Analog Filter Design

Part. 3: Time Continuous Filter Implementation
Active Filters

Motivations

- Inductors are generally difficult to miniaturize
  L ~ (coil area) x (number of coils)^2 x (magnetic permeability)
  Integrated inductors limited to a few nH (max)
  Stray magnetic field cause unwanted coupling
- Resistors and capacitors can be easily integrated: feasible ranges are much wider than for inductors
- Active Filters Target: Synthesis of arbitrary transfer functions using only resistors, capacitors and active elements.

Design approaches for active filters

- Cascade of Biquadratic (Biquad) and Bilinear cells
- State Variable Filters (MLF: Multiple Loop Feedback circuits)
- Simulation of LC filters with active RC networks

Cascade of Biquad (Bilinear) functions

- Biquad Transfer Function

$$H_{BQ}(f) = \frac{c_2 s^2 + c_1 s + c_0}{s^2 + d_1 s + d_0} = H_0 \frac{b_2 s^2 + b_1 \frac{\omega_z}{Q_z} s + b_0 \omega_z^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

b_2, b_0 : 0, 1
b_1: 0, ±1

- Bilinear Transfer Function

$$H_{BL}(f) = H_0 \frac{b_1 s + b_0 \omega_z}{s + \omega_p}$$

b_1 : 0, 1
b_0: 0, ±1

“Bits” b_2, b_1, b_0 determine which terms are present in the numerator

Poles and Zeroes in a Biquad

$$H_{BQ}(f) = H_0 \frac{s^2 + b_1 \frac{\omega_z}{Q_z} s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2} = \frac{(s - s_z)(s - s_z^*)}{(s - s_p)(s - s_p^*)}$$

case $b_2, b_0\ b_1 \neq 0$

$$\begin{cases} \omega_z = |s_z| & \frac{1}{2Q} = -b_1 \frac{\mathrm{Re}(s_z)}{|s_z|} = -b_1 \cos(\theta_z) \\ \omega_p = |s_p| & \frac{1}{2Q} = -\frac{\mathrm{Re}(s_p)}{|s_p|} = -\cos(\theta_p) \end{cases}$$

Notable cases

$$\frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Low pass

$$\frac{s^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

High pass

$$\frac{\frac{\omega_p}{Q_p} s}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Band pass

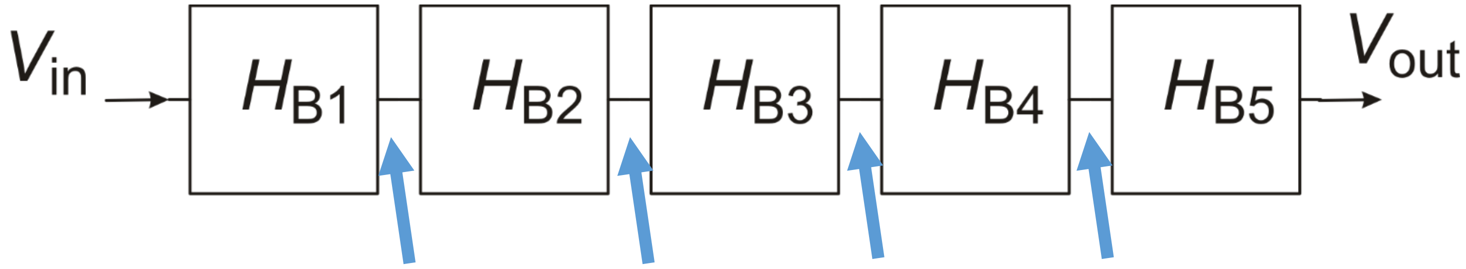
$$\frac{s^2 + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Band Stop

$$\frac{s^2 - \frac{\omega_p}{Q_p} s + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

All pass
(phase equalizer)

Sequencing criteria for biquad cascades



Requirement: $Z_{out} << Z_{in}$

Degrees of Freedom:

