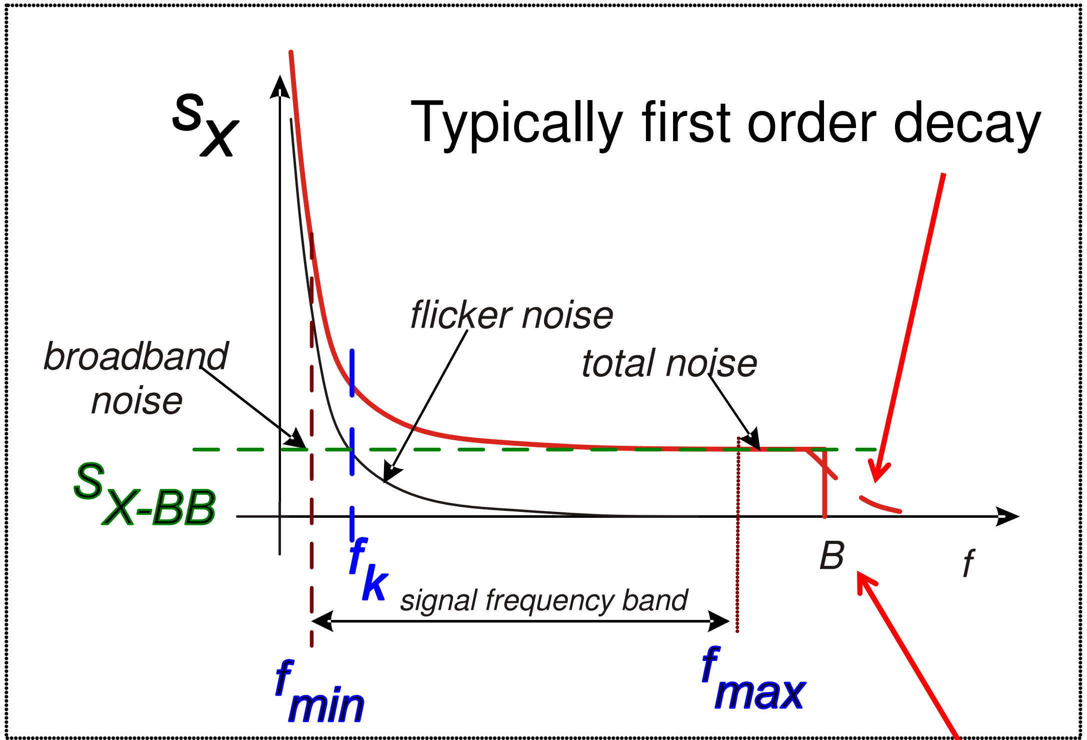
Practical Rules for noise calculations

Idealized amplifier noise density

Noise PSD in Low-Pass Systems



B=equivalent noise bandwidth

PSD: Power Spectral Density

$$S_{XF}(f) = \frac{k_F}{f^{\gamma}} \qquad \gamma \cong 1$$

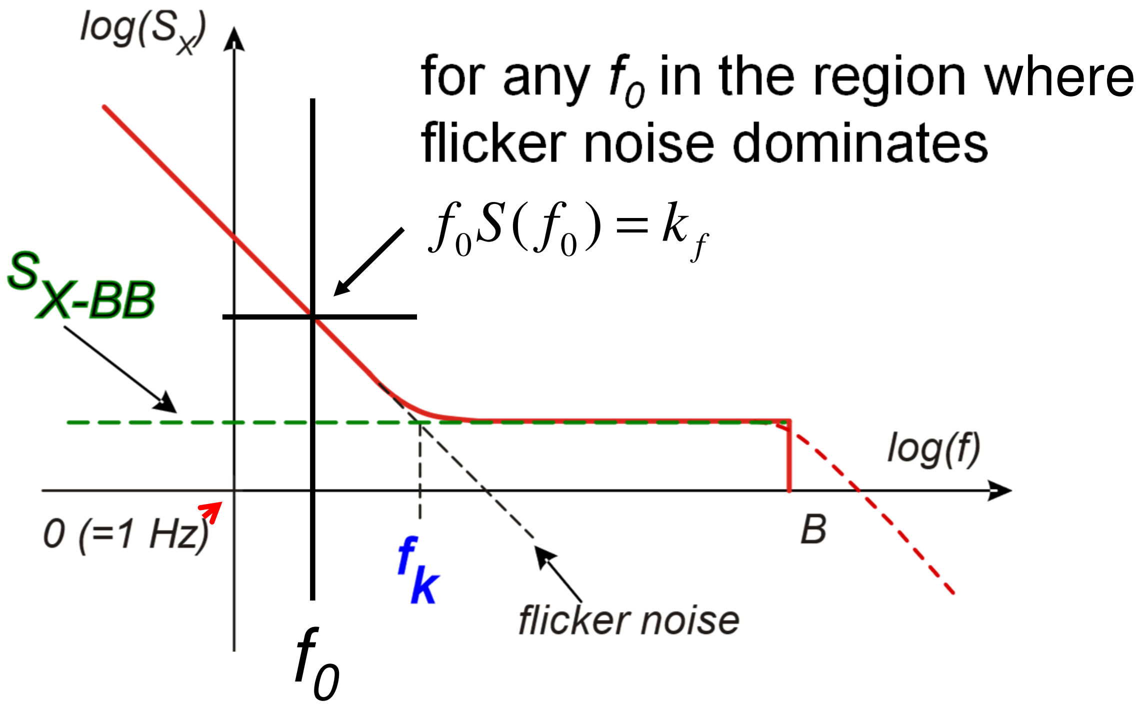
$$S_{X-BB}(f) = \text{constant} = S_{X-BB}$$

Definition of flicker corner frequency

$$f_k : \quad S_{XF}(f_k) = S_{X-BB}(f_k)$$

$$f_k S_{X-BB} = k_F$$

Amplifier noise spectrum in logarithmic axes



If the noise cut off is determined by a first order low-pass function:

$$B = \frac{\pi}{2} f_{-3dB}$$

Total rms noise in the signal bandwidth

$$x_{rms} = \sqrt{\int_{f_{min}}^{f_{max}} S_X(f)df} = \sqrt{\int_{f_{min}}^{f_{max}} S_{XBB}(f)df + \int_{f_{min}}^{f_{max}} S_{XF}(f)df}$$

