Analog Filter Design

Part. 5: Analog Discrete Time Filters: Switched Capacitor Filters

First SC circuits: simulations of resistances

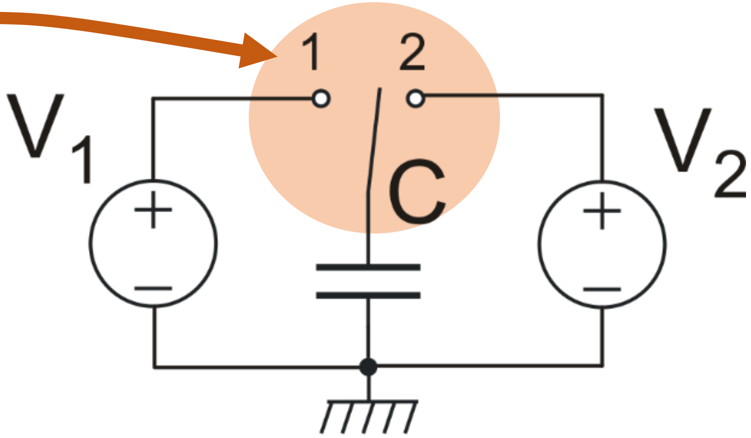
- In integrated RC active filters, the resistances may be the largest components, especially when low frequency singularities are required, as in audio analog processors.
- Singularities in RC filters are proportional to 1/RC factors. Since R and C are marked by non-correlated process variations, the spread in filter characteristic frequencies can be very large (up to 20 %).
- Resistances simulated with switches and capacitors are given by expressions like:

$$R_{eq} = \frac{1}{f_{ck} C}$$

where f_{ck} is the clock frequency. With the small capacitors available on chip it is possible to obtain very large resistors.

Switched Capacitor resistance: principle

Alternation between position 1 and 2 at the clock frequency f_{ck}

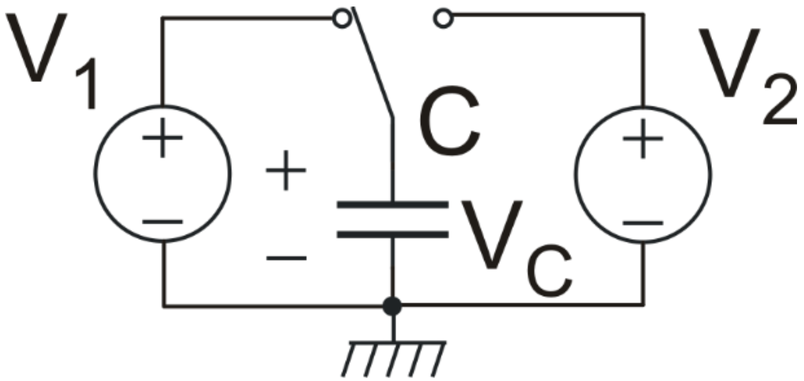


$$\Delta Q_C = C\left(V_C^{(final)} - V_C^{(initial)}\right)$$

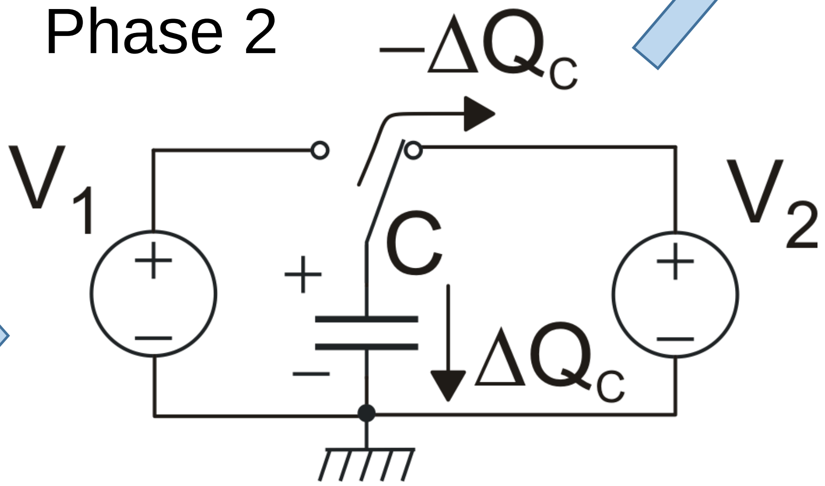
$$-\Delta Q_C = C\left(V_1 - V_2\right)$$

$$I_{eq} = -\Delta Q \cdot f_{CK} = C f_{ck}\left(V_1 - V_2\right)$$

Phase 1

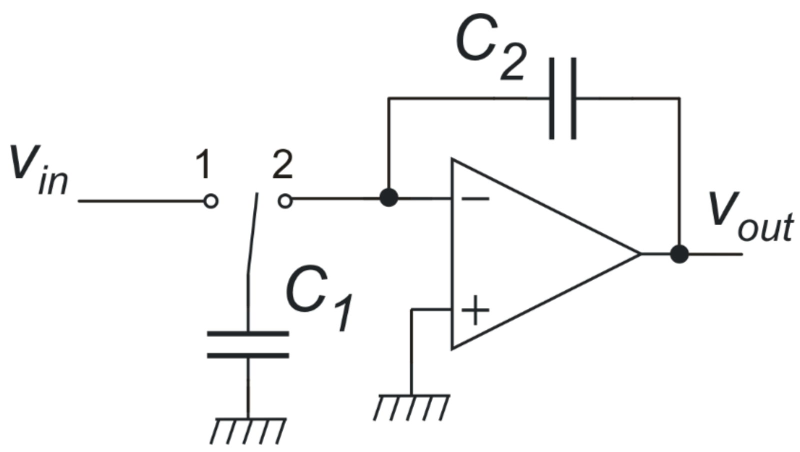


Phase 2

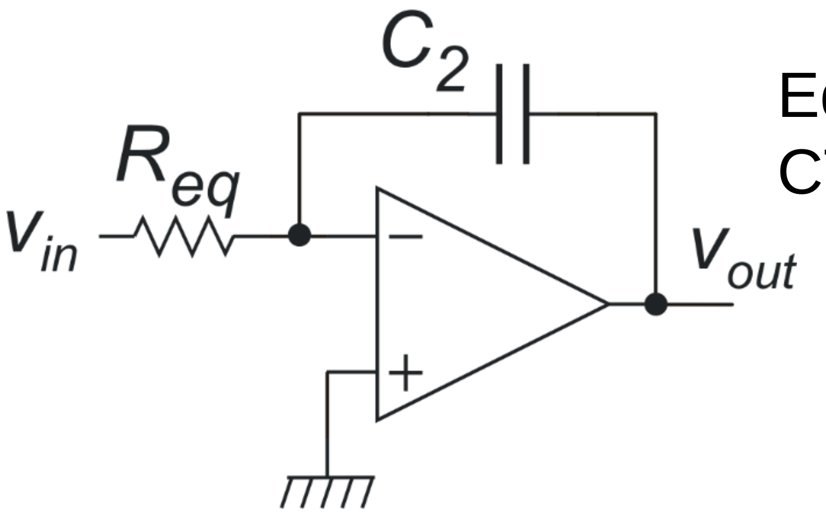


$$R_{eq} = \frac{1}{f_{ck} C}$$

Simple integrator based on SC resistor



Simple SC integrator

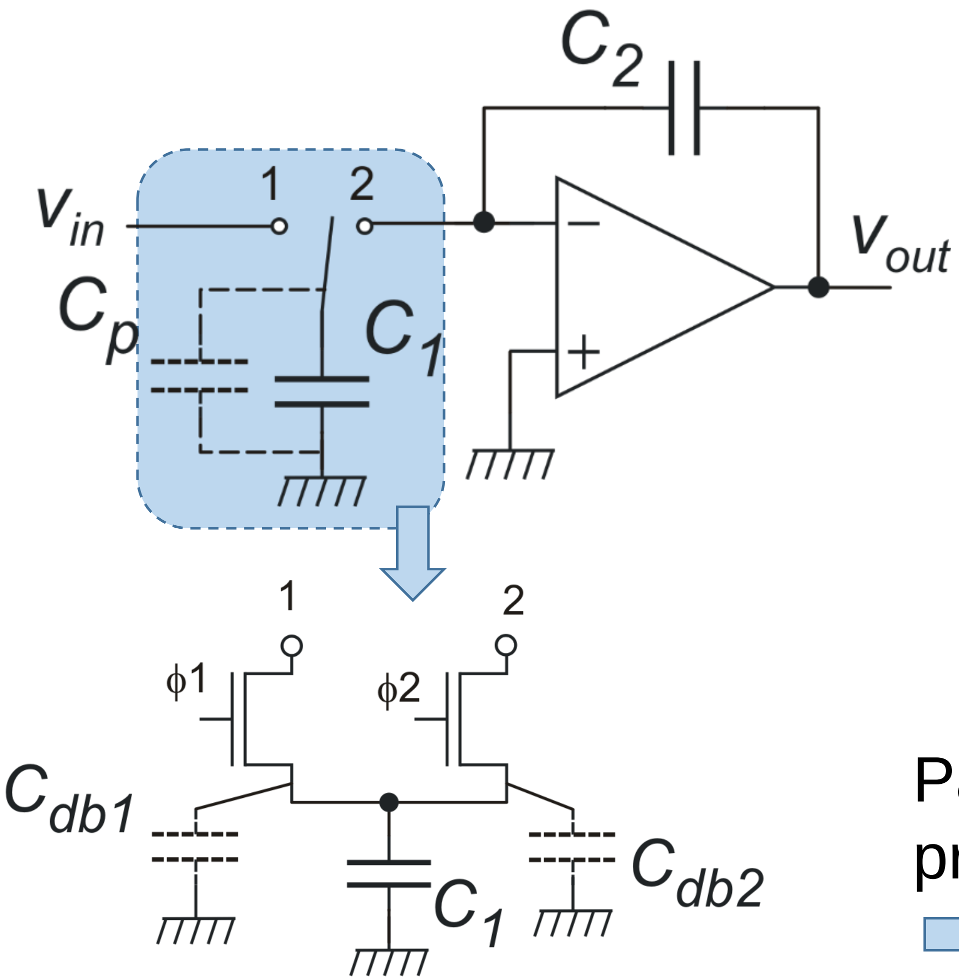


Equivalent CT Integrator

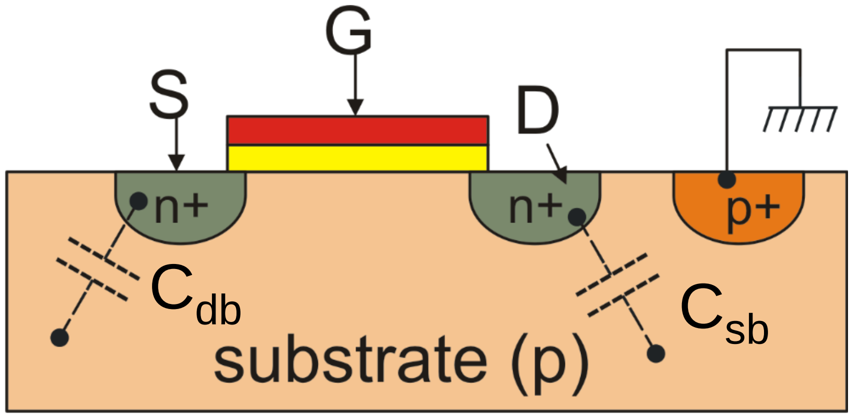
$$v_{out} = -\frac{\omega_0}{s} \qquad \omega_0 = \frac{1}{R_{eq} C_2} = \frac{C_1}{C_2} f_{ck}$$

