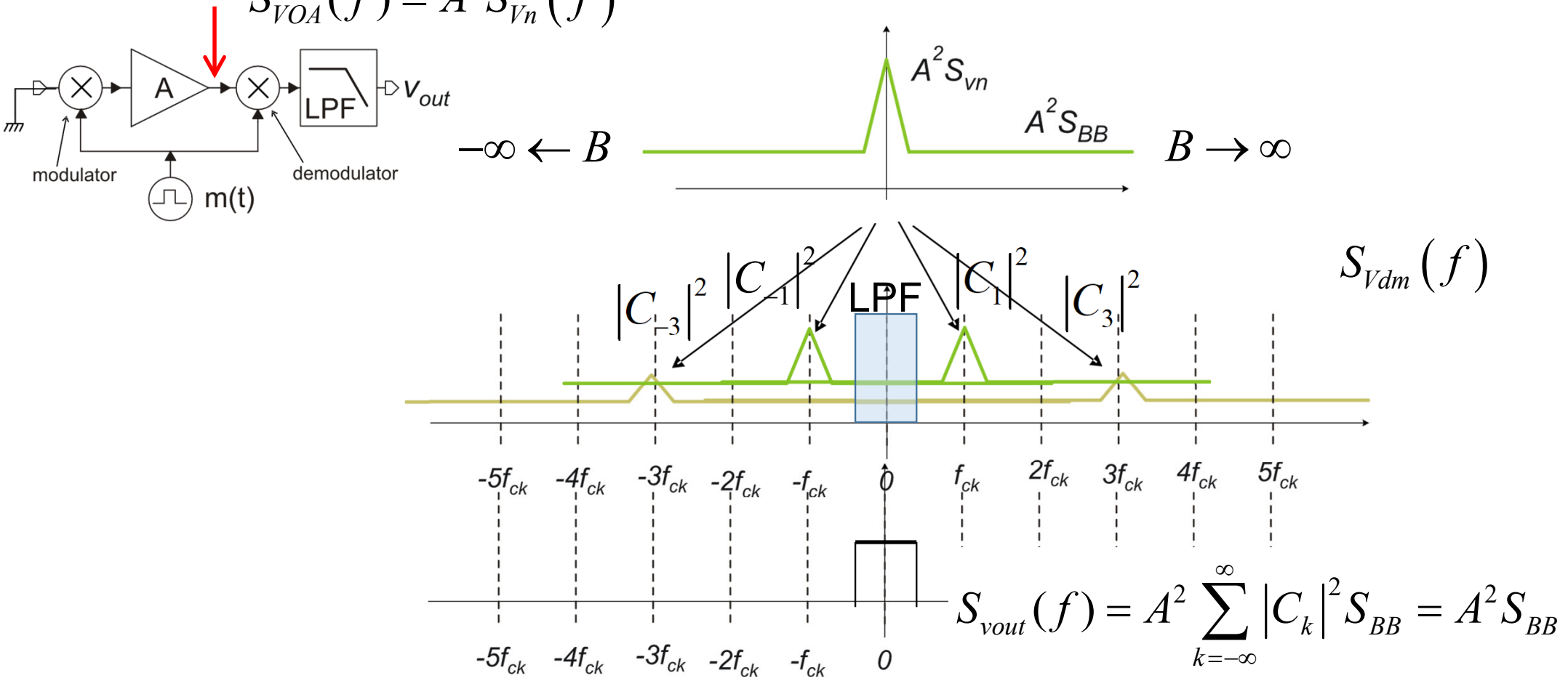
$$\text{Power} = \langle m^2(t) \rangle = 1$$

Application of chopper modulation in the case of INFINITE bandwidth and null delay does not alter the function and gain of the original amplifier

$$V_{out}(f) = AV_S(f)$$

Effect of CHS on the noise spectrum

$$S_{VOA}(f) = A^2 S_{Vn}(f)$$



Effective noise PSD referred to the input

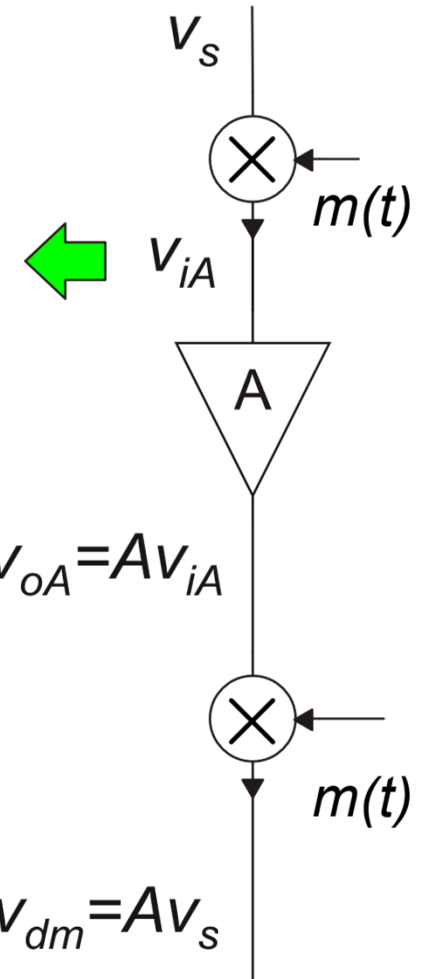
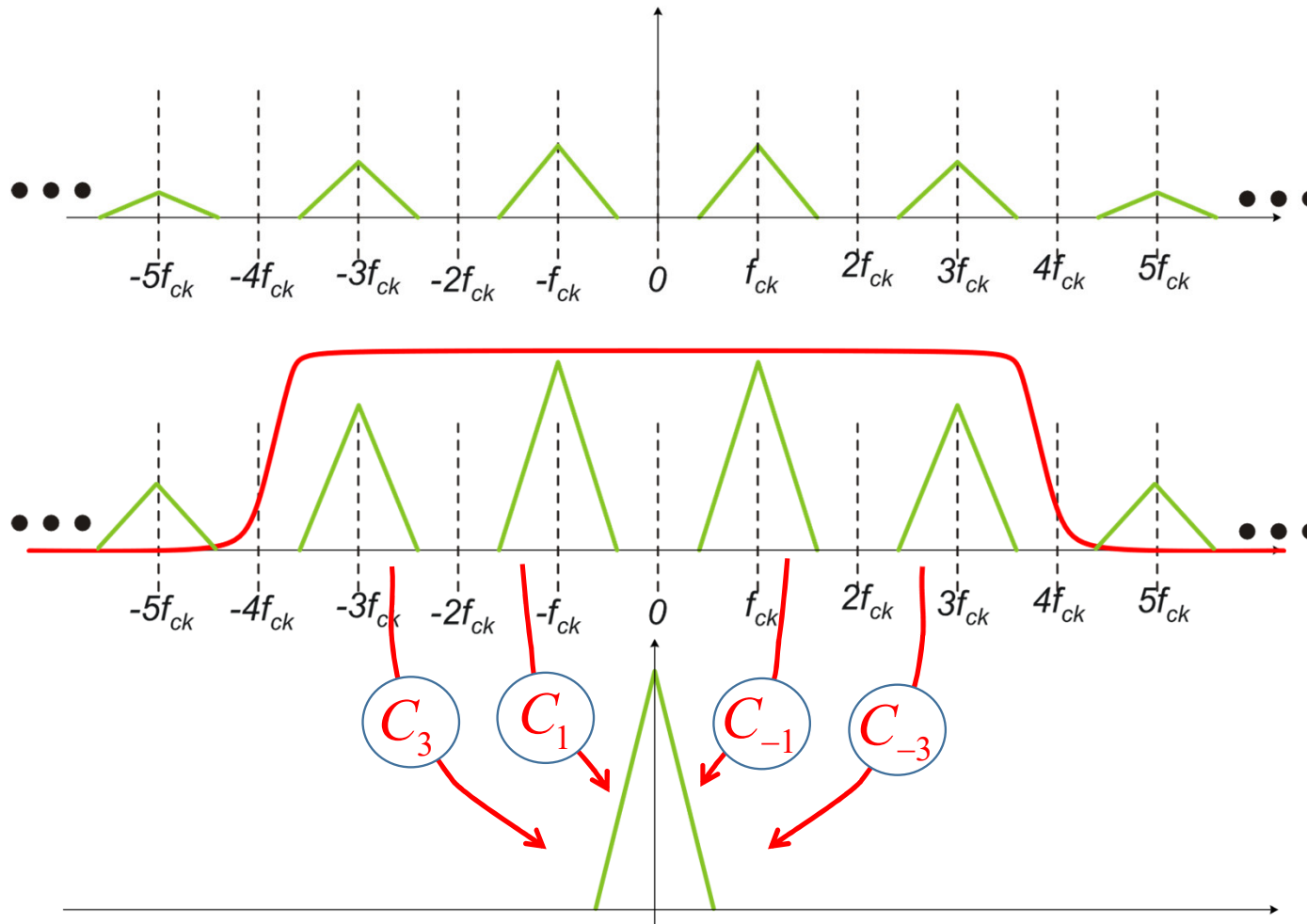
$$v_{n-eff} = \frac{v_{n-out}}{A}$$