- Poles – Zeroes pairing (when zeroes are present)
- Physical position of each biquad in the cascade
- Pass-Band gain of each individual element of the cascade

Sequencing criteria: Rules of Thumb

Targets
- Maximize the Dynamic Range (DR)
- Minimize the transmission sensitivity (to component variations)
- Minimize the pass-band attenuation
- Simplify the tuning procedure

- Pairing: couple together poles and zeroes which are closer in the s-plane (flatter response, less component spread)
- Position: Place the biquads with lower Q closer to the inputs
  Keep biquads with similar frequency of maximum as far away as possible
  If possible, place LP Biquads first and HP or BP Biquads last
- Gain distribution: balance the signal amplitude over the various biquads

Biquad implementations

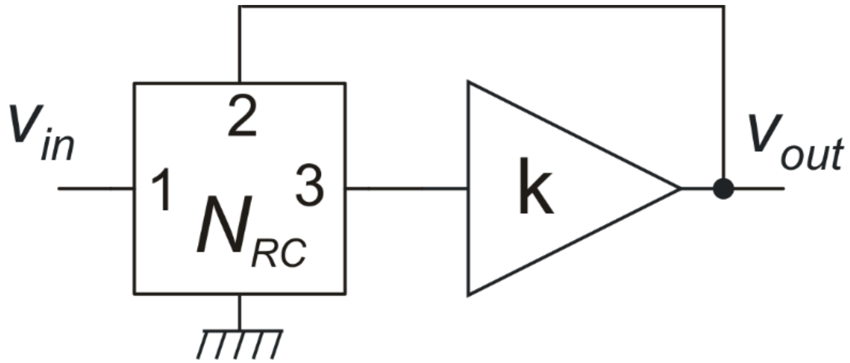
- Op-amp Based:

- SAB (Single Opamp Biquad)
  - **Finite Gain SABs – positive feedback**
  - Finite Gain SABs – negative feedback
  - **Infinite Gain SABs**
- Multiple op-amp Biquads (e.g. MFL)

- OTA based (Gm-C filters)

SABs

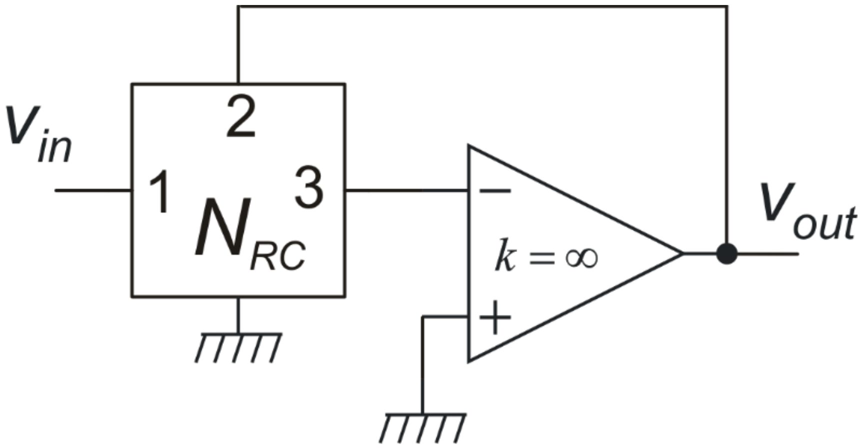
Finite gain



$$\begin{cases} I_3 = y_{13}V_1 + y_{23}V_2 + y_{33}V_3 = 0 \\ V_3 = V_2 / k \end{cases}$$