- Important: The unity gain angular frequency of the integrator (ω_0) depends only on capacitance ratios and the clock frequency. Ratios can be fabricated with high precision and accurate frequencies can be obtained from crystal oscillators
- **Filter with precise corner frequencies can be obtained.**

Effect of parasitic capacitances



$$\omega'_0 = \frac{C_1 + C_p}{C_2} f_{ck}$$

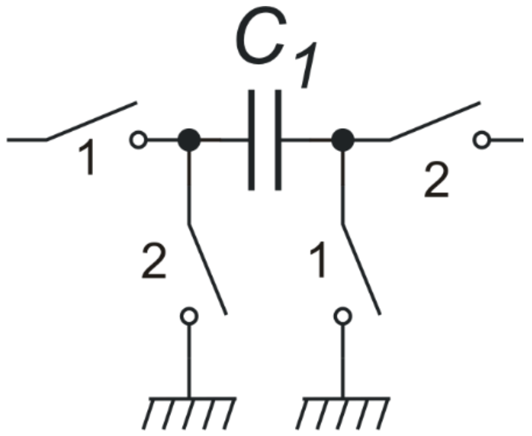


Parasitic capacitances are not well predictable and generally non linear

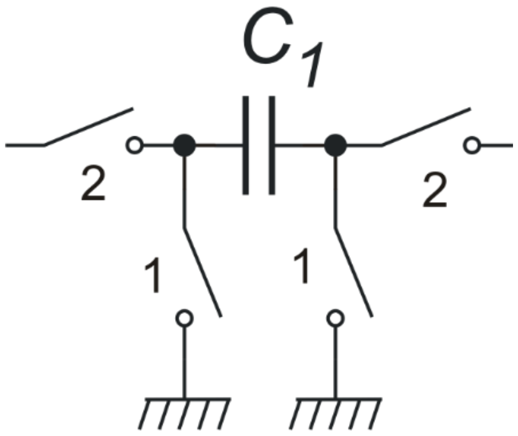
- Inaccuracy of the corner frequencies
- Distortion

Parasitic Insensitive (PI) SC integrator

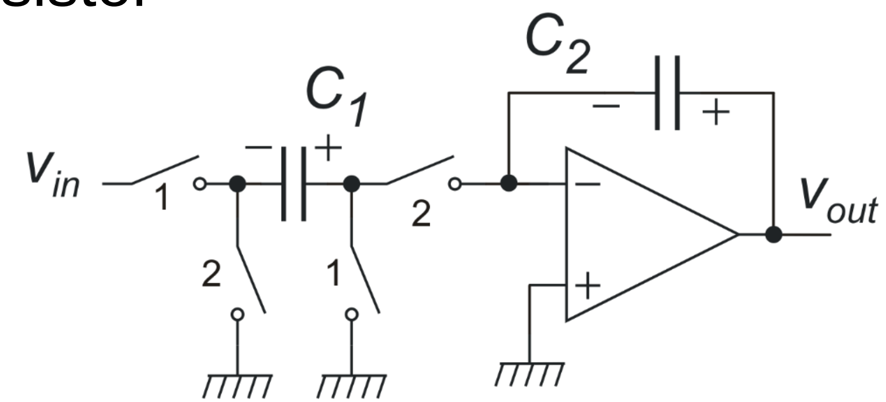
- Parasitic Insensitive SC resistors



Negative SC resistor



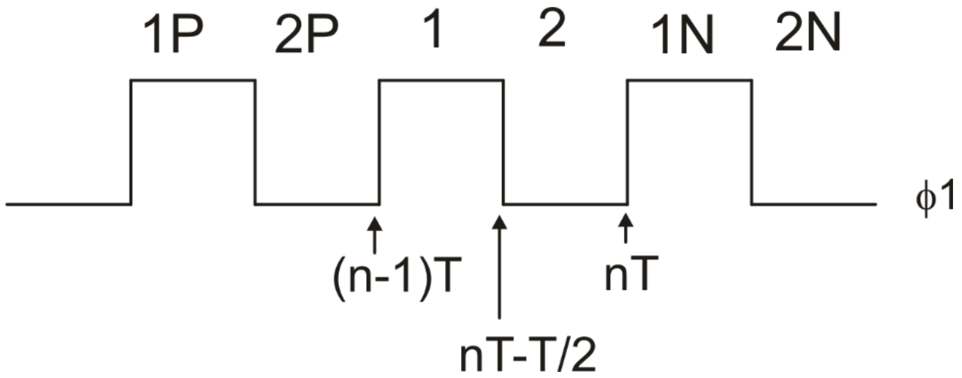
Positive SC resistor

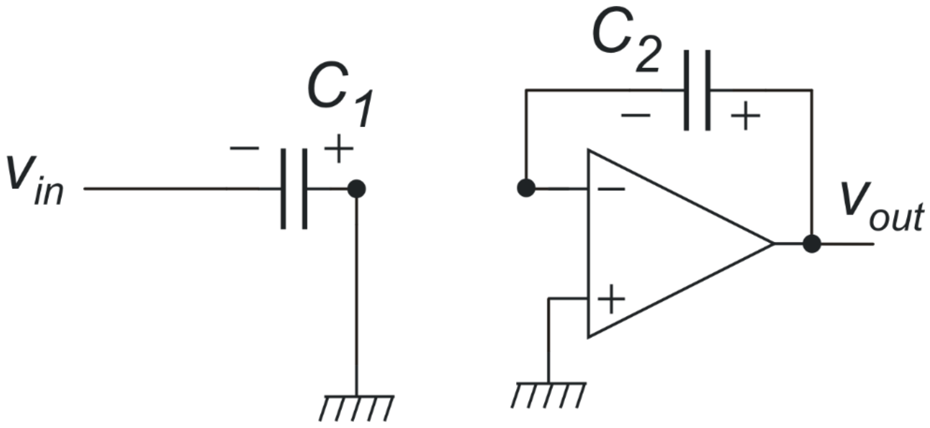


Example: Integrator with negative SC Res.

Parasitic Insensitive (PI) SC integrator

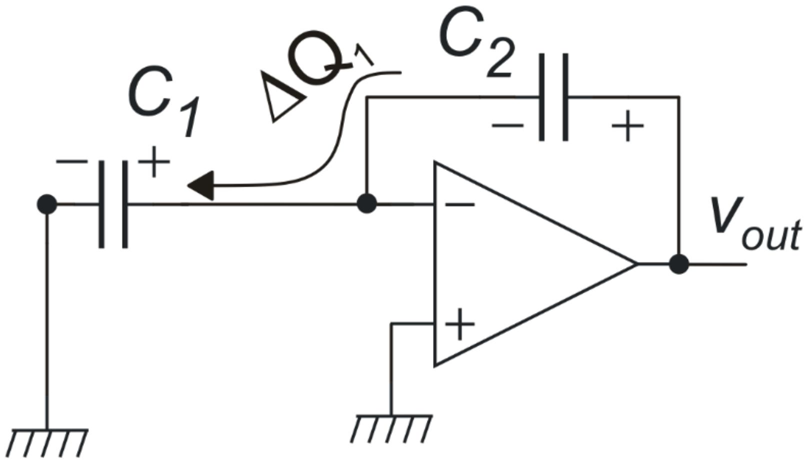
- Integrator with negative resistor: phases





$$V_{C1}^{(1)} = -v_{in}^{(1)}$$

$$V_{C2}^{(1)} = V_{C2}^{(2P)} \quad \text{(hold)}$$



$$V_{C1}^{(2)} = 0$$

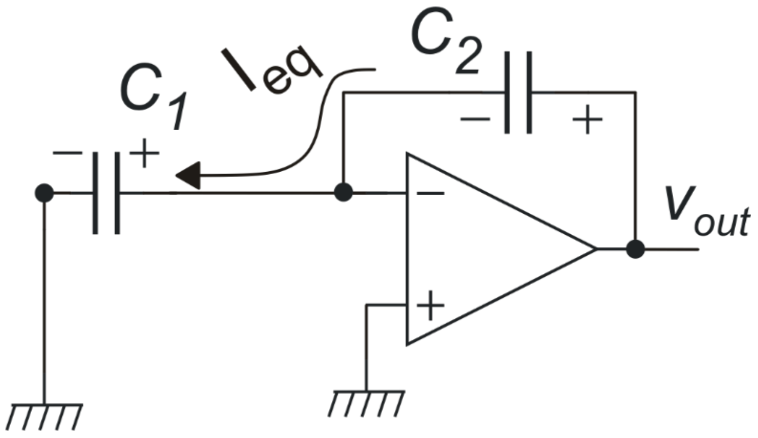
$$V_{C2}^{(2)} = V_{C2}^{(2P)} + \frac{\Delta Q_{C1}}{C_2}$$

$$\Delta Q_{C1} = C_1\left(V_{C1}^{(2)} - V_{C1}^{(1)}\right) = v_{in}^{(1)} C_1$$