$$S_{vn-eff}(f) = \frac{S_{vout}(f)}{A^2} = \frac{A^2 S_{BB}}{A^2} = S_{BB}$$

Application of chopper modulation in the case of INFINITE bandwidth result in cancellation of the flicker noise and in a residual noise in the signal bandwidth just equal to the **broadband noise PSD** of the original amplifier

No Noise foldover occurs!

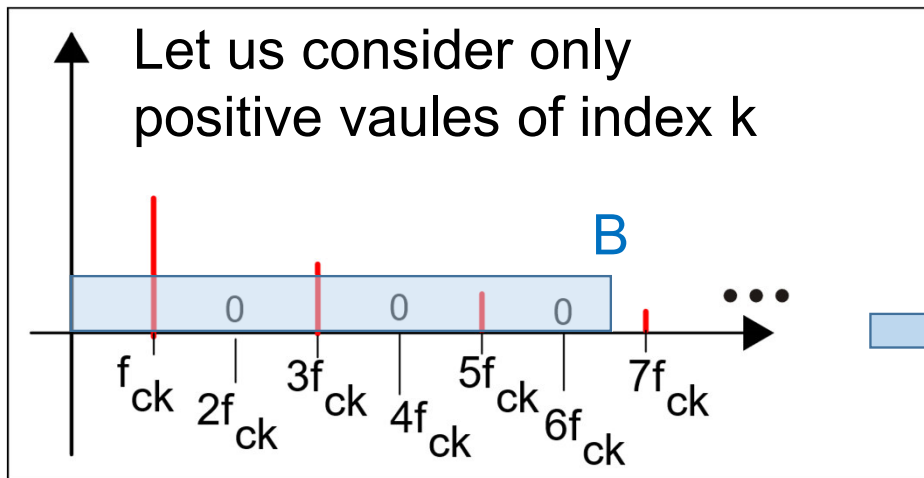
Effect of finite amplifier bandwidth



Effect of finite amplifier bandwidth

$$V_{out}(f) = A \left[\sum_{k=-N}^N |C_k|^2 \right] V_s(f)$$

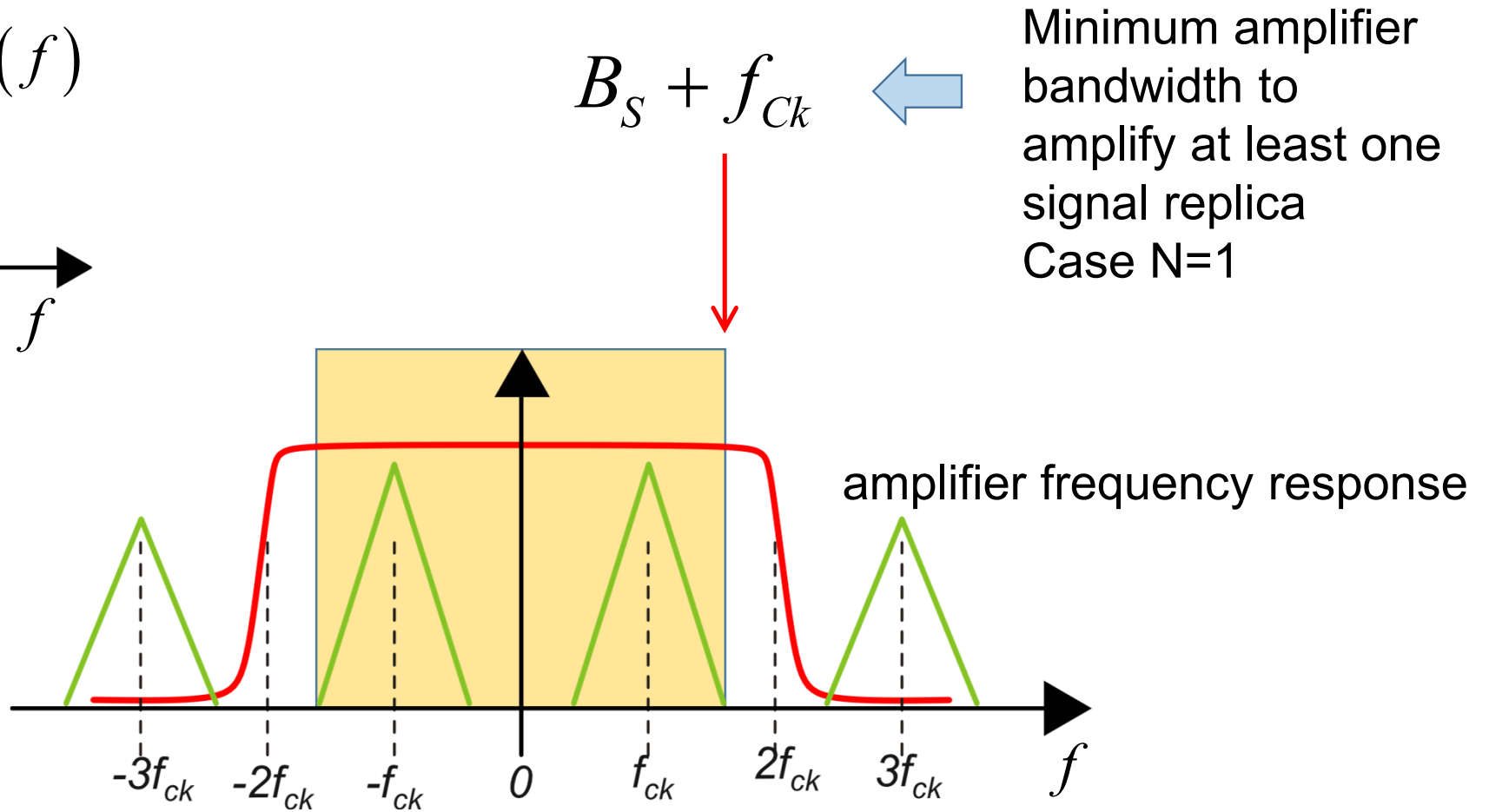
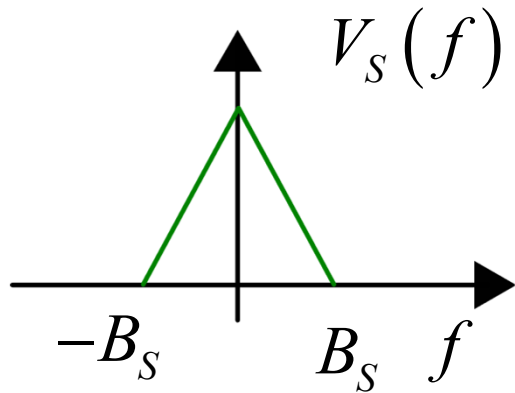
Summation is now limited to a finite number of terms