$$\frac{V_{out}}{V_{in}} = \frac{V_2}{V_1} = \frac{-y_{13}}{y_{23} + y_{33}/k}$$

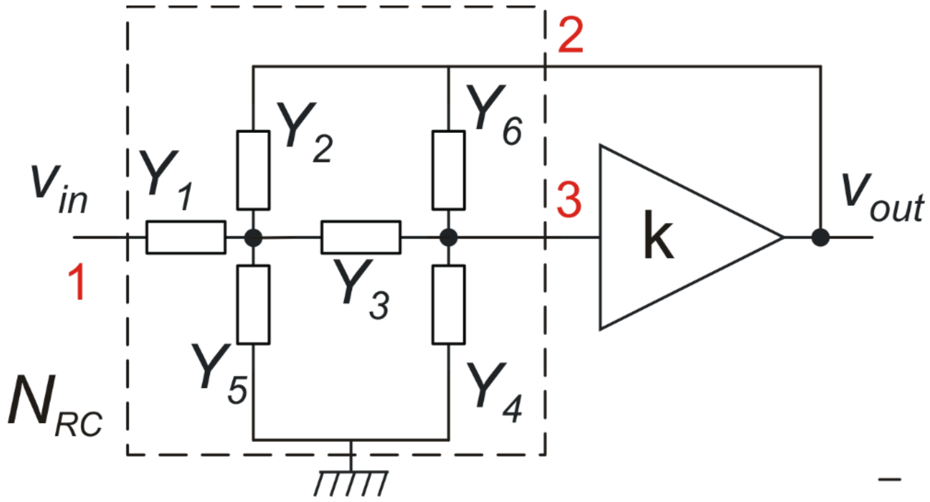
Infinite gain



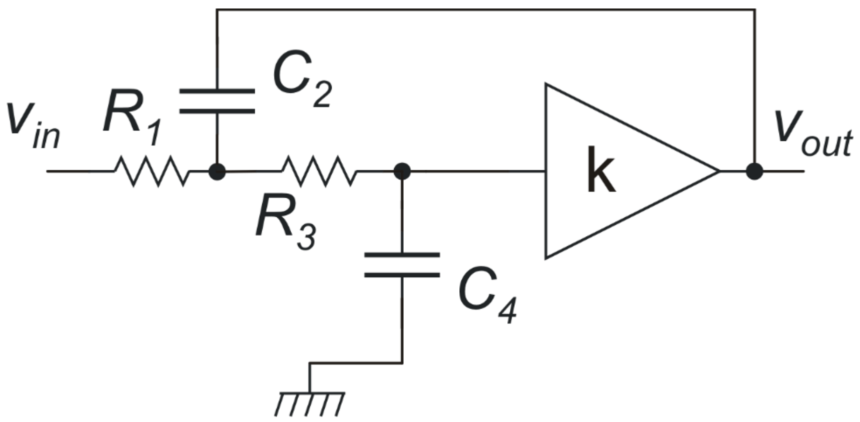
$$I_3 = y_{13}V_1 + y_{23}V_2 = 0$$

$$\frac{V_{out}}{V_{in}} = \frac{V_2}{V_1} = \frac{-y_{13}}{y_{23}}$$

Example: Sallen-Key Biquads



SK General configuration



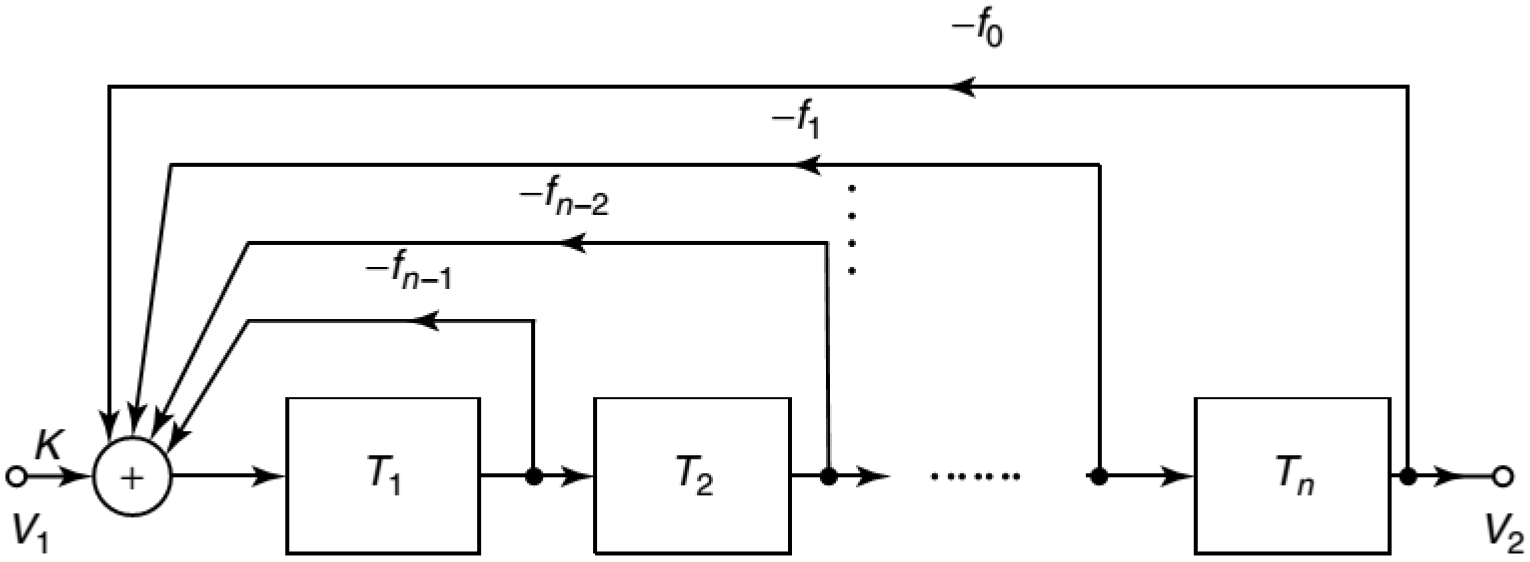