$$\boxed{V_{out}^{(2)} = V_{out}^{(2P)} + \frac{C_1}{C_2} v_{in}^{(1)}}$$

$$V_{out} = V_{C2}$$

Equivalent currents

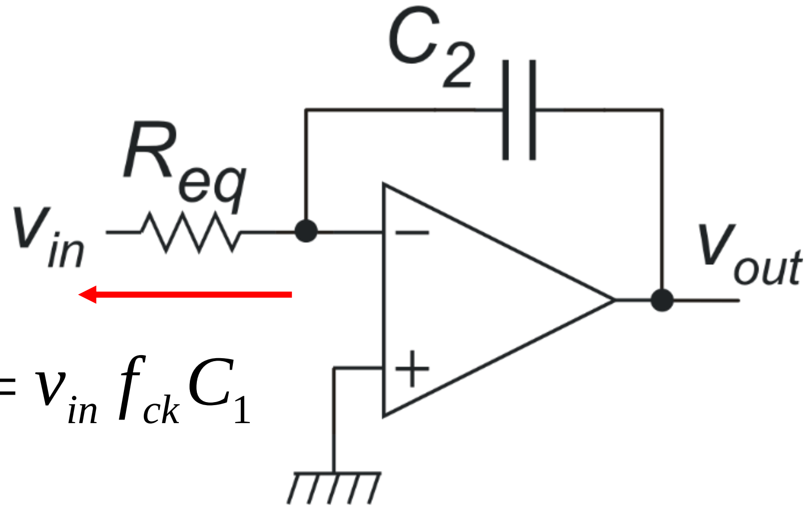


$$\Delta Q_{C1} = v_{in}^{(1)} C_1$$

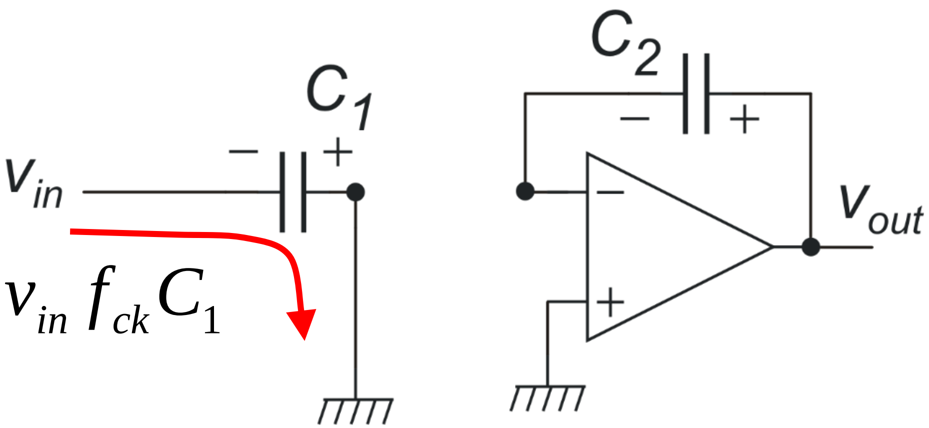
$$I_{eq} = f_{CK} \Delta Q_{C1} = v_{in} f_{ck} C_1$$

$$R_{eq} = -\frac{1}{f_{ck} C_1} < 0$$

In terms of charge delivered to the amplifier



$$I_{eq} = v_{in} f_{ck} C_1$$

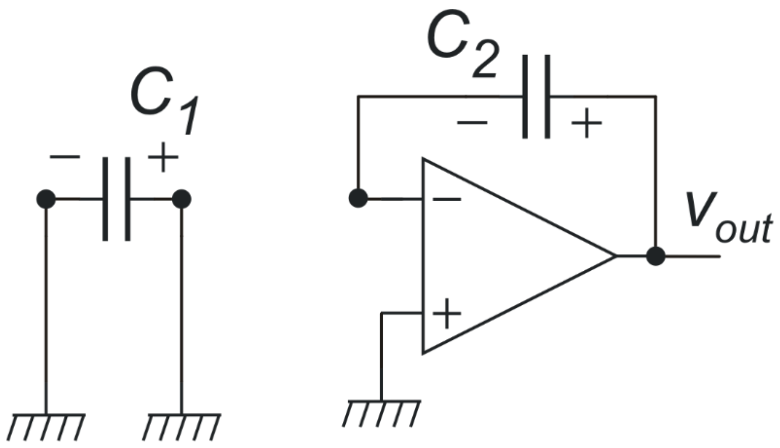


$$I_{eq} = v_{in} f_{ck} C_1$$

$$R_{eq} = \frac{1}{f_{ck} C_1} > 0$$

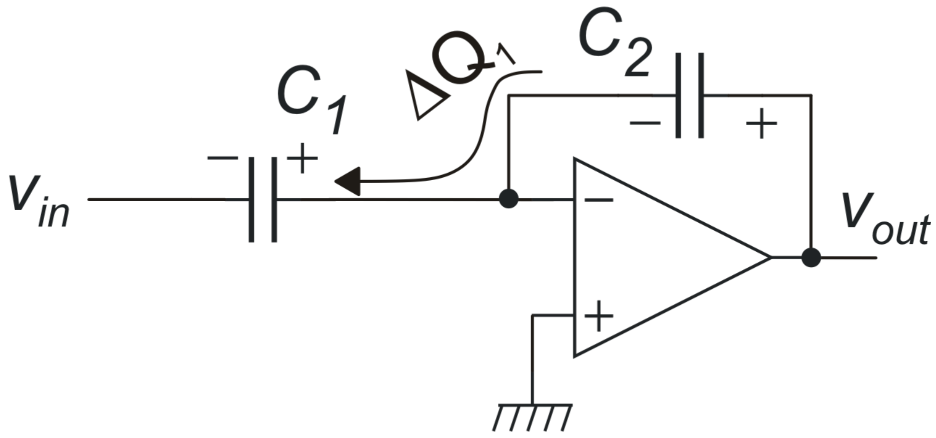
Seen by the source v_{in}

Parasitic Insensitive (PI) SC integrator



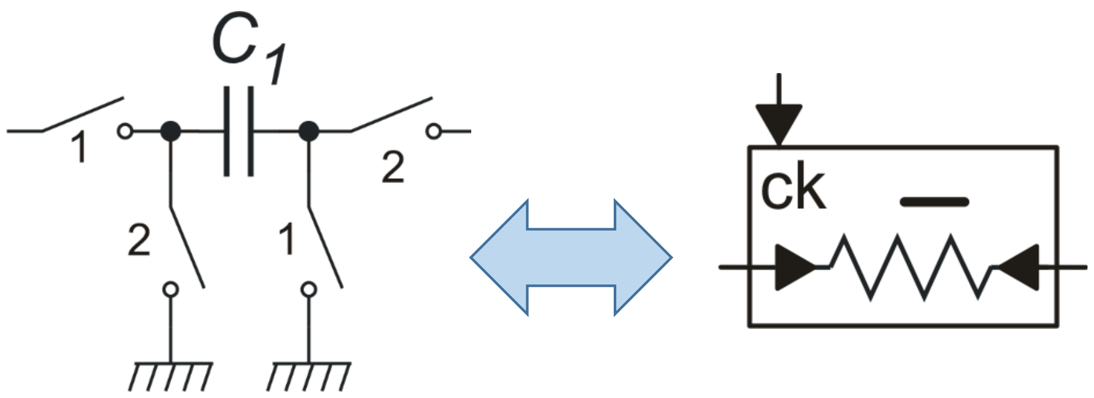
$$V_{C1}^{(1)} = 0$$
$$V_{C2}^{(1)} = V_{C2}^{(2P)} \quad \text{(hold)}$$

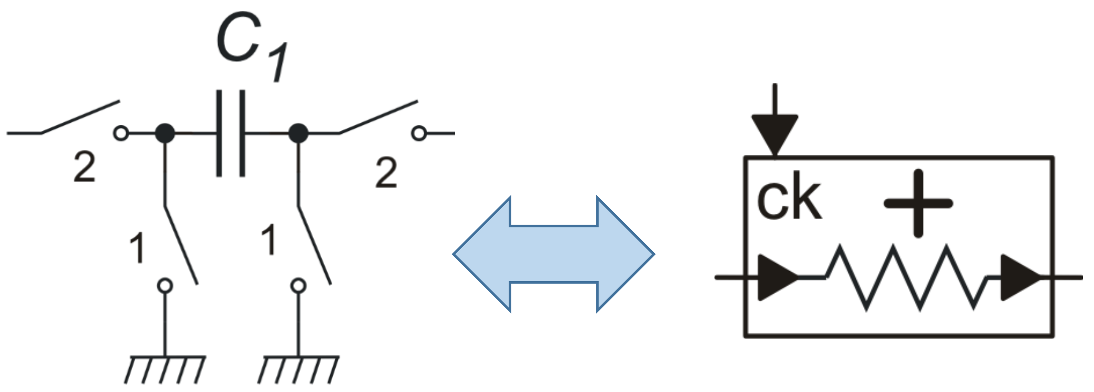
$$V_{out}^{(2)} = V_{out}^{(2P)} - \frac{C_1}{C_2} v_{in}^{(2)}$$



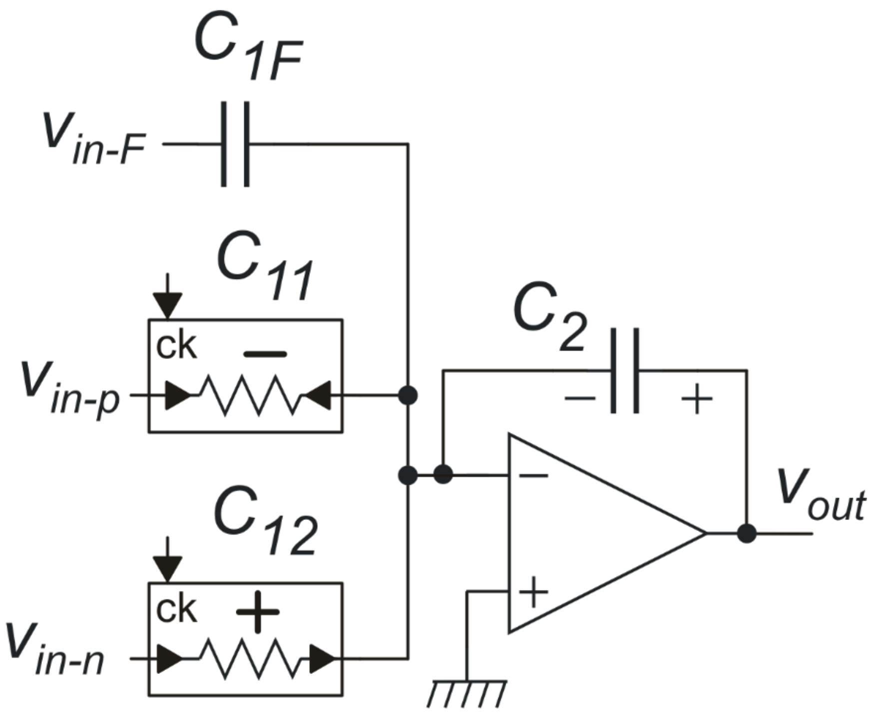
$$V_{C1}^{(2)} = -v_{in}^{(2)}$$
$$V_{C2}^{(2)} = V_{C2}^{(2P)} + \frac{\Delta Q_{C1}}{C_2}$$
$$\Delta Q_{C1} = C_1\left(V_{C1}^{(2)} - V_{C1}^{(1)}\right) = -v_{in}^{(2)} C_1$$