$(f_{max} < B)$

Broad-band

$$\int_{f_{min}}^{f_{max}} S_{XBB}(f)df = S_{XBB}\left(f_{max} - f_{min}\right)$$

$$S_{XF}(f) = \frac{k_F}{f}$$

Flicker

$$\int_{f_{min}}^{f_{max}} S_{XF}(f)df = k_F \ln\left(\frac{f_{max}}{f_{min}}\right) = k_F \log_{10}\left(\frac{f_{max}}{f_{min}}\right)\frac{1}{\log_{10}(e)} \cong k_F 2.3 \cdot n_{dec}$$

n_{dec} =number of decades from f_{min} to f_{max}

Flicker noise for signal bands that include *dc*

$$\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df = k_F \ln\left(\frac{f_{\max}}{f_{\min}}\right)$$

If f_{min} =0, the integral is infinity

Many signals of interest include dc. This is true for practically most signals produced by sensors like temperature, pressure, acceleration etc.

→ For these cases, do we have **infinite noise magnitude** ?

The solution to this paradox is that, in practical cases, speaking of a real dc component is meaningless, since it would be constant across an infinite interval of time.

For every practical scenario, there is always a **finite** "observation time period", across which we require a signal to be constant to state that this is a dc component.

Flicker noise for signal bands that include *dc*

Then, we use the flicker noise expression:

$$\int_{f_{\min}}^{f_{\max}} S_{XF}(f)\,df = k_F \ln\left(\frac{f_{\max}}{f_{\min}}\right) \qquad \text{with} \qquad f_{\min} \approx \frac{1}{T_{obs}}$$

Where T_{obs} is the "observation time".

If the signal band includes dc, we generally set T_{obs} = 10s-100s, resulting in f_{min}= 0.1-0.01 Hz.

Example

$$\int_{f_{min}}^{f_{max}} S_{XF}(f)df = k_F 2.3 \cdot n_{dec}$$

Specifications: f_{max}=1kHz, f_{min}=0 (dc)

For T_{obs}=100 s, f_{min}=0.01 Hz $\quad$ $n_{dec}=5$

For $T_{obs}=10^5$ s (> 1 day) s, f_{min}=10 μHz $\quad$ $n_{dec}=8$

The flicker component to $<(x_n)^2>$ is increased by 60% and the *rms* component by 26%

In terms of resolution, this is quite a negligible increase. To have an increase of 1 unit in the ENOB associated to the DR we need an increase of 100 % in x_{rms} , i.e. a factor of 4 in $<(x_n)^2>$. The presence of a significant contribution from S_{XBB} makes this flicker increment even less important.

The choice of T_{obs} (f_{min}) is not critical !

Something more about the broad-band component

$$\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df = S_{XBB}\left(f_{\max} - f_{\min}\right)$$

S_{XBB} has units $[X]^2$/Hz where [X] are the units of quantity X.
For example if X is a voltage, we have **V^2/Hz**
This is not the specification that is generally used in practical cases (e.g. amplifier datasheets).

What is generally given, is the square root of the PSD:

$$\sqrt{S_X}$$

Units: $[X]/\sqrt{Hz}$

$$\sqrt{S_X}$$

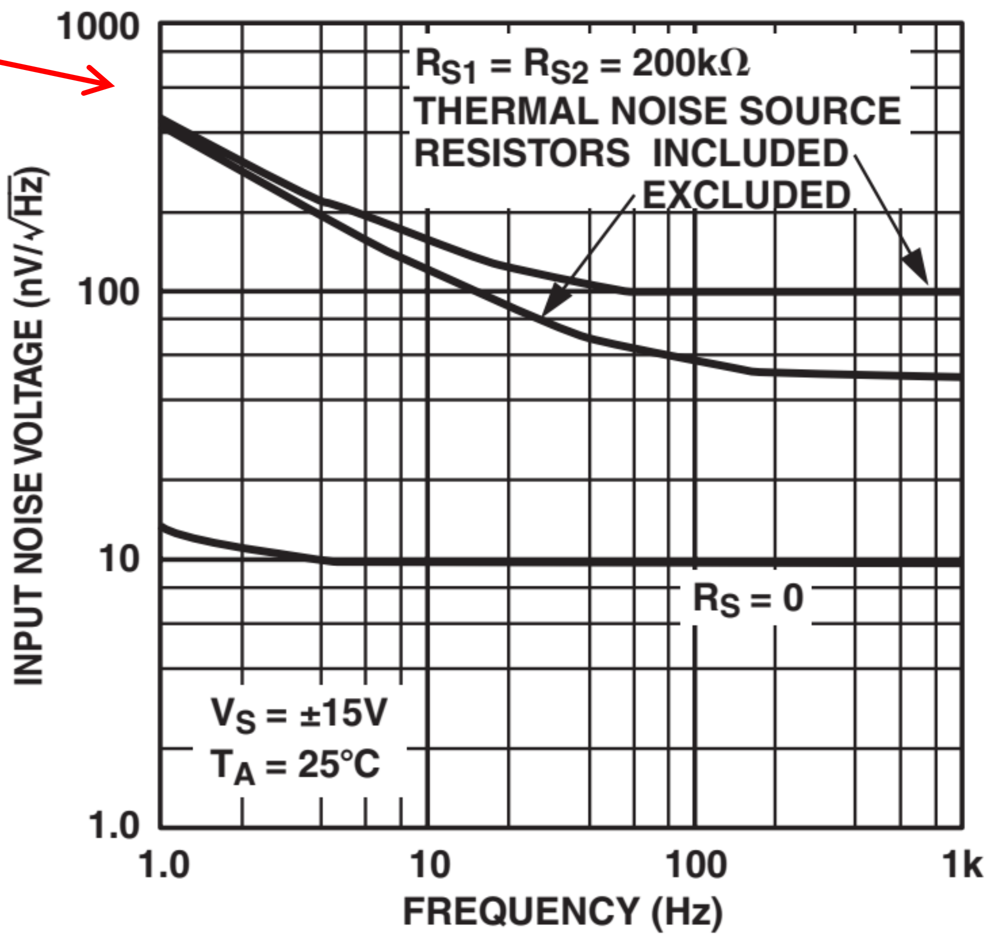
$$nV/\sqrt{Hz}$$

or nV/sqrt(Hz)