SK- Low pass filter

$$\frac{V_{out}}{V_{in}} = \frac{KY_1Y_3}{(Y_1+Y_2+Y_5)(Y_3+Y_4+Y_6)+Y_3(Y_4+Y_6)-K\{Y_6(Y_1+Y_2+Y_3+Y_5)+Y_2Y_3\}}$$

Multiple Feedback Loop Filters

- Cascaded Biquads: Feedback exist only inside blocks
- MFL Filters: Feedback involve all stages together

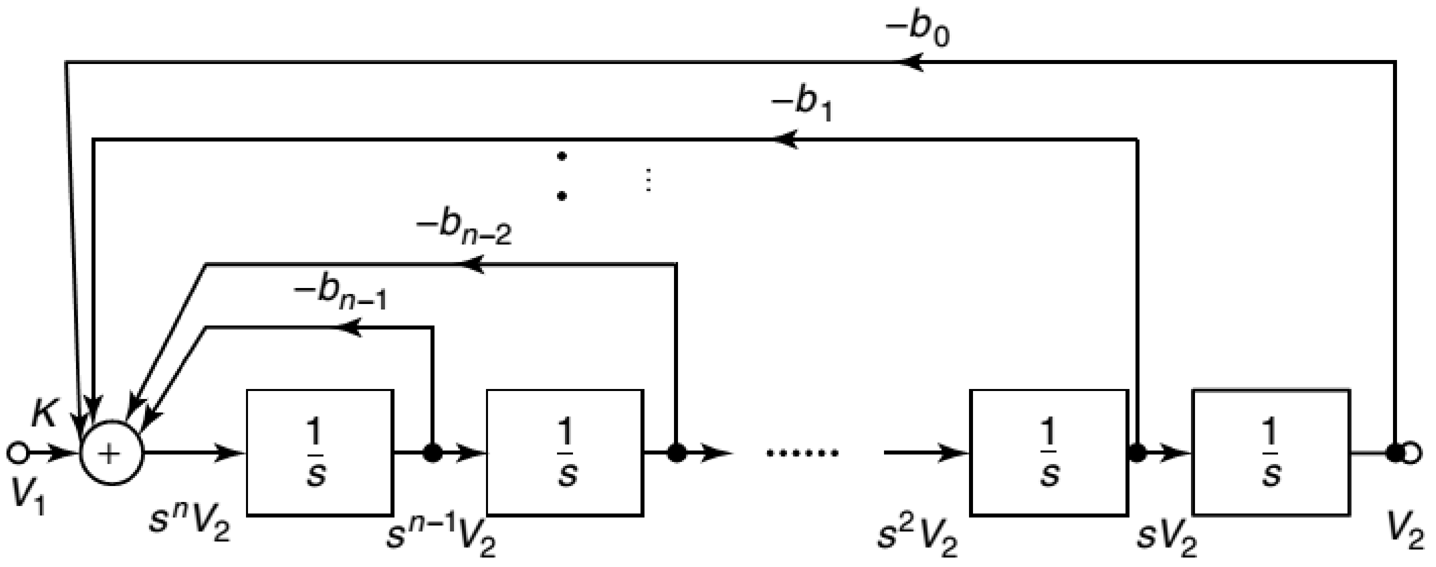
⇒ More Interaction: less sensitivity to component variations