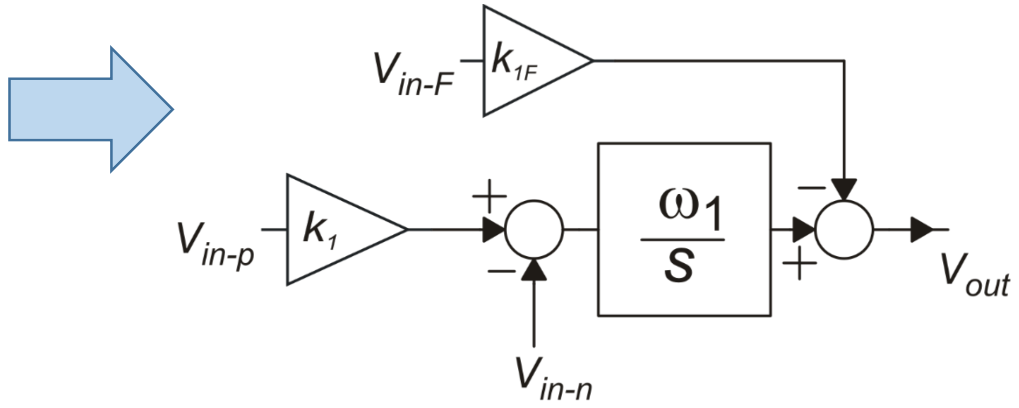
PI-SC resistors: symbols used in this course





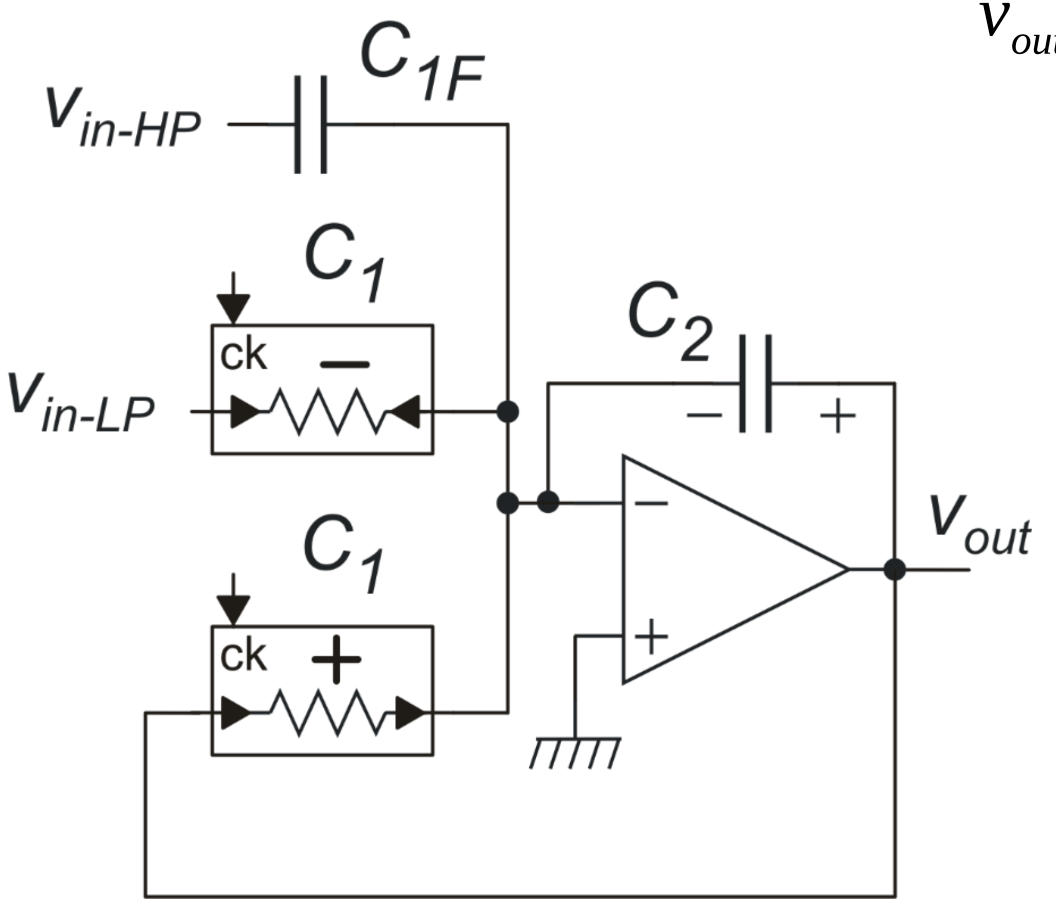
Example 1: versatile integrator





$$\omega_1 = f\frac{C_{12}}{C_2} \qquad k_1 = \frac{C_{11}}{C_{12}} \qquad k_{1F} = \frac{C_{1F}}{C_2}$$

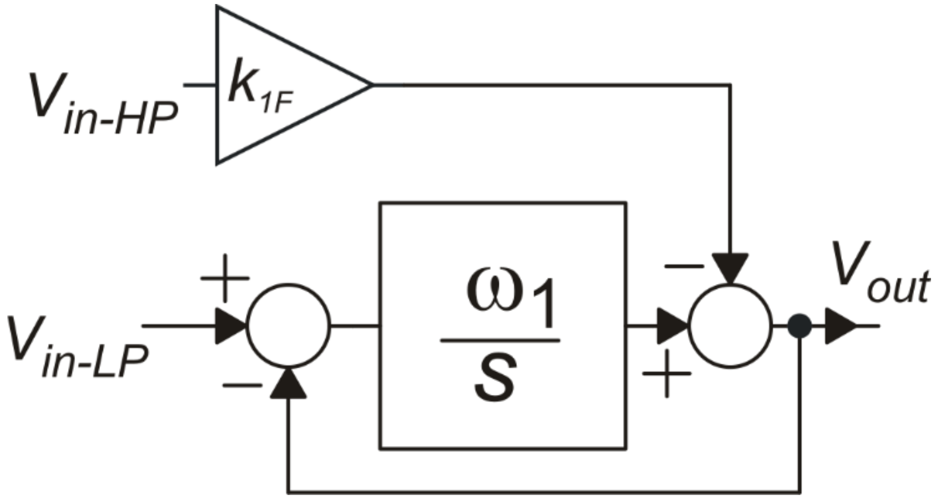
Example 2 First order filter



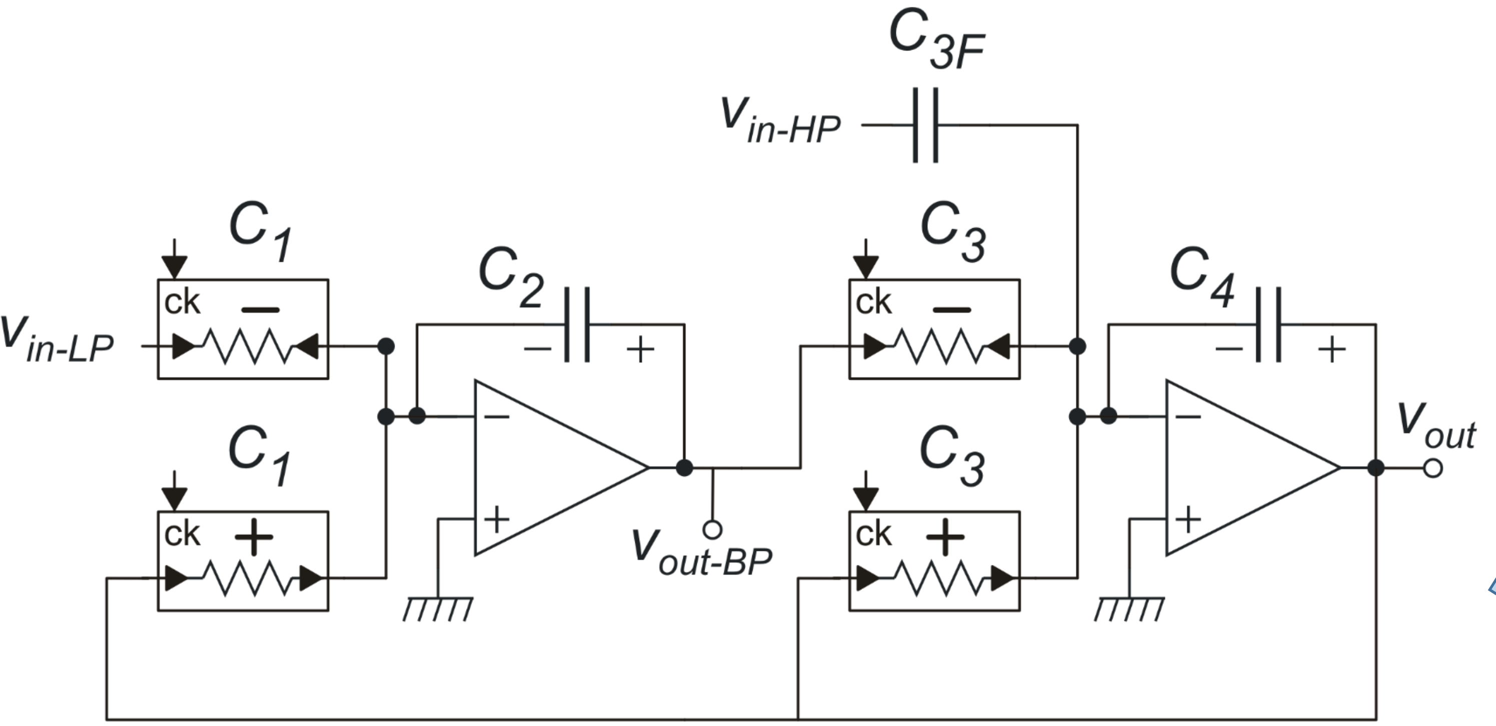
$$v_{out} = \frac{\omega_1}{s+\omega_1} v_{in-LP} - \frac{s k_{1F}}{s+\omega_1} v_{in-HP}$$