«noise density»

(instead of :
noise **power** spectral density)

Example



Practical *rms* noise calculation:

$$x_{rms} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_X(f)df} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df + \int_{f_{\min}}^{f_{\max}} S_{XF}(f)df}$$

$$x_{rms-BB} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df}\,; \qquad x_{rms-F} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df}$$

$$x_{rms} = \sqrt{x_{rms-BB}^2 + x_{rms-F}^2}$$

$$\sqrt{1+\left(\frac{1}{5}\right)^2} = 1.0198$$

It is sufficient that one of the two contribution is 5 times smaller than the other to get practically negligible (with a 2 % error)

Example

Includes both flicker and BB noise

Input Noise Voltage	e_n p-p	0.1 Hz to 10 Hz[3]	0.38	0.65	μV p-p
Input Noise Voltage Density	e_n	f_O = 10 Hz	10.5	20.0	nV/√Hz

$$x_{rms-BB} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XBB}(f)df} = \sqrt{S_{XBB}\left(f_{\max} - f_{\min}\right)} = \sqrt{S_{XBB}}\sqrt{\left(f_{\max} - f_{\min}\right)}$$

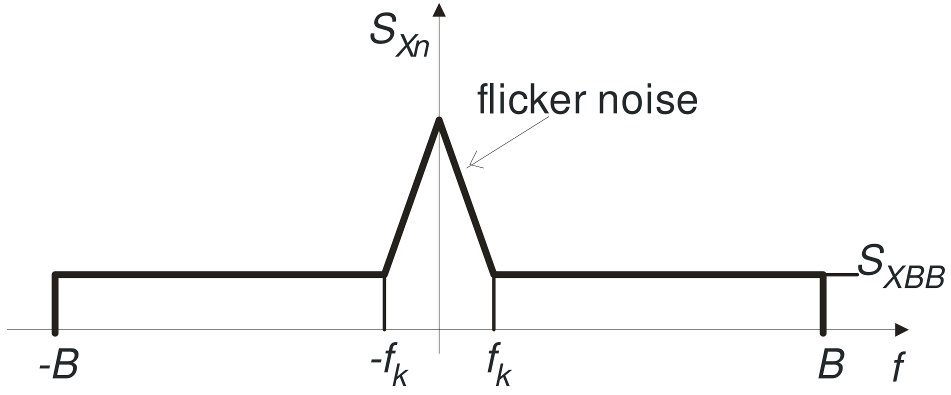
$$x_{rms-F} = \sqrt{\int_{f_{\min}}^{f_{\max}} S_{XF}(f)df} = \sqrt{2.3k_F n_{dec}} \cong 1.5\sqrt{k_F}\sqrt{n_{dec}}$$

> X_{pp-F} over two decades

$$x_{n-pp} = 4x_{n-rms} = 4\sqrt{x_{rms-BB}^2 + x_{rms-F}^2} = \sqrt{\left(4x_{rms-BB}\right)^2 + \left(4x_{rms-F}\right)^2}$$

$$x_{n-pp} = \sqrt{x_{pp-BB}^2 + x_{pp-F}^2}$$

Schematic two-sided representation of amplifier noise



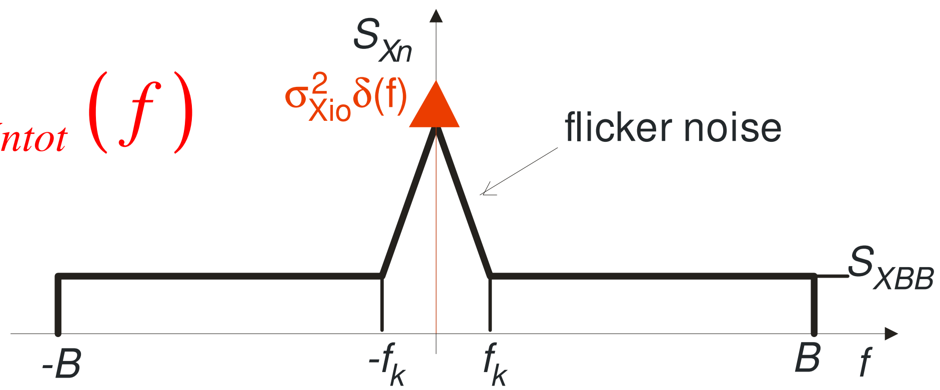
$$x_{ntot} = x_n + x_{io}$$

Total additive error: offset + noise

x_{io} is a stationary, non-ergodic stochastic process.
Noise and offset are independent processes

$$R_{Xntot}(\tau) = R_{Xn}(\tau) + \sigma^2_{Xio}$$

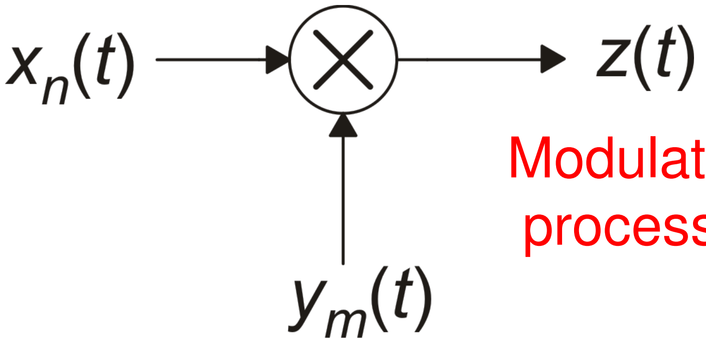
$S_{Xntot}(f)$



Generalized spectrum that represents noise and offset together

Modulation of a stochastic process

Stationary stochastic process (e.g. noise)



Modulated process

Modulating signal (e.g. local oscillator)

We can consider y_m a stochastic process by adding some randomness. It is sufficient to introduce a random delay to make *z(t)* stationary. Or we can refer to the formalism of <u>cyclostationary</u> processes

If y_m is a periodic deterministic signal, *Z(t)* is generally no more a stationary process

The auto-correlation function is no more a simple function of the delay τ and a PSD cannot be calculated

But ...