$T_i(s)$ can be:
- Integrators
- Lossy Integrators
- Biquads

“Follow the Leader Filter” (FLF) architecture”

State variable filters
(MFL filters based on Integrators)

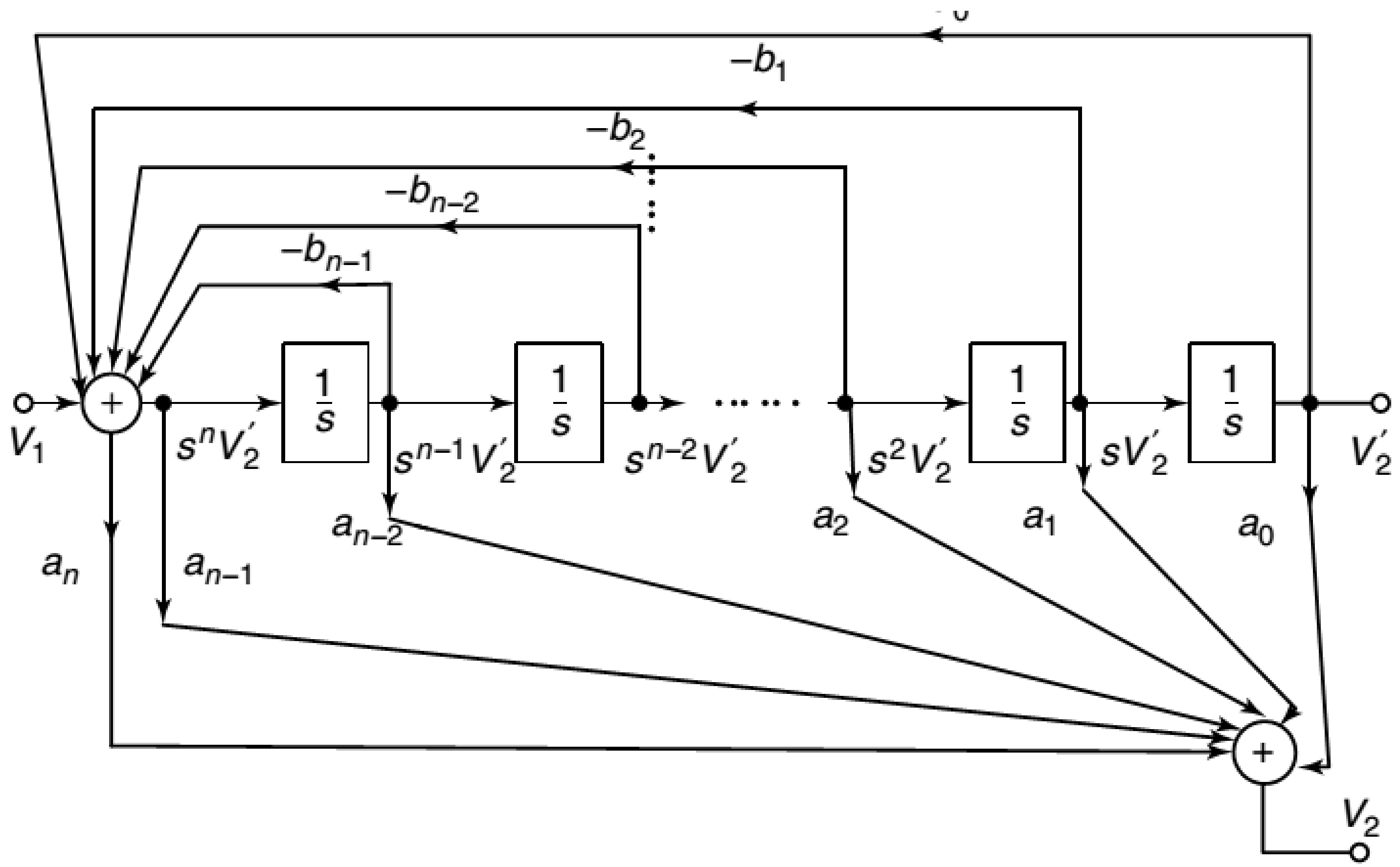


$$V_2 = V_1\left(\frac{1}{s}\right)^n - b_0\left(\frac{1}{s}\right)^n V_2 - b_1\left(\frac{1}{s}\right)^{n-1} V_2 \;\ldots\ldots - b_{n-1}\left(\frac{1}{s}\right)$$

$$\frac{V_2}{V_1} = \frac{K}{s^n + b_{n-1}s^{n-1} \ldots. + b_1 s + b_0}$$