$$\omega_1 = \frac{1}{R_{eq} C_2} = f_{ck} \frac{C_1}{C_2}$$

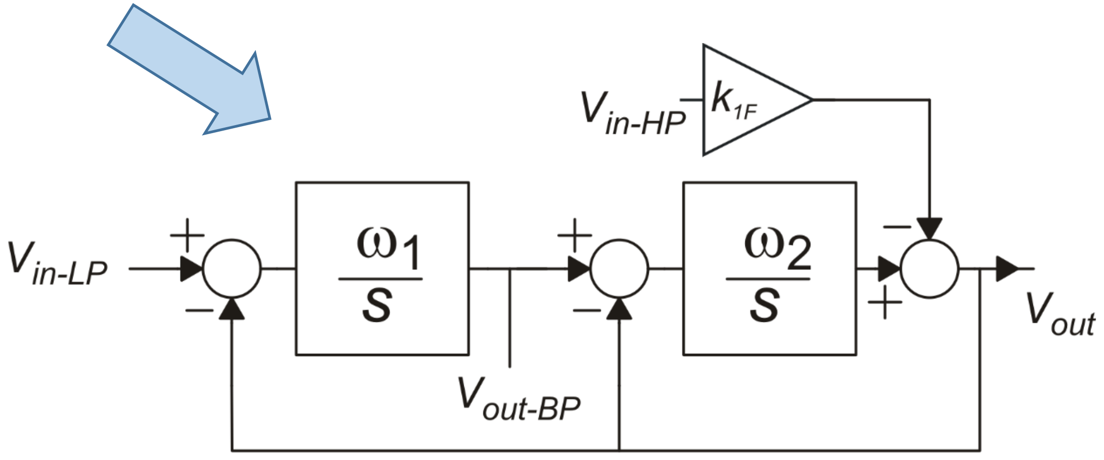
$\cong$



Example 3: Universal SC Biquad Filter

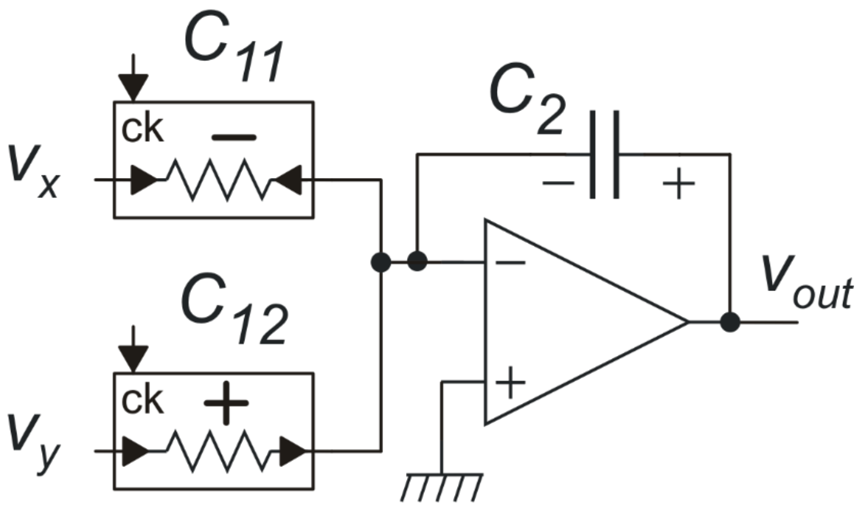


$$\omega_P = \sqrt{\omega_1 \omega_2} \quad Q_P = \sqrt{\frac{\omega_1}{\omega_2}}$$



$$\omega_1 = f_{ck} \frac{C_1}{C_2} \quad \omega_2 = f_{ck} \frac{C_3}{C_4} \quad k_{1F} = \frac{C_{3F}}{C_4}$$

Discrete time nature of SC filters: Integrator



$$V_{out}^{(2)} = V_{out}^{(2P)} + \frac{C_1}{C_2} v_{in}^{(1)}$$

negative res.
non-inverting

$$V_{out}^{(2)} = V_{out}^{(2P)} - \frac{C_1}{C_2} v_{in}^{(2)}$$

positive res.
inverting

$$V_{out}(nT) = V_{out}[(n-1)T] - \frac{C_{12}}{C_2} v_y(nT) + \frac{C_{11}}{C_2} v_x\left(nT - \frac{T}{2}\right)$$

defining

$$v_{xD}(t) \equiv v_x\left(t - \frac{T}{2}\right)$$

$$V_{out}(n) = V_{out}(n-1) - \frac{C_{12}}{C_2} v_y(n) + \frac{C_{11}}{C_2} v_{xD}(n)$$

Discrete time nature of SC filters: Integrator

$$V_{out}(n) = V_{out}(n-1) - \frac{C_{12}}{C_2} v_y(n) + \frac{C_{11}}{C_2} v_{xD}(n)$$

$$V_{out}(z) = z^{-1} V_{out}(z) - \frac{C_{12}}{C_2} V_y(z) + \frac{C_{11}}{C_2} V_{xD}(z)$$

$$V_{out}(z) = \frac{1}{1-z^{-1}} \frac{C_{12}}{C_2} V_y(z) + \frac{1}{1-z^{-1}} \frac{C_{11}}{C_2} V_{xD}(z)$$

$$H(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

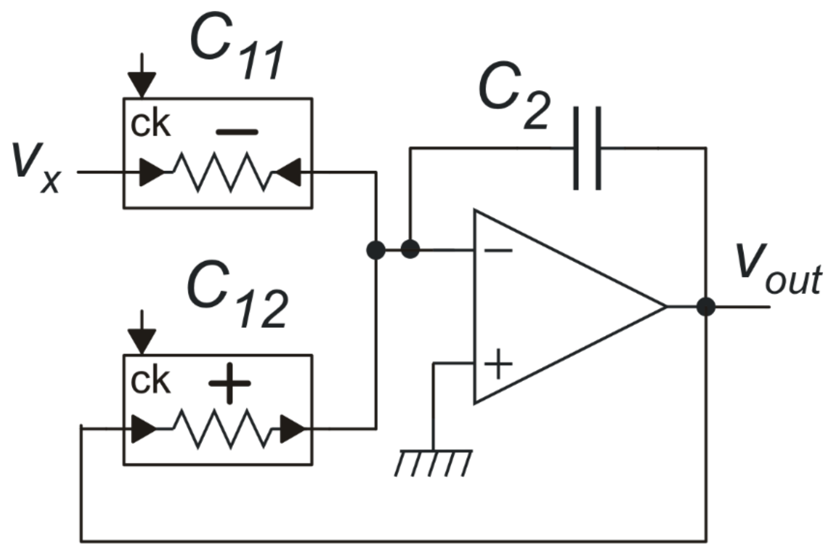
Backward Euler DT integrator
(non-delayed integrator)

Compare with:

$$H(z) = \frac{z^{-1}}{1-z^{-1}} = \frac{1}{z-1}$$

Standard (forward) Euler DT integrator
(delayed form)

First order filter: discrete time nature

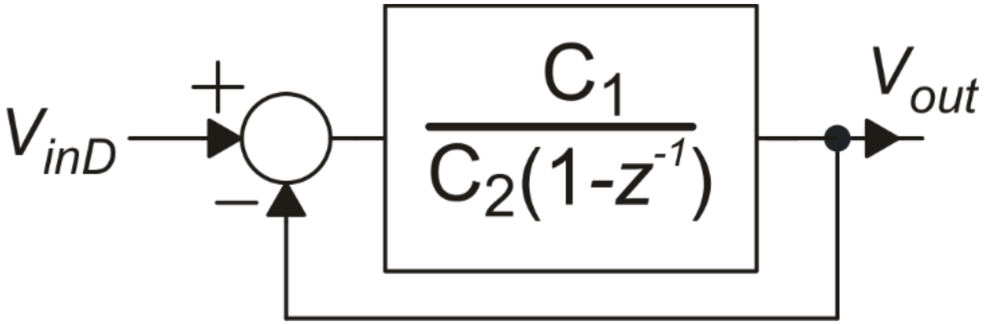


$$V_{out}(z) = \frac{k_C}{1-z^{-1}} v_{inD}(z) - \frac{k_C}{1-z^{-1}} V_{out}(z)$$

$$V_{out}(z) = \frac{k_C}{(1+k_C) - z^{-1}} v_{inD}(z)$$

$$z_p = \frac{1}{1+k_C} \qquad 0 < z_p < 1$$

always stable !



Step response $V_{out}(n) = \left(1 - z_p^n\right) u(n)$

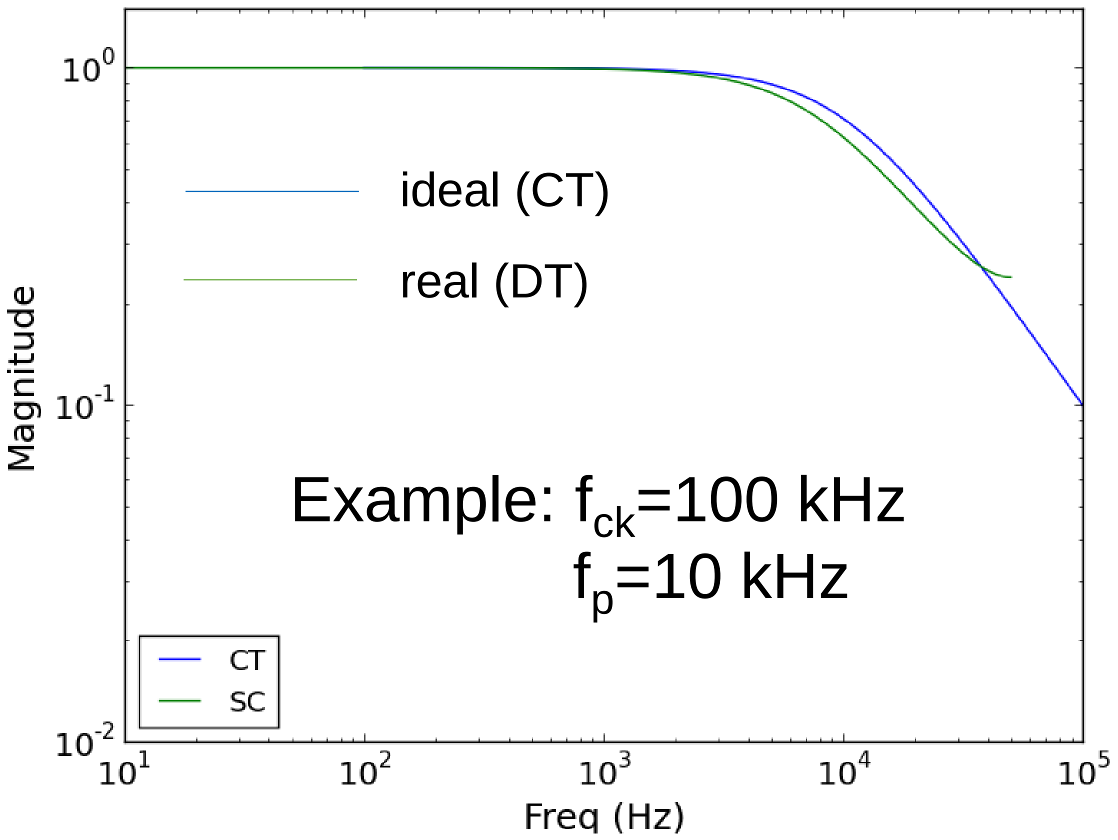
always monotonic !