$$N = \left\lfloor \frac{B}{f_{ck}} \right\rfloor$$

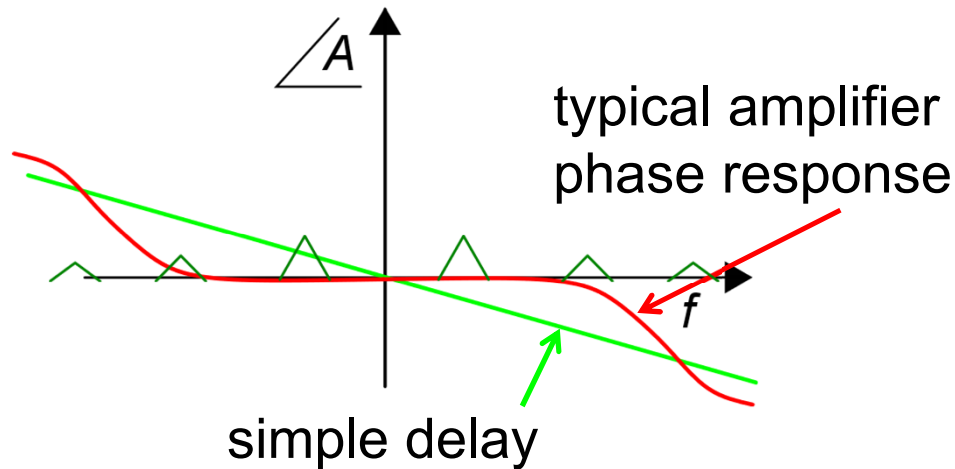
$$V_{out}(f) = A\alpha \cdot V_s(f) \quad \left[\sum_{k=-N}^N |C_k|^2 \right] = \alpha < 1 \quad A_{eff} = A\alpha < A$$

Minimum amplifier bandwidth



Minimum amplifier bandwidth to amplify at least one signal replica
Case N=1

Effect of phase delays

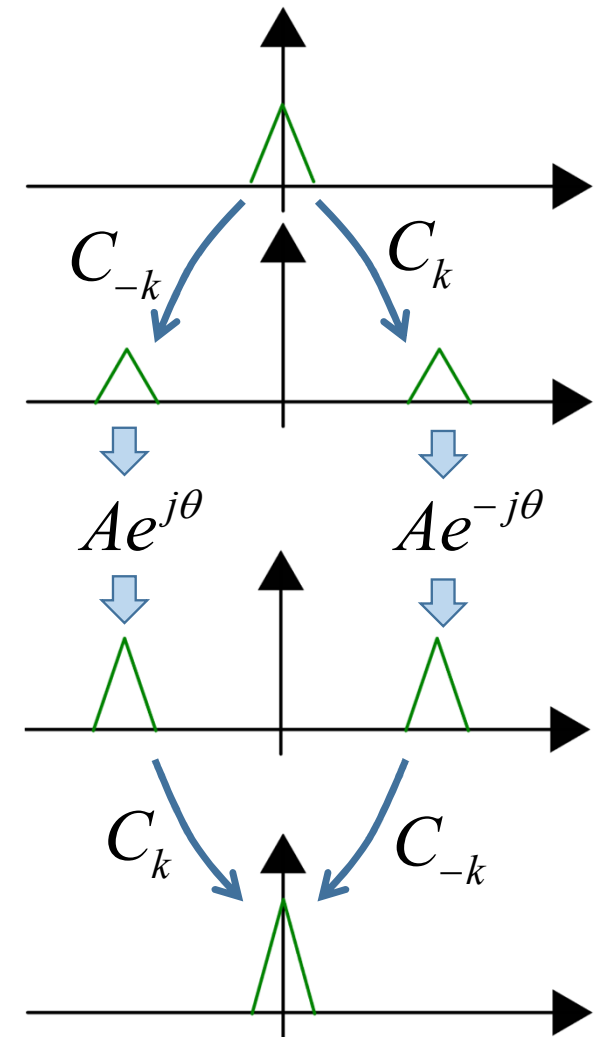


$$V_S(f) A |C_k|^2 (e^{j\theta} + e^{-j\theta}) = V_S(f) A |C_k|^2 \cdot 2 \cdot \cos(\theta)$$