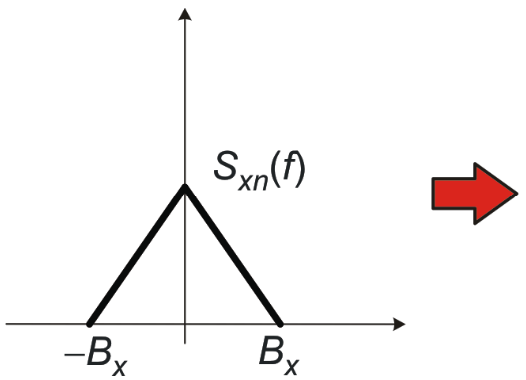
Modulation of stochastic processes

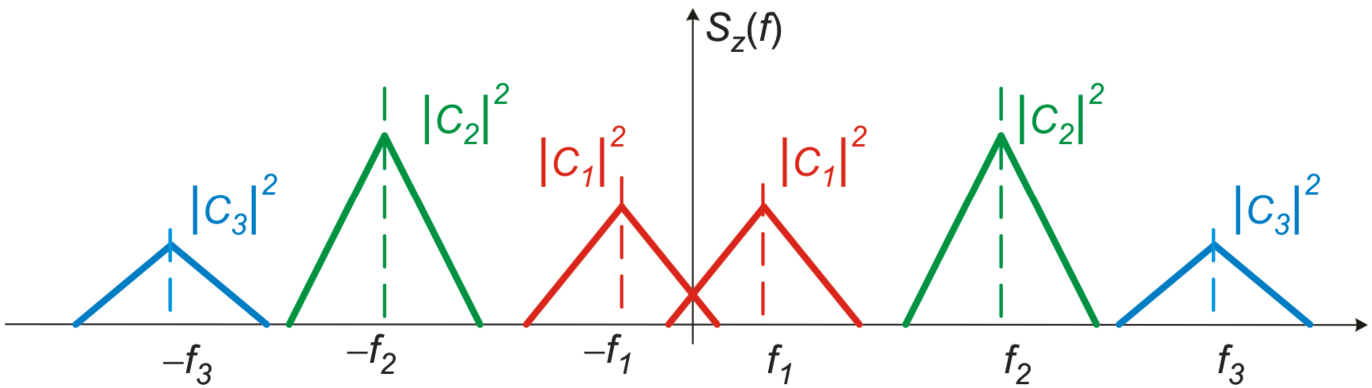
$$y_m(t) = \sum_{k=-n}^{n} C_k e^{j\omega_k t}$$

Complex Fourier series representation of the modulating signal $y_m(t)$

$$C_{-k} = C_k^* \qquad \omega_k = k\omega_0$$

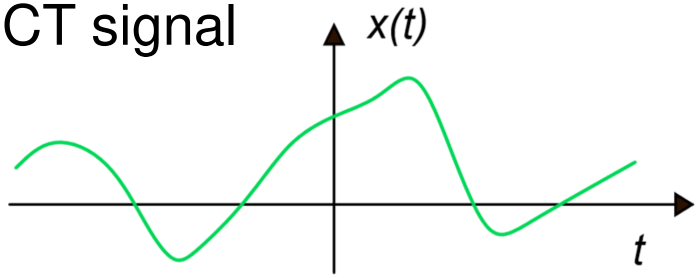
$$S_Z(f) = \sum_{i=-\infty}^{\infty} |C_i|^2 S_{Xn}(f - k\omega_i)$$