First order LP filter: frequency response

$$z^{-1} = e^{-j\omega T} \cong 1 - j\omega T$$

$$H(e^{j\omega T}) \cong \frac{k_C}{k_C + j\omega T} e^{-j\omega T/2}$$

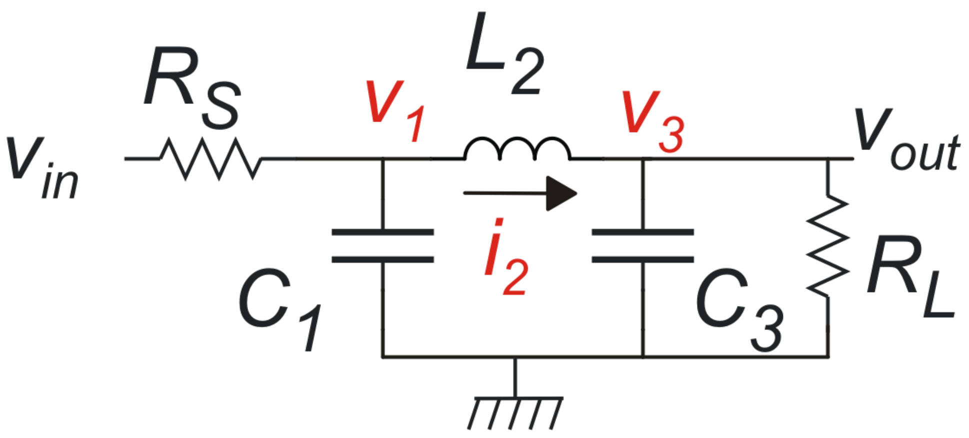
$$\omega_p = \frac{k_C}{T} = f_{ck} \frac{C_1}{C_2}$$



Filter synthesis by means of LC ladder network simulation with SC integrators

- Advantage: low sensitivity with respect to component value variations

Example



$$v_1 = \frac{1}{sC_1}\left(\frac{v_{in} - v_1}{R_S} - i_2\right)$$

$$i_2 = \frac{1}{sL_2}(v_1 - v_3)$$

$$v_{out} \equiv v_3 = \frac{1}{sC_3}\left(i_2 - \frac{v_3}{R_L}\right)$$

In order to obtain an homogeneous variable set, it is convenient to define:

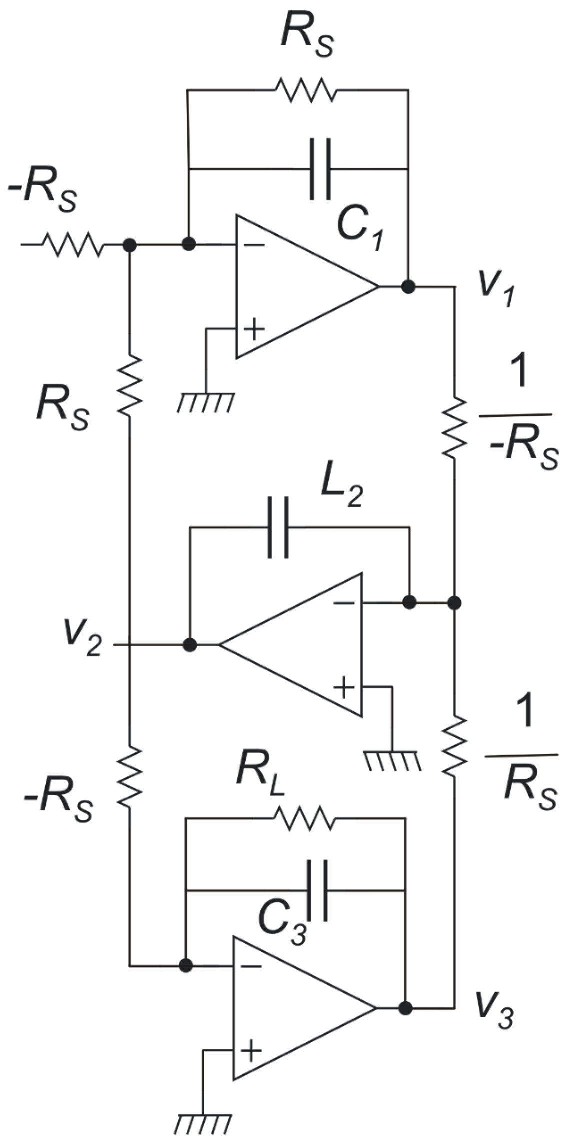
$$v_2 \equiv R_S i_2$$

Example: ladder LC network simulation

$$v_1 = \frac{1}{sC_1R_S}\left(v_{in} - v_1 - v_2\right)$$

$$v_2 = \frac{R_S}{sL_2}\left(v_1 - v_3\right)$$

$$v_{out} \equiv v_3 = \frac{1}{sC_3R_L}\left(v_2\frac{R_L}{R_S} - v_3\right)$$



-) Indicated values represent numerical identities (dimensions are not relevant)

-) Resistors are implemented with either positive or negative parasitic insensitive switched capacitors resistances.

SC Filters that do not require the equivalent resistance approximation

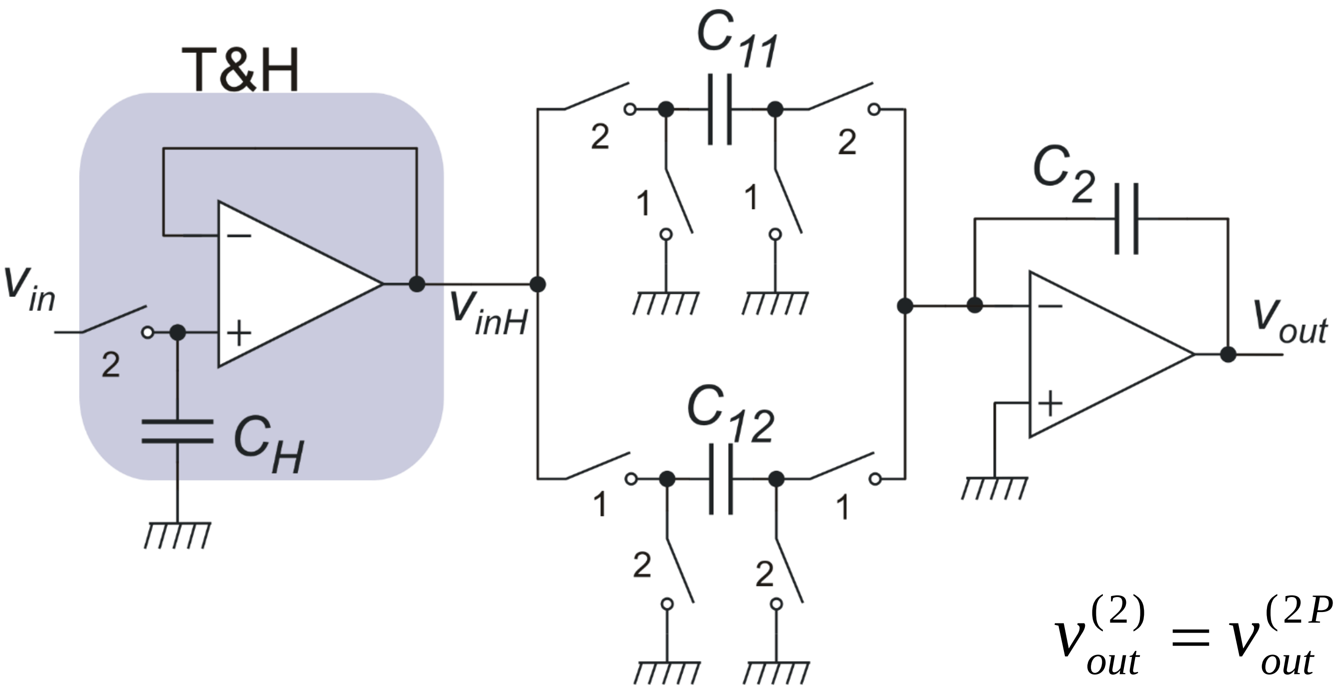
- These filters are obtained by direct implementation of the H(z) transfer function.

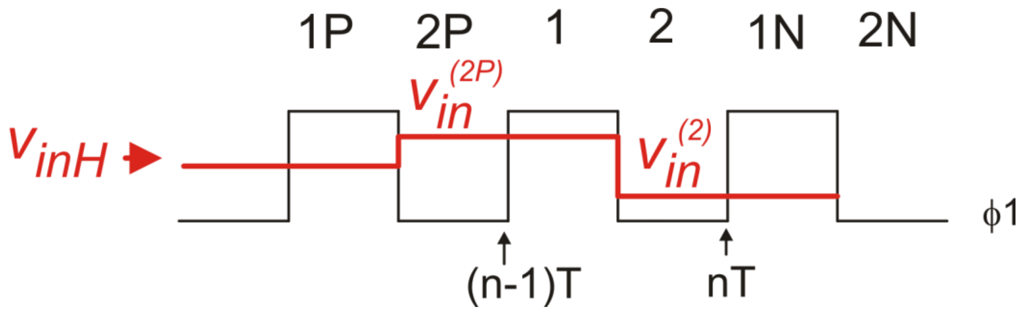
- The H(z) can be obtained by means of:

  -) conversion of a CT transfer function into the DT domain, by substituting *"s"* with a proper rational function of "*z*" (e.g. bilinear transformation);

  -) synthesis with the typical approaches of digital filters (e.g. FIR filters)

Example: synthesis of a bilinear integrator





$$v_{inH}^{(1)} = v_{in}^{(2P)}$$
$$v_{inH}^{(2)} = v_{in}^{(2)}$$

$$v_{out}^{(2)} = v_{out}^{(2P)} - \frac{C_{12}}{C_2} v_{inH}^{(1)} - \frac{C_{11}}{C_2} v_{inH}^{(2)}$$

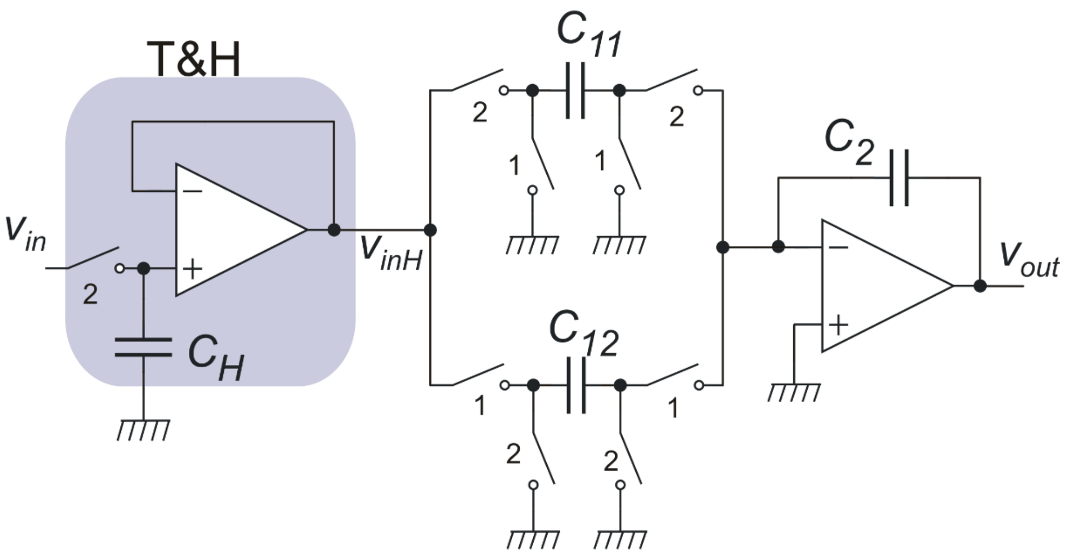
$$v_{out}^{(1)} = v_{out}^{(2P)} - \frac{C_{12}}{C_2} v_{inH}^{(1)}$$

$$v_{out}^{(2)} = v_{out}^{(1)} - \frac{C_{11}}{C_2} v_{inH}^{(2)}$$

$$v_{out}^{(2)} = v_{out}^{(2P)} - \left(\frac{C_{12}}{C_2} v_{in}^{(2P)} + \frac{C_{11}}{C_2} v_{inH}^{(2)} \right)$$