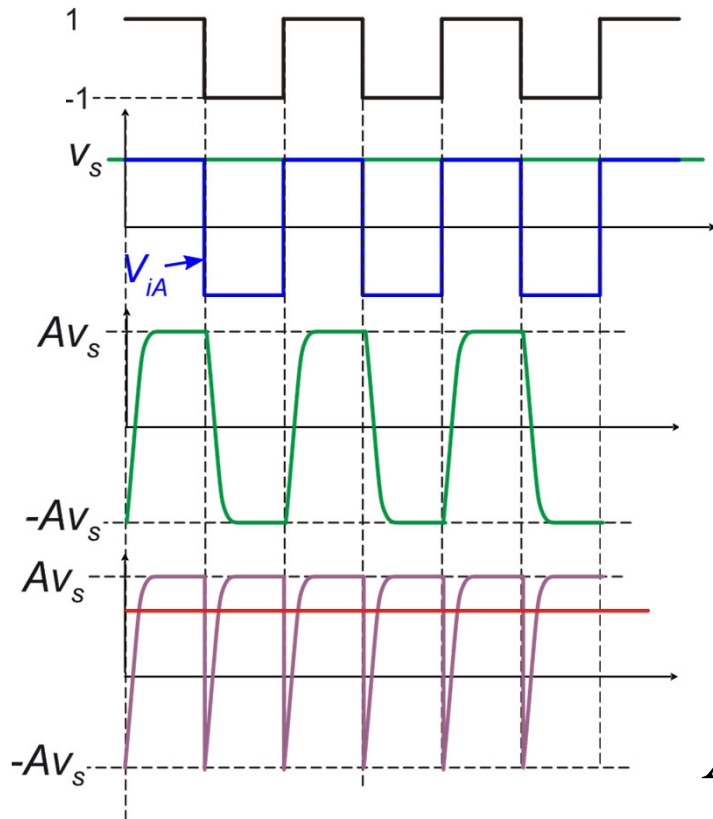
Replicas that undergo a phase shift are attenuated when they are brought back to baseband.

For $\theta=90^\circ$ \Rightarrow total cancellation of the contribution

For $\theta=180^\circ$ \Rightarrow the sign of the contribution is reversed

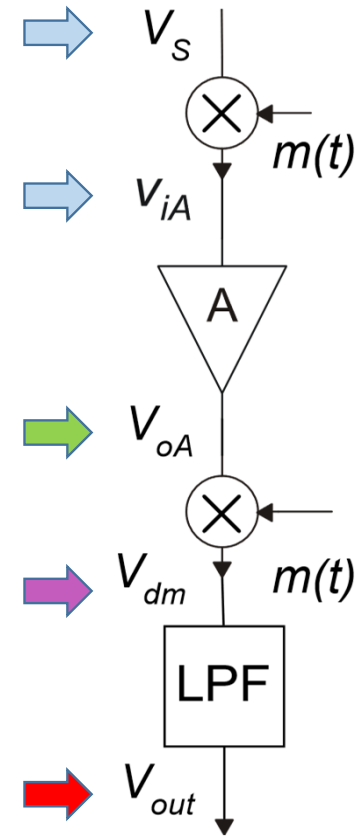


Time domain analysis with finite amplifier bandwidth

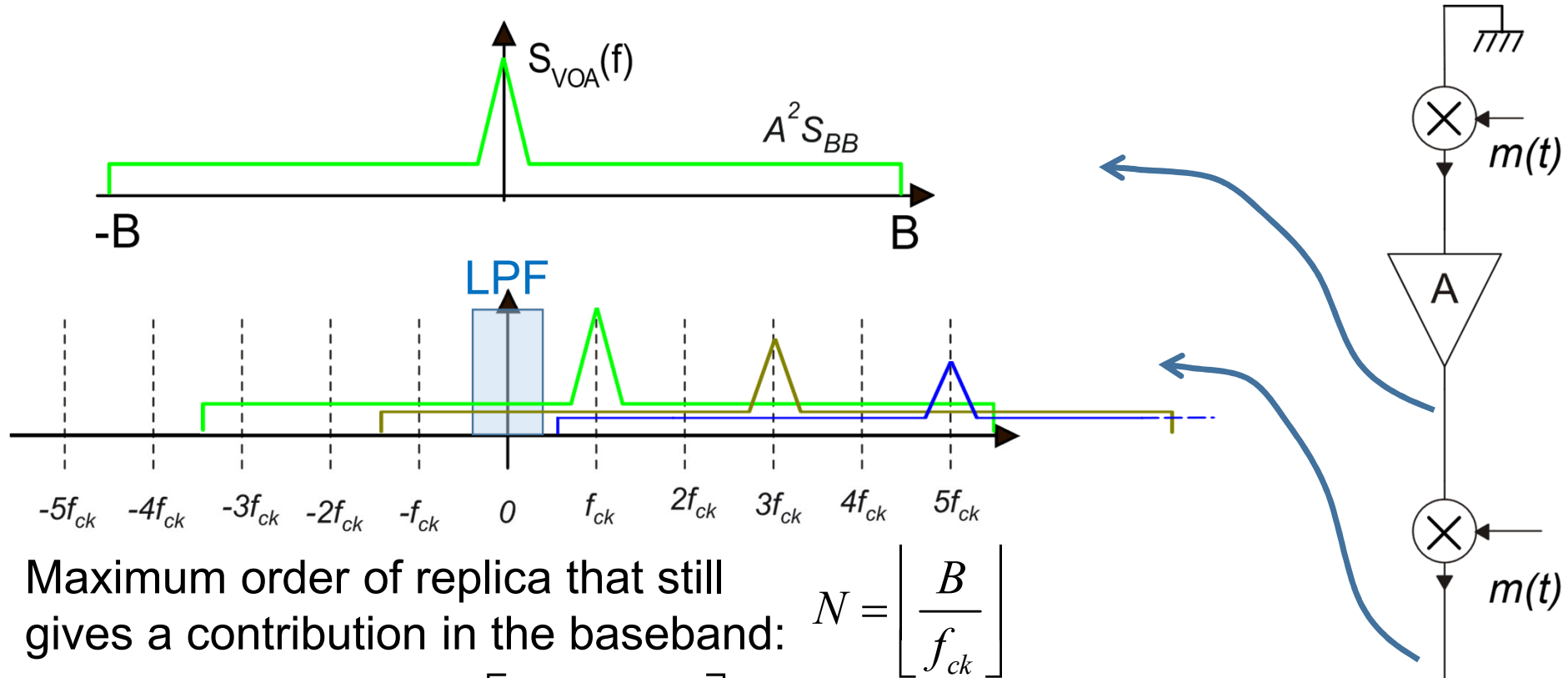


$$V_{out} < AV_s$$

$$A_{eff} = A\alpha < A$$



Effect of the amplifier finite bandwidth on the output noise PSD



$$S_{Vout}(f) = S_{BB} A^2 \left[\sum_{k=-N}^N |C_k|^2 \right] = S_{BB} \alpha A^2$$