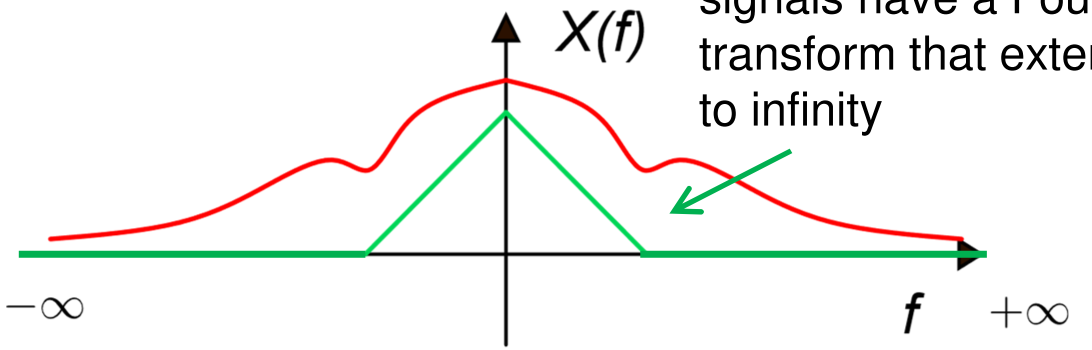




Discrete-time (DT) and continuous-time (CT) signals

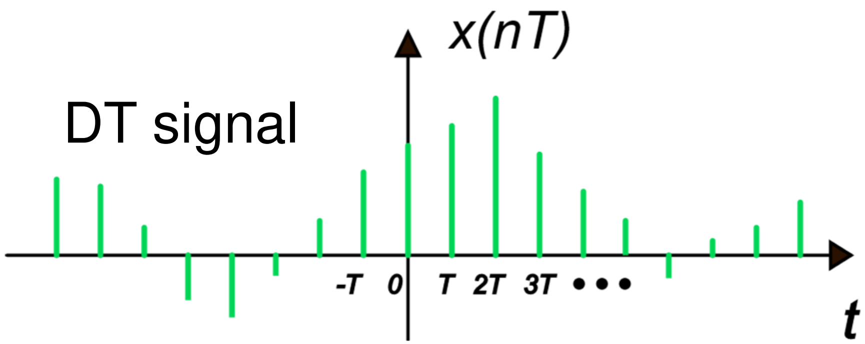
CT signal



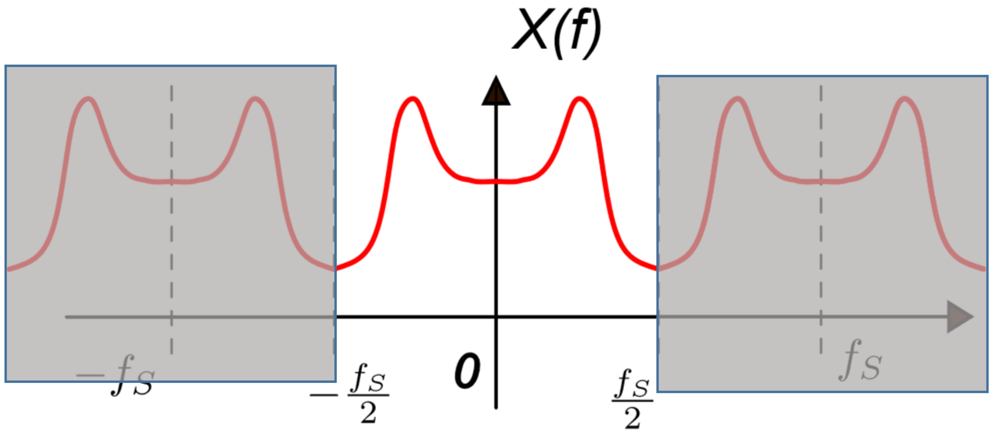


Even band-limited signals have a Fourier transform that extends to infinity

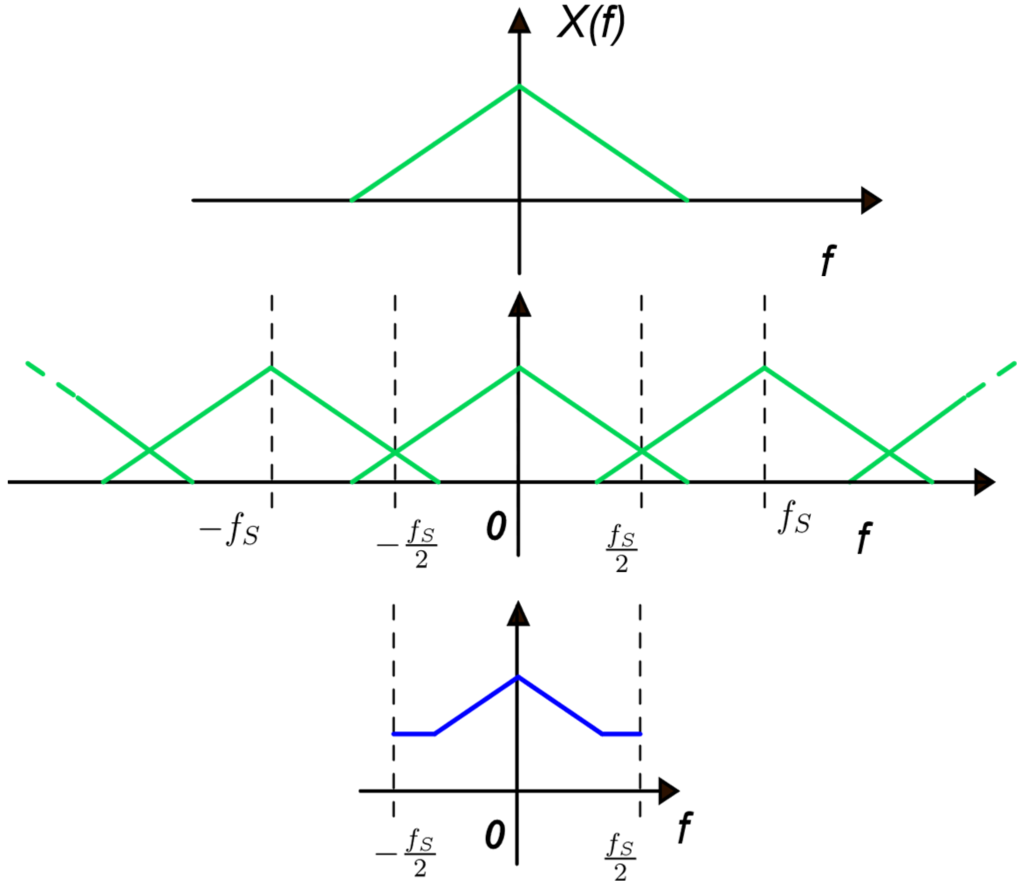
DT signal



…. and their Fourier transform



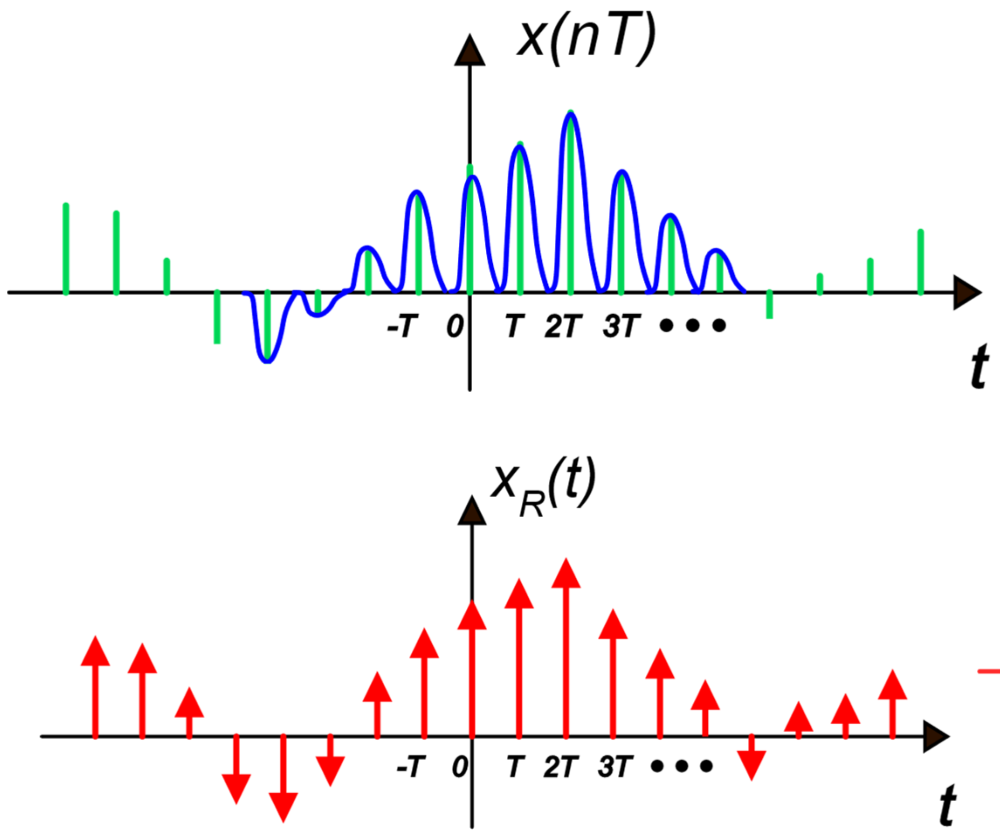