Example: synthesis of a bilinear integrator

$$v_{out}^{(2)} = v_{out}^{(2P)} - \left(\frac{C_{12}}{C_2} v_{in}^{(2P)} + \frac{C_{11}}{C_2} v_{in}^{(2)} \right)$$



$$H(z) = -\frac{\dfrac{C_{11}}{C_2} + \dfrac{C_{12}}{C_2} z^{-1}}{1 - z^{-1}}$$

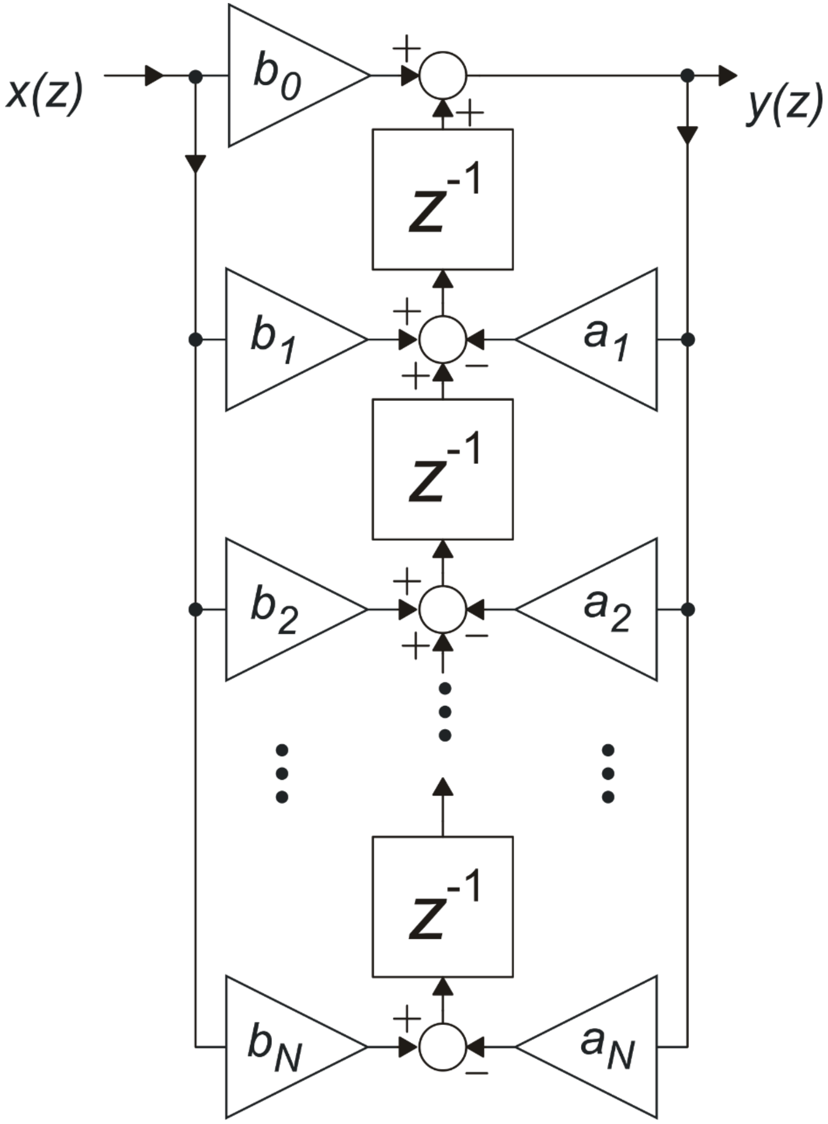
bilinear integrator
(inverting)

for: $C_{11} = C_{12}$

$$H(z) = -\frac{C_{11}}{C_2} \frac{1 + z^{-1}}{1 - z^{-1}}$$

$$= -\omega_0 \frac{T}{2} \frac{1 + z^{-1}}{1 - z^{-1}} \Leftrightarrow -\frac{\omega_0}{s}$$

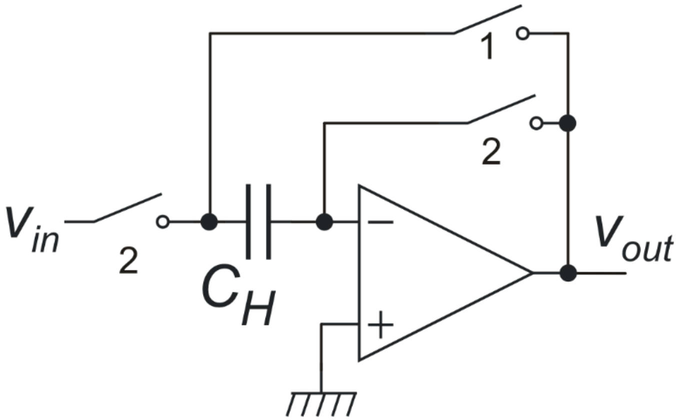
Direct synthesis



$$H(z)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\ldots+b_Mz^{-M}}{1+a_1z^{-1}+a_2z^{-2}+\ldots.+a_Nz^{N}}$$

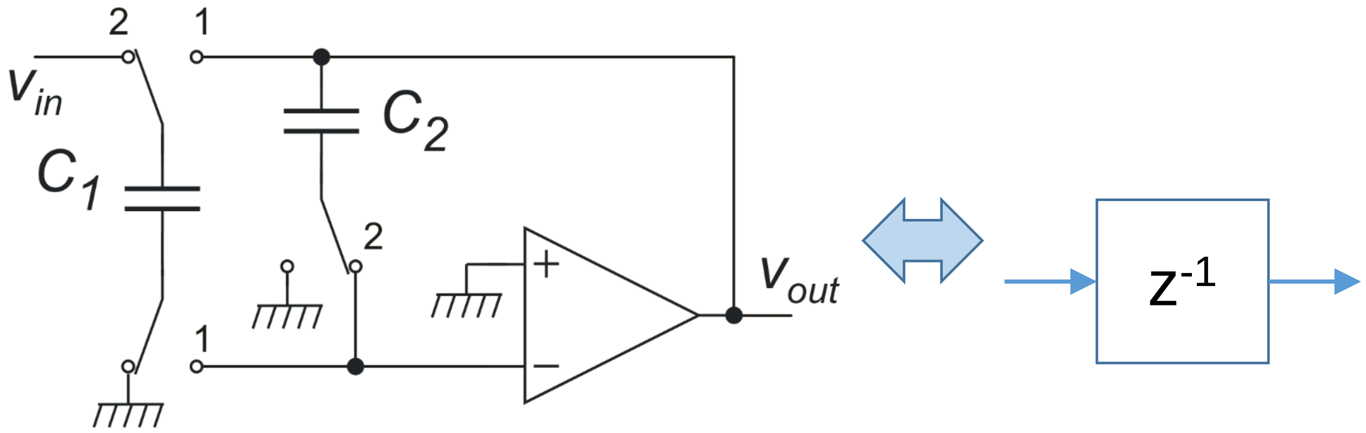
Delay lines and adders (summing amplifiers) are necessary

Direct synthesis: analog DT delay lines

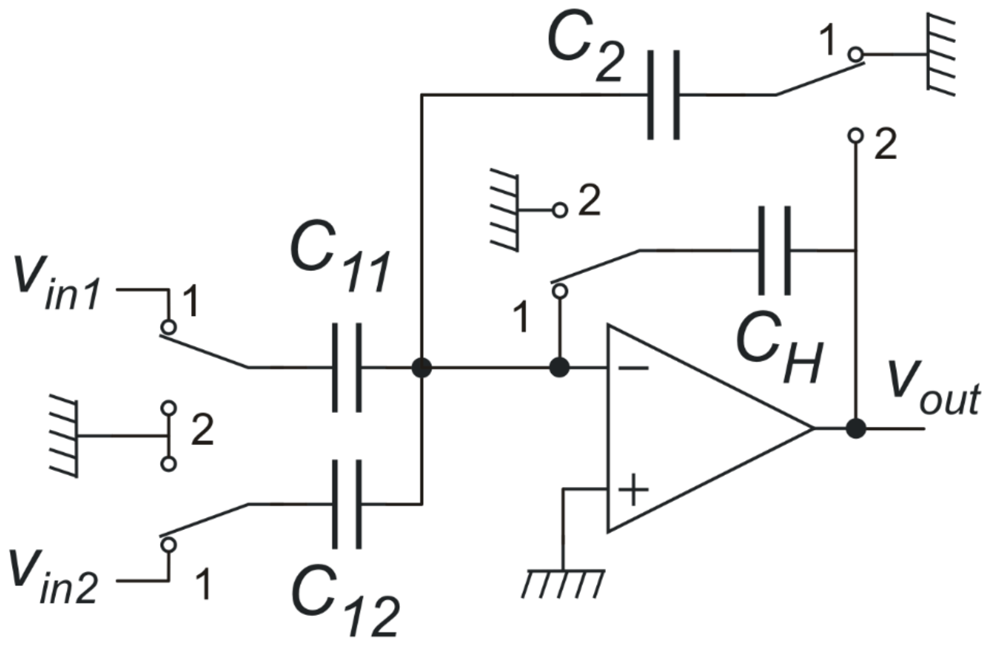


Half-period delay line
with return to zero in phase 2

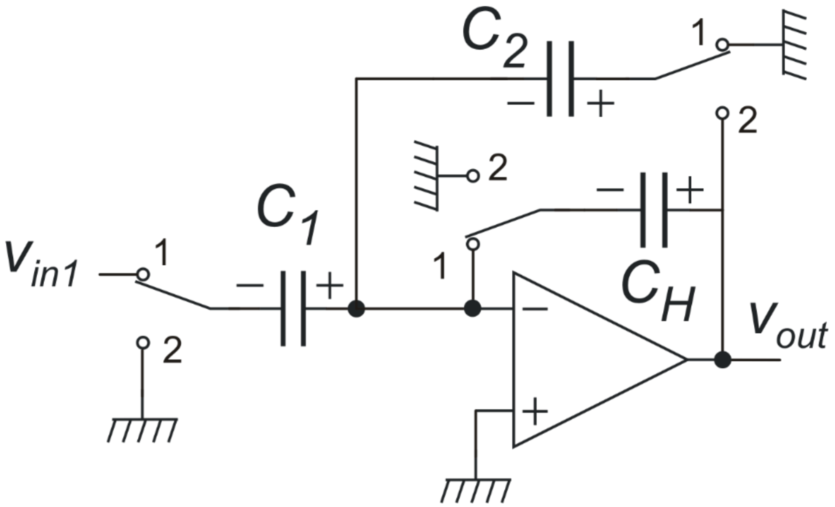
Sample and Hold
(zero tracking time)