Finite bandwidth: effective input referred noise density

Amplifier effective gain: $A_{eff} = A\alpha < A$

Output noise PSD: $S_{Vout}(f) = S_{BB}\alpha A^2$

$$\left[\sum_{k=-N}^N |C_k|^2 \right] = \alpha < 1$$

$$N = \left\lfloor \frac{B}{f_{ck}} \right\rfloor$$

$$S_{vn-eff}(f) = \frac{S_{vout}(f)}{A_{eff}^2} = \frac{\alpha A^2 S_{BB}}{(\alpha A)^2} = \frac{S_{BB}}{\alpha} > S_{BB}$$

N	1	3	5	15
α	0.8106	0.9006	0.9331	0.9747
$1/\alpha$	1.234	1.110	1.072	1.026

AZ, CDS and CHS compared

Residual noise at low frequencies

$$\text{AZ: } \frac{2B}{f_{ck}} S_{BB}$$

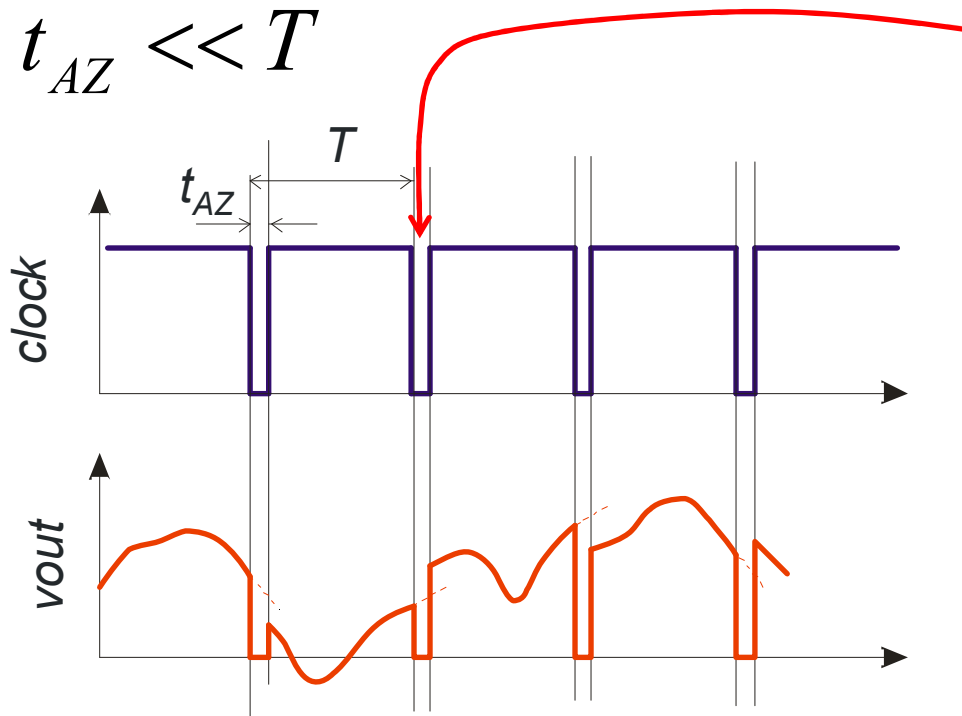
$$\text{CDS: } \frac{4B}{f_{ck}} S_{BB}$$

$$\text{CHS: } \cong S_{BB}$$

The CHS technique gives the lower residual noise in the signal bandwidth, for the same broadband S_{BB} of the original amplifier

The AZ and CDS techniques suffer from noise foldover, which is represented by the ratio B/f_{ck} . The minimum value of this ratio is not the same for the two techniques

B/f_{ck} ratio requirements for AZ



In the AZ phase the amplifier passes from full output signal to a small value ($-Av_n$). At the end of the AZ phase, the residual error should be small, otherwise part of the output signal is sampled together with the noise/offset

$$t_{set} \leq t_{AZ} \ll T$$

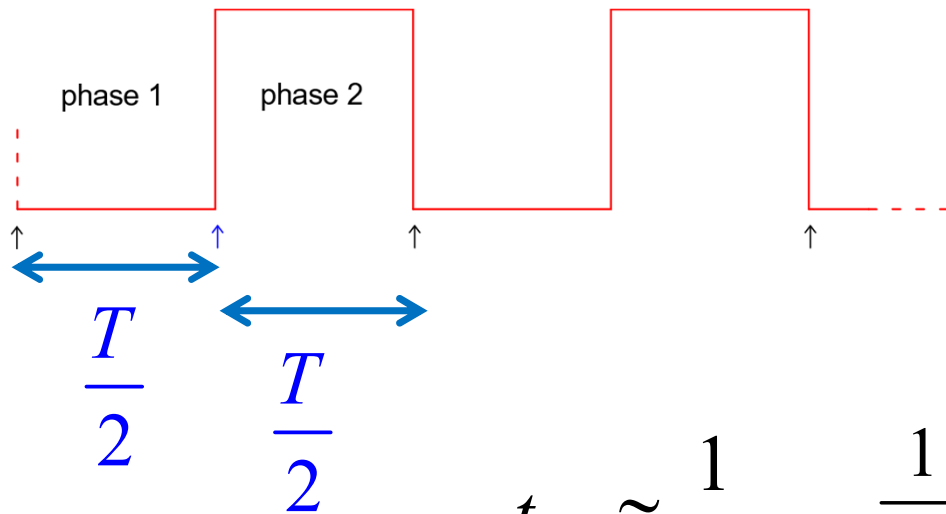
$$\frac{1}{t_{set}} \gg \frac{1}{T} = f_{ck}$$

$$t_{set} \approx \frac{1}{B}$$

$$B \gg f_{ck}$$

E.g.: $\frac{B}{f_{ck}} \approx 100 - 1000$

B/f_{ck} ratio requirements for CDS



In the CDS the amplifier must settle to the final value in a period equal to half the clock cycle ($T/2$).

$$t_{set} < \frac{T}{2}$$

$$t_{set} \cong \frac{1}{B}$$

$$\frac{1}{t_{set}} > \frac{2}{T} = 2f_{ck}$$

$$B > 2f_{ck} \quad \frac{B}{f_{ck}} > 2$$

In practice, the requirement for small residual error (high accuracy) and the occurrence of the slew rate phenomenon impose larger value for B/f_{ck} . Generally:

$$\min\left(\frac{B}{f_{ck}}\right) \cong 3$$

B: amplifier bandwidth

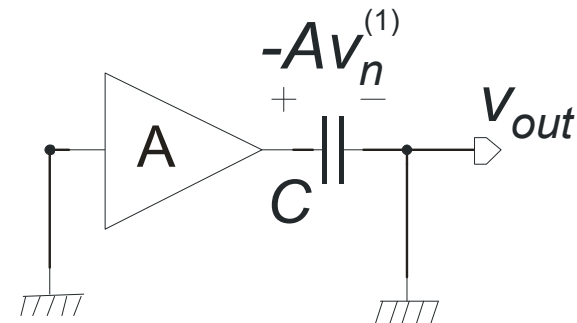
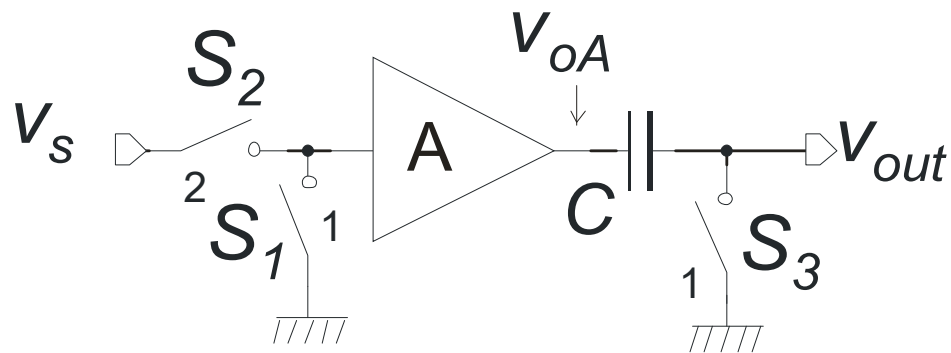
S_{BB} : amplifier broadband noise

AZ, CDS and CHS compared

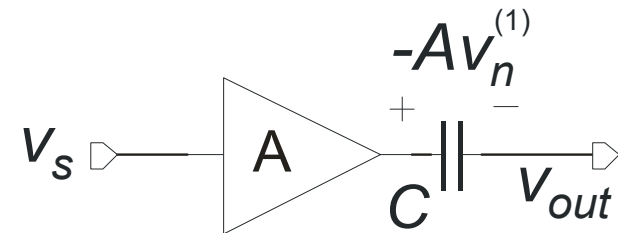
Method	Signal bandwidth (B_S)	Residual baseband noise ($f < f_{ck}/2$)	f_{ck} constraints	Particular characteristics
AZ	$B_S = B$	$\frac{2B}{f_{ck}} S_{BB}$	$f_{ck} \ll B$	Maintains the original time continuous frequency response of the amplifier.
CDS	$B_S < f_{ck}/2$	$\frac{4B}{f_{ck}} S_{BB}$	$f_{ck} < B/3$	Fully sampled data system.
CHS	$B_S < f_{ck}$	S_{BB}	$f_{ck} + B_S < B$ $B_S < f_{CK}$	Requires fully-differential architecture and the presence of an effective low pass filter.

Simple example of circuitual implementations: Open-Loop Offset Compensation

- AZ Amplifier**



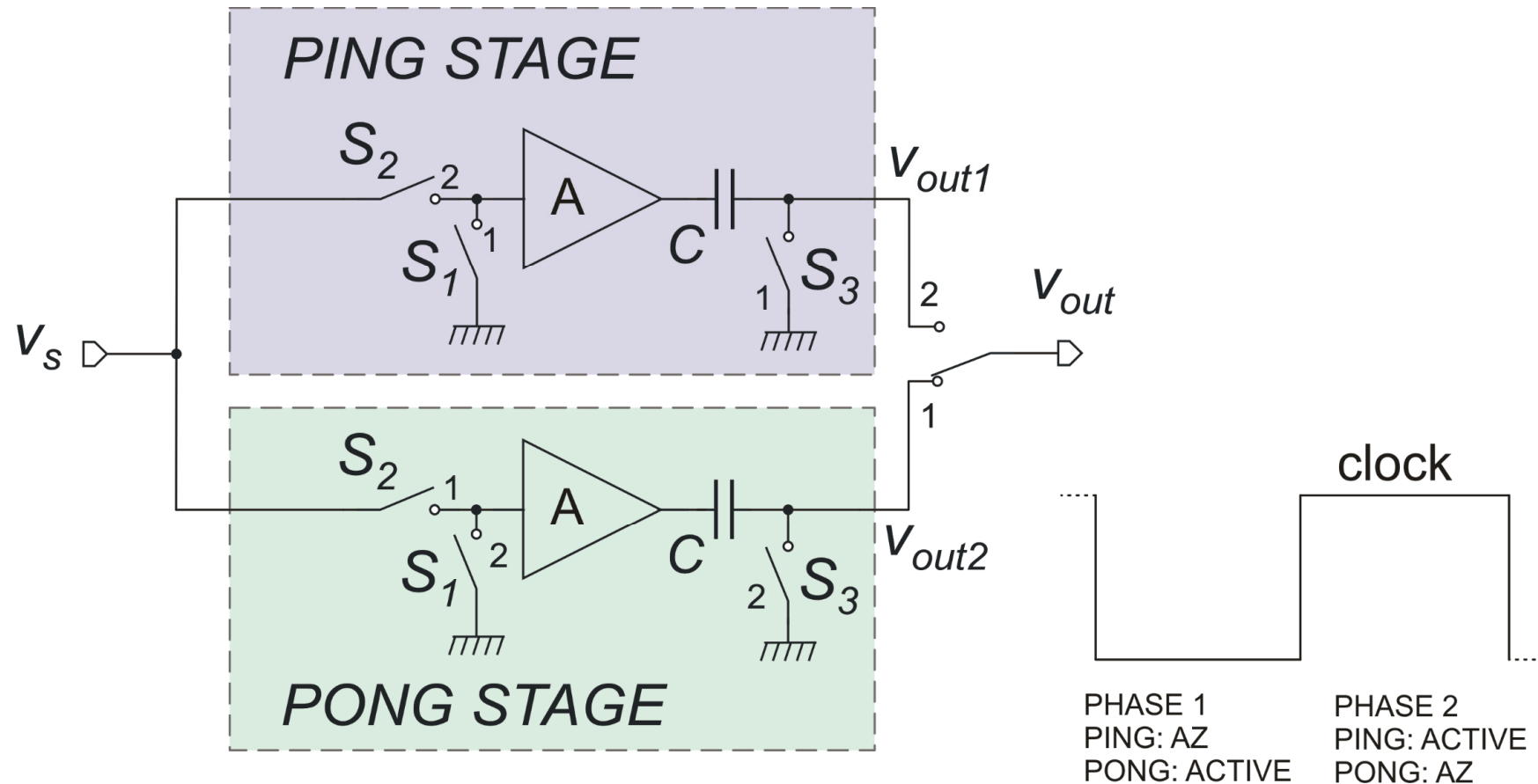
Phase 1



Phase 2

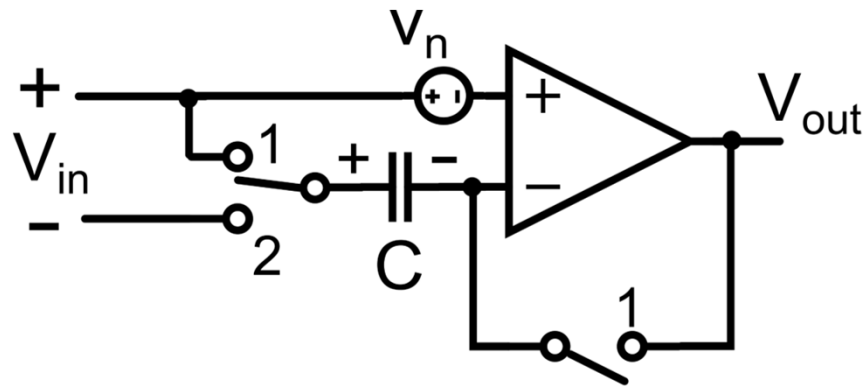
$$V_{out}(t) = A \left[V_s(t) - \left(v_n(t) - v_n^{(1)} \right) \right]$$