DT signals from sampling of CT signals



Place a replica of the original spectrum across each multiple of f_S

Add the replicas only across the DT frequency interval

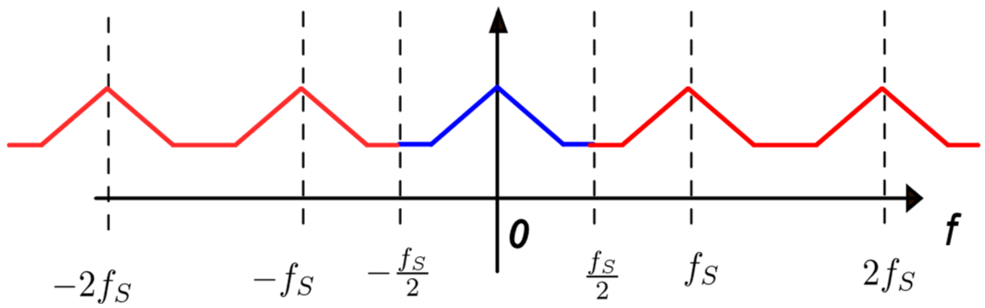
CT signals from DT ones



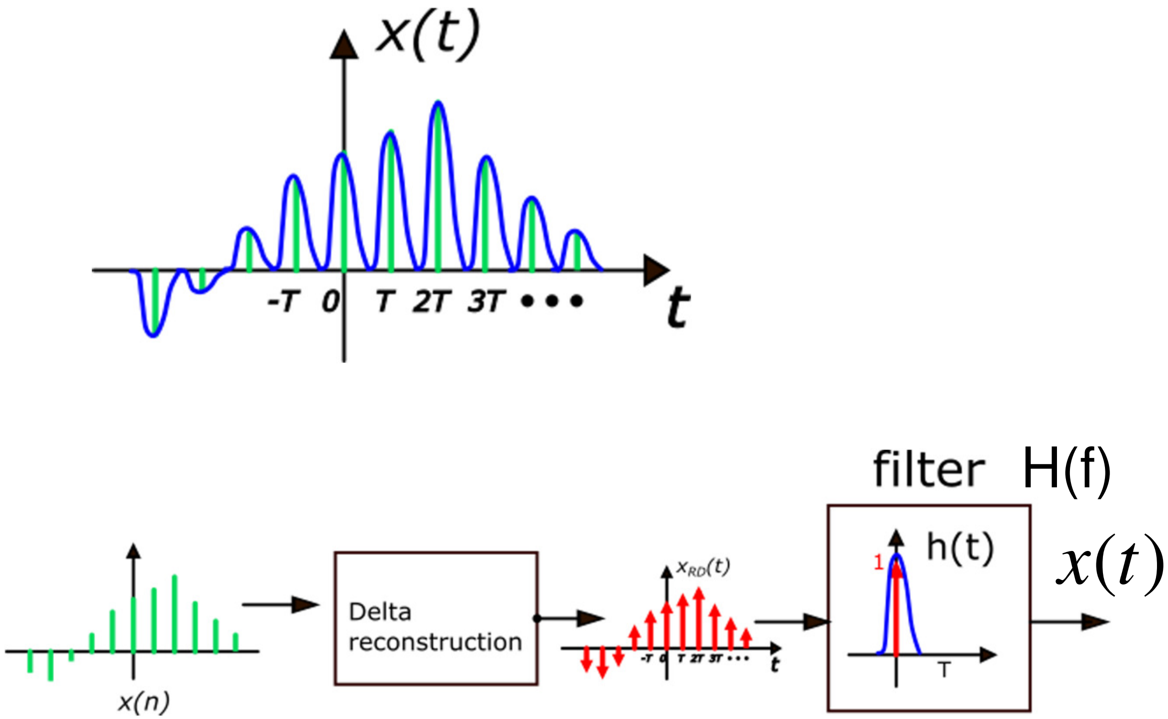
For our purposes, we are interested in:

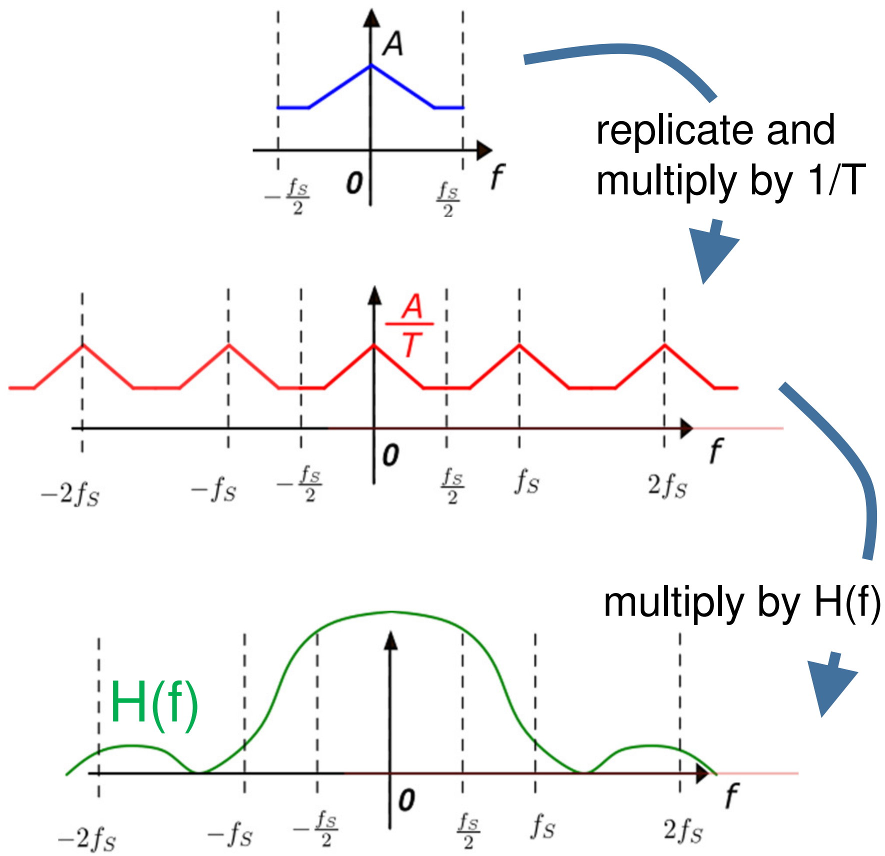
- Reconstruction with delta functions δ(t)
- Hold - reconstruction