Summing amplifier

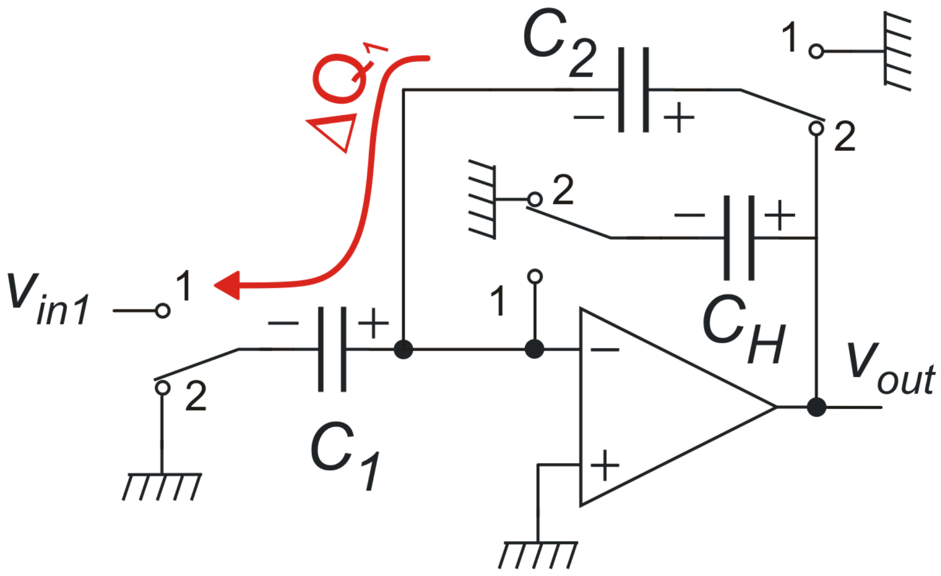


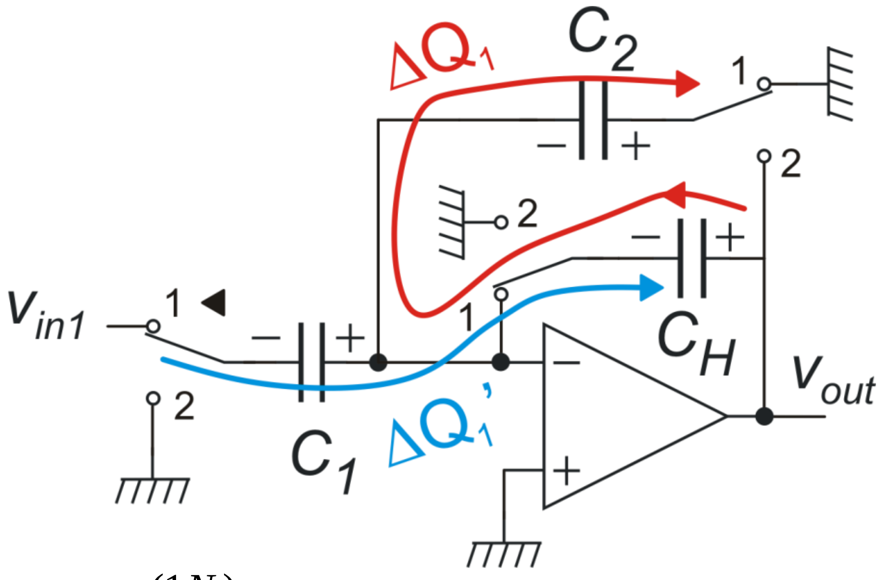
Simplified case: single input



Phase 1

Summing amplifier: analysis





$$\Delta Q_1 = C_1 v_{in}^{(1)}$$

$$v_{out}^{(2)} = \frac{C_1}{C_2} v_{in}^{(1)}$$

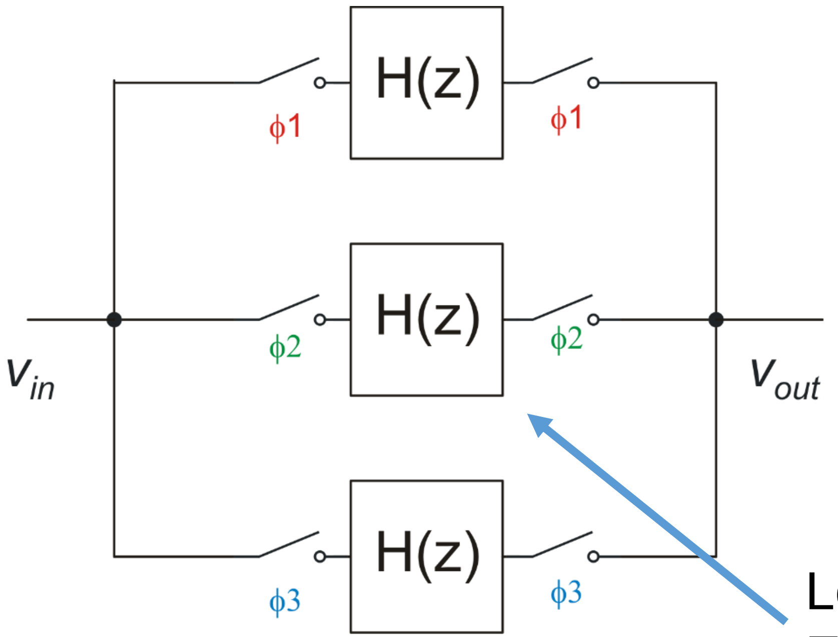
$$\Delta Q_1' = C_1 v_{in}^{(1N)}$$

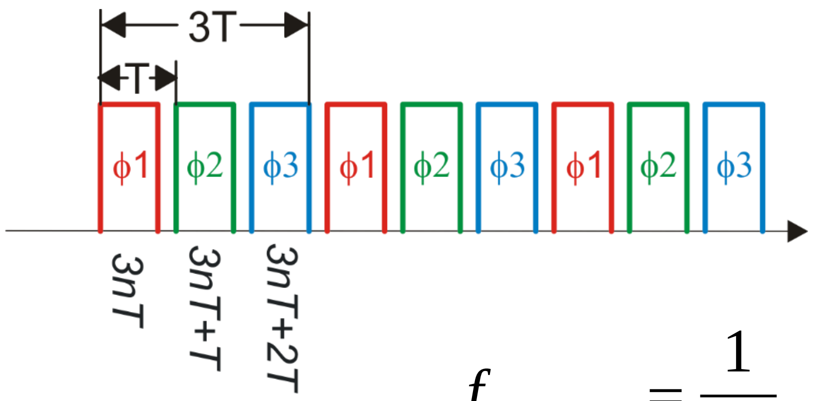
$$v_{out}^{(1N)} = v_{out}^{(2)} + \frac{C_1}{C_H}\left(v_{in}^{(1)} - v_{in}^{(1N)}\right)$$

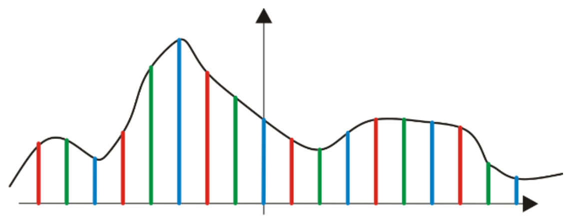
If v_{in} does not change much across a period, the output voltage is maintained in phase 1

Multipath filters

- The target is obtaining a band-pass filter with a very narrow band (i.e. an high $Q=B/f_0$) i.e. a very selective filters.
- Synthesis of very selective Band-Pass filters by means of traditional techniques is very difficult due to component inaccuracy and active element non-idealities (e.g. amplifier gain)
- Multipath filters uses N low pass filters (in this example N=3) fed with decimated sample sequences, in order to explicitly produce aliasing.



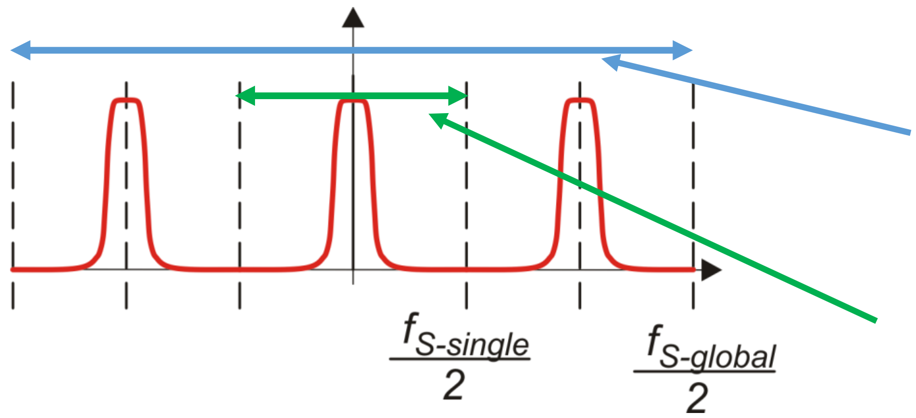




$$f_{S-single} = \frac{1}{3T} \qquad f_{S-global} = \frac{1}{T} = 3\,f_{S-single}$$

Low pass filters (nominally identical).
Each filter receive samples at frequemcy $f_{S\text{-}single}=f_{S\text{-}global}/3$

Multipath Filters



- Due to aliasing, the low pass response is duplicated around $f_{S\text{-}single}$.
- This would be meaningless for a single filter, since signals around $f_{S\text{-}single}$ are beyond the Nyquist limit
- Using all the three filters together with delayed phases is equivalent to sampling at $f_{S\text{-}global.}$ Now, signals at $f_{S\text{-}single}$ are within the Nyquist limit
- The replica of the response around $f_{S\text{-}single}$ can be made very narrow, by simply reducing the bandwidth of the individual low pass filters.