The ping-pong approach to reduce the B/f_{ck} ratio in AZ systems



Simple example of circuitual implementations: Closed-Loop Offset Compensation

- AZ Amplifier**



Phase 1:

$$V_C(t) = v_n(t)$$

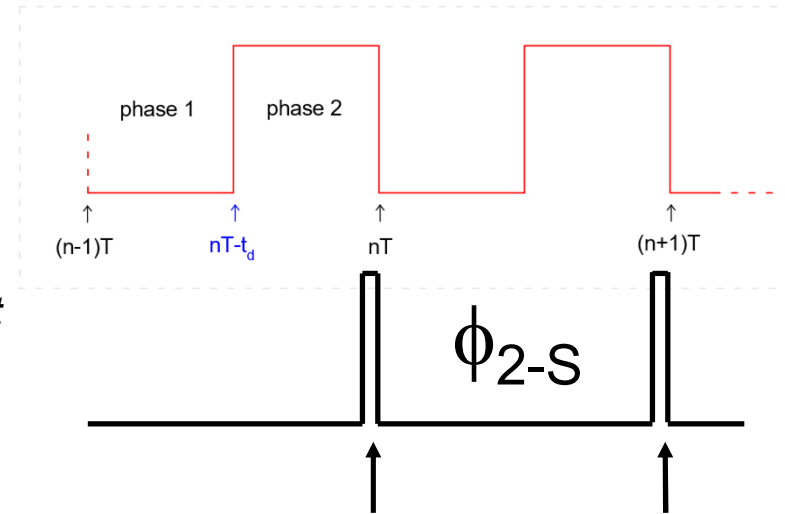
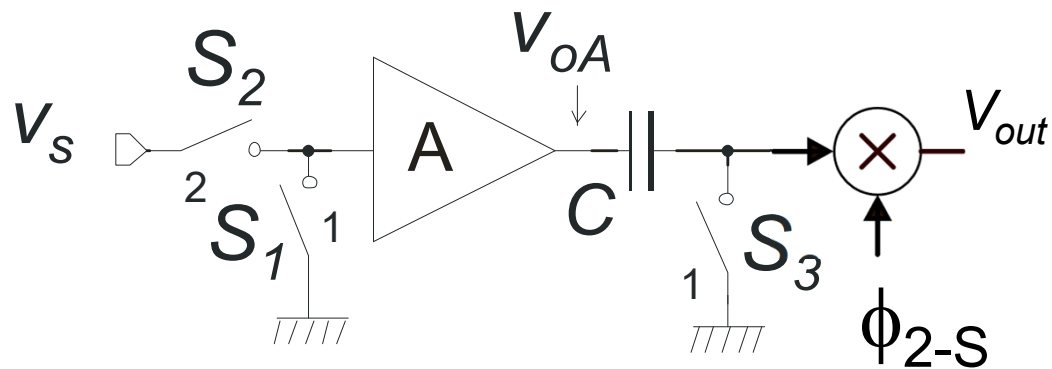
Phase 2:

$$V_{out}(t) = A \left[V_S(t) - v_n(t) - V_C^{(1)} \right]$$

$$V_{out}(t) = A \left[V_S(t) - \left(v_n(t) - v_n^{(1)} \right) \right]$$

Simple example of circuitual implementations: Open-Loop Offset Compensation

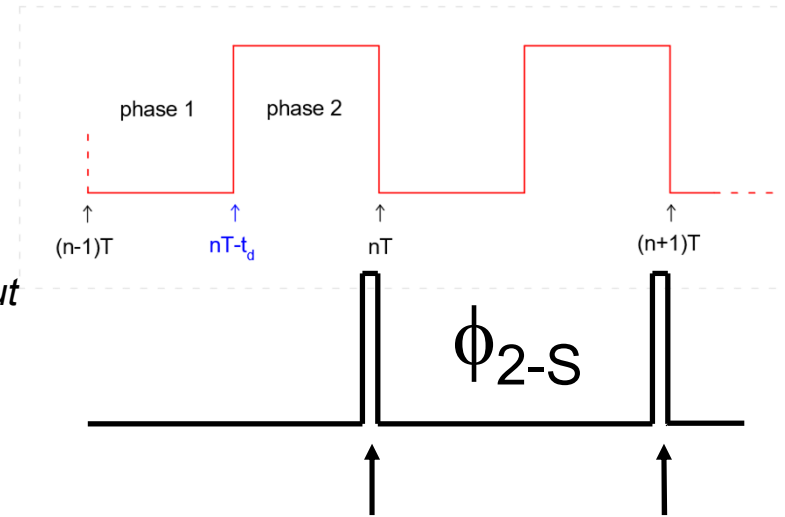
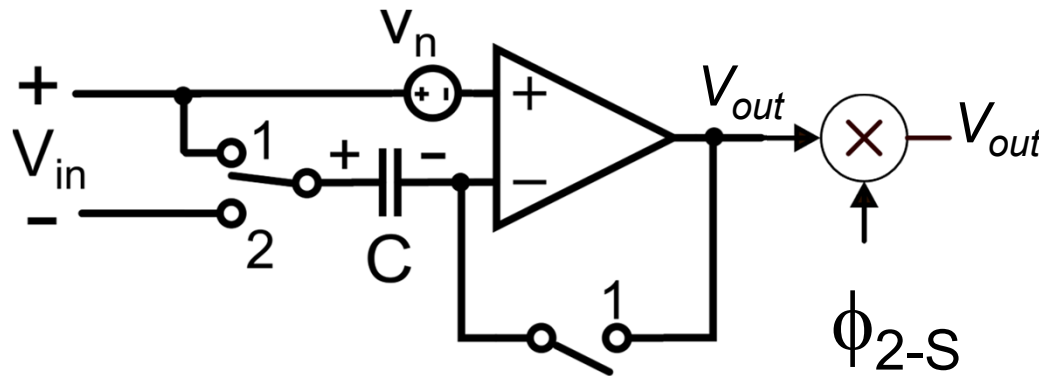
- CDS Amplifier**



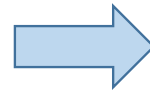
$$V_{out}(t) = A \left[V_s(t) - \left(v_n(t) - v_n^{(1)} \right) \right] \quad \longrightarrow \quad V_{out}^{(2)} = A \left[V_S^{(2)} - \left(v_n^{(2)} - v_n^{(1)} \right) \right]$$

Simple example of circuitual implementations: Closed-Loop Offset Compensation

- CDS Amplifier**



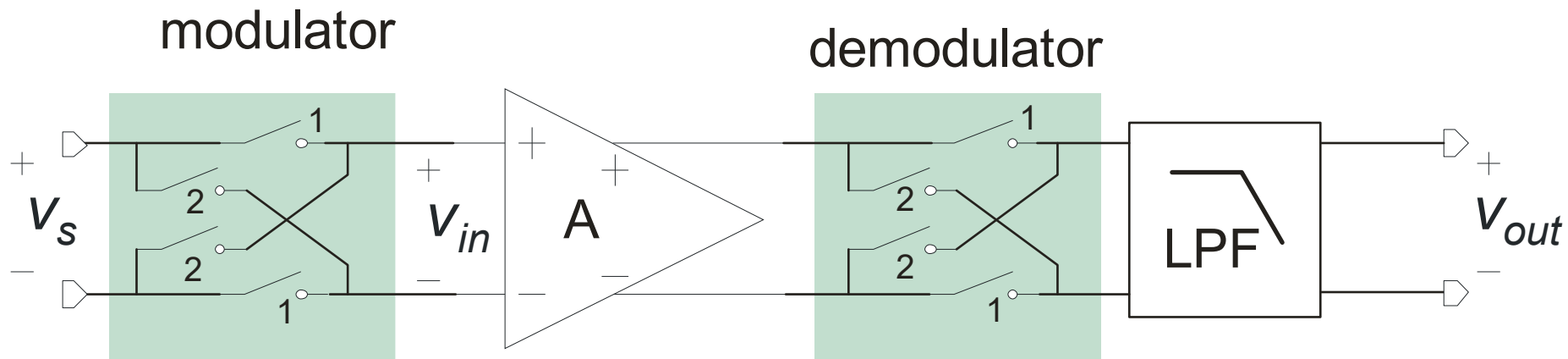
$$V_{out}(t) = A \left[V_s(t) - \left(v_n(t) - v_n^{(1)} \right) \right]$$



$$V_{out}^{(2)} = A \left[V_s^{(2)} - \left(v_n^{(2)} - v_n^{(1)} \right) \right]$$

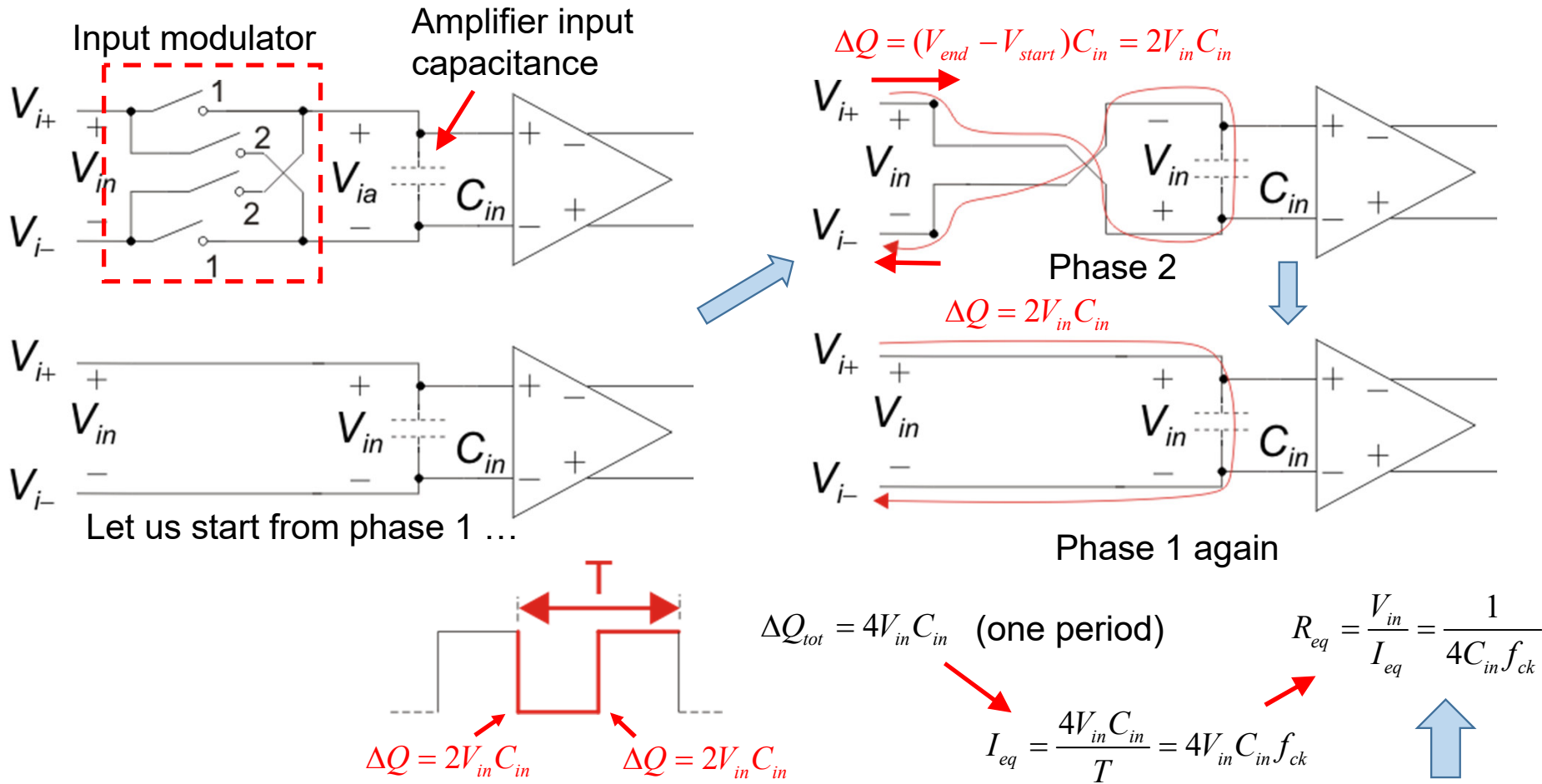
Simple example of circuitual implementations

- **Chopper Amplifier**



- A differential input and differential output facilitate the implementation of the modulator and demodulator, respectively (fully-differential amplifier)
- The amplifier gain cannot be arbitrarily high, otherwise the amplified offset could saturate the amplifier. Typical values of A are < 1000

Finite input resistance of chopper amplifiers



Residual offset