Replicas are multiplied by $\frac{1}{T}$



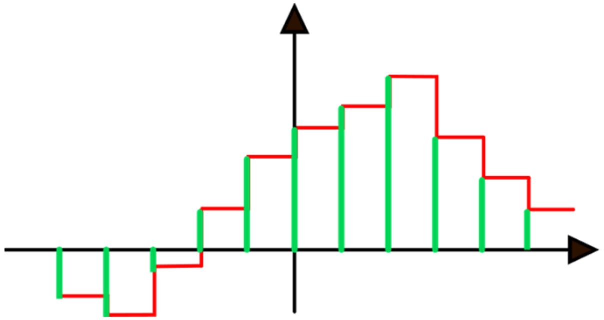
DT-CT reconstruction with an arbitrary pulse





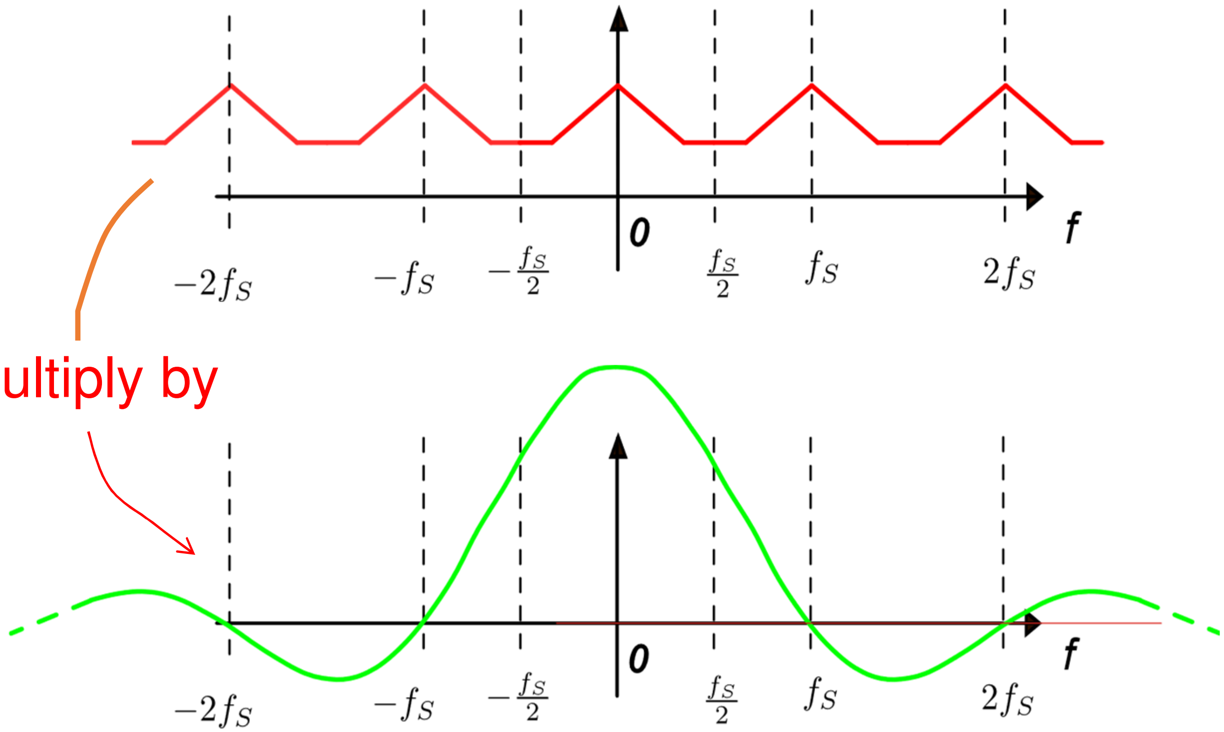
Theoretical (non real) process

Hold reconstruction

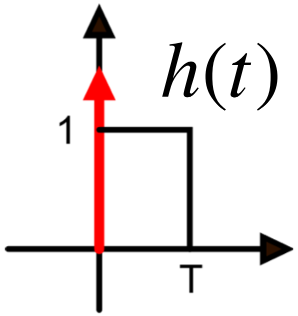


It is the simpler way to convert a digital signal (data sequence) into an analog one by means of a DAC

$$\text{replicas} \times \frac{1}{T}$$



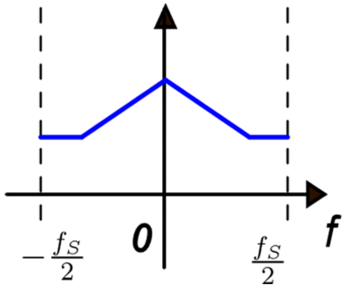
Reconstruction pulse



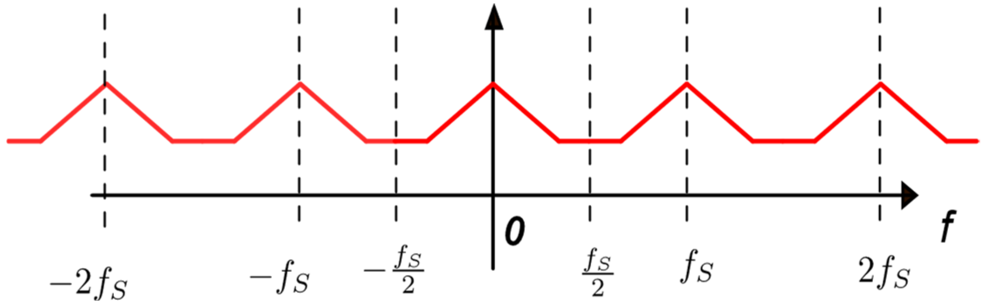
$$H(f) = T \cdot e^{-j\pi fT} \operatorname{sinc}(\pi fT)$$