➡ Low pass, all poles filters

Multi Feedback – Multi Feed Forward

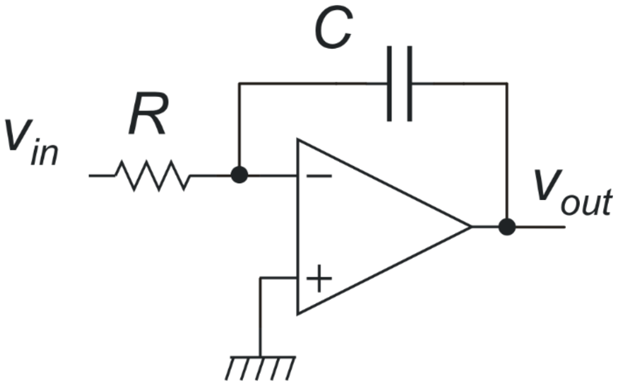


$$\frac{V_2}{V_1} = \frac{a_n s^n + a_{n-1} s^{n-1} \dots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} \dots + b_1 s + b_0}$$

Arbitrary Functions
(poles and zeros)

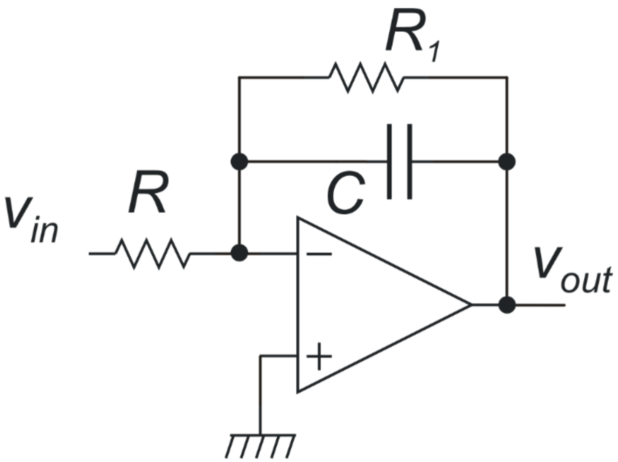
Integrators: Op-amp based solution

Integrator (inverting)



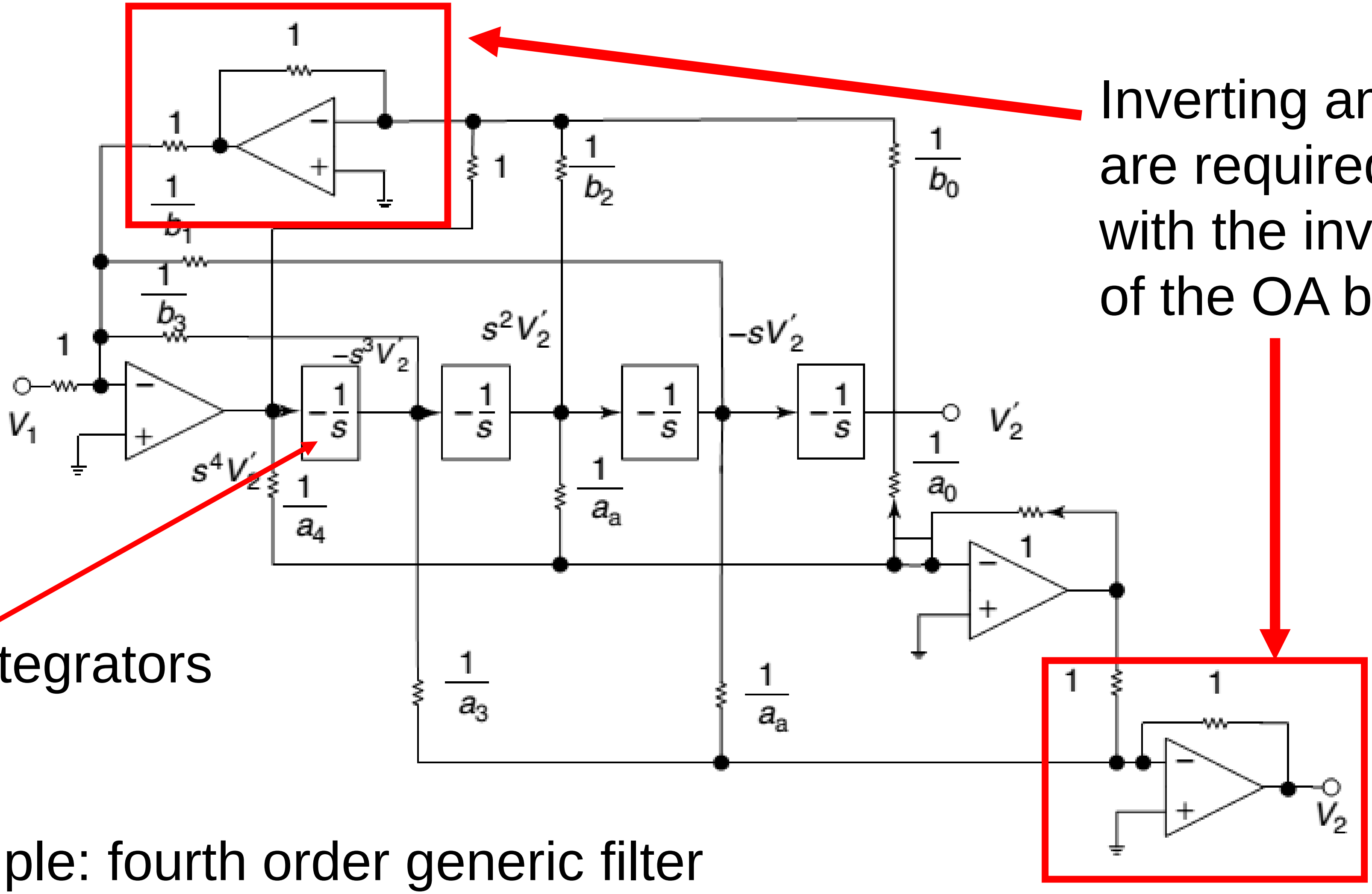
$$\frac{v_{out}}{v_{in}} = -\frac{1}{RC}\frac{1}{s}$$

Lossy Integrator (inverting)



$$\frac{v_{out}}{v_{in}} = -\frac{R_1}{R}\left(\frac{\frac{1}{R_1 C}}{s+\frac{1}{R_1 C}}\right)$$

Example: State variable Filter with Op-amp Integrators

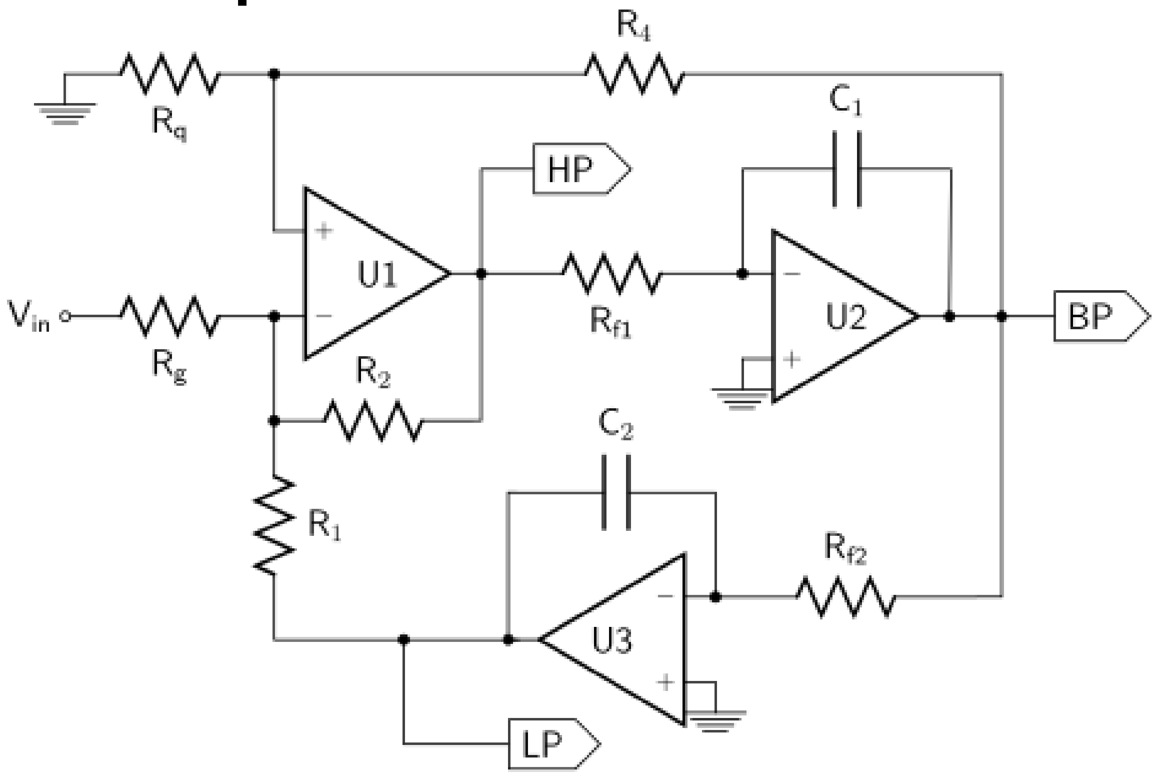


Inverting amplifiers are required to cope with the inverting function of the OA based Integrators

Opamp integrators (inverting)

Example: fourth order generic filter

MLF: Example Universal 2nd order Filter