$$\operatorname{sinc}(x) = \frac{\sin(x)}{x}$$

Hold - reconstruction: summary

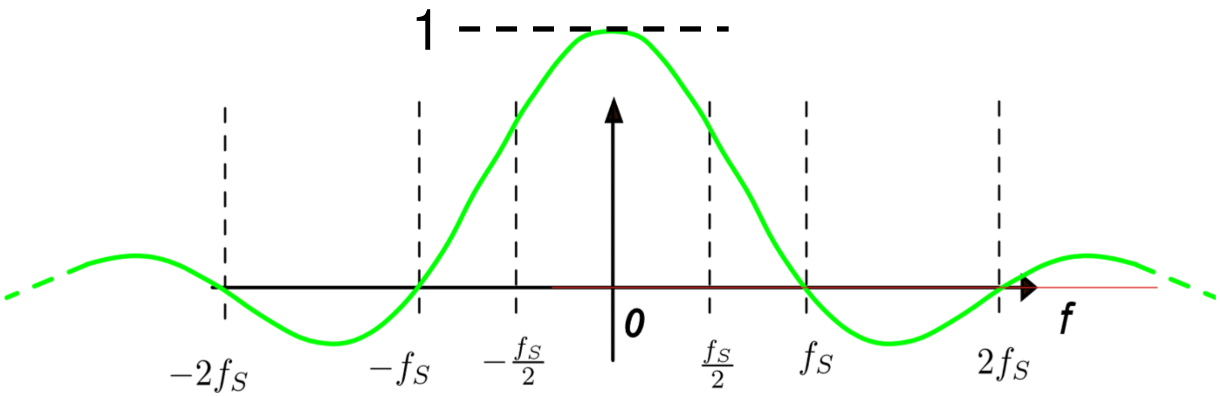


Start from the discrete time fourier transform



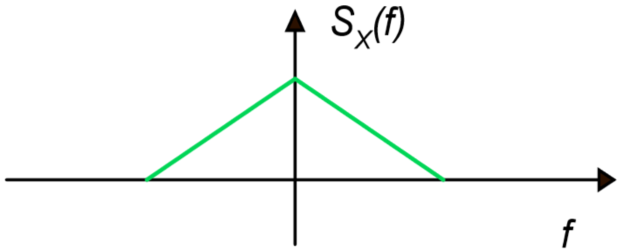
Replicate the spectrum (non scaled) across any multiple of f_s

Multiply by: $e^{-j\pi fT}\mathrm{sinc}(\pi fT)$

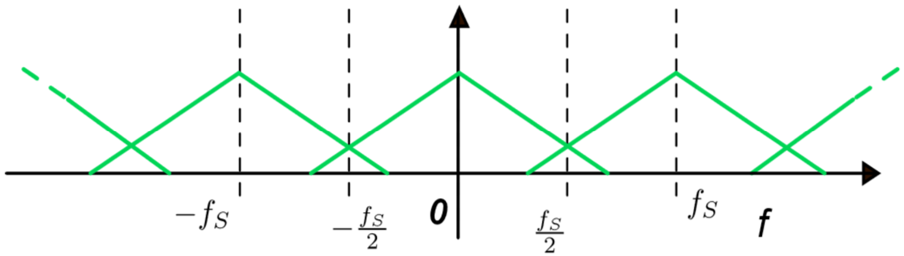


Sampling and holding a stochastic process

PSD of the CT stochastic process

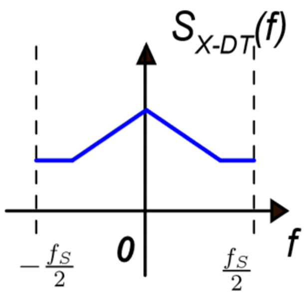


The PSD is replicated and the replicas are added

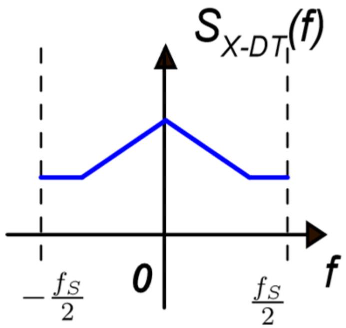


The procedure is similar to the case of a deterministic signal but for a stochastic process it is the PSD to be involved

Only the spectrum in the $-f_S/2$, $+f_S/2$ interval is retained

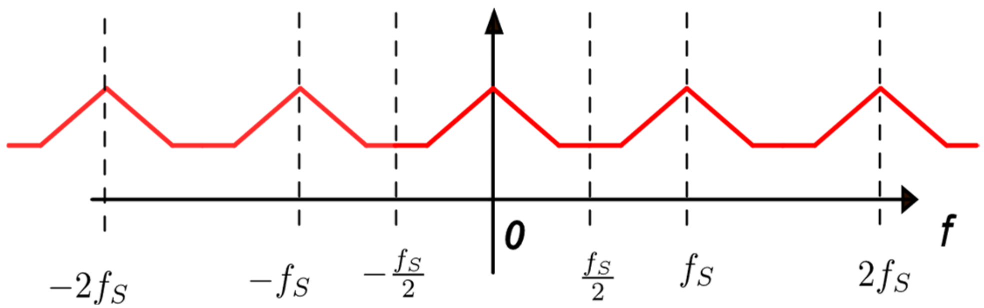


Reconstruction by hold-operation



PSD of the DT stochastic process

Replicate with f_s step



multiply by $|H(s)|^2 = \mathrm{sinc}^2(\pi f T)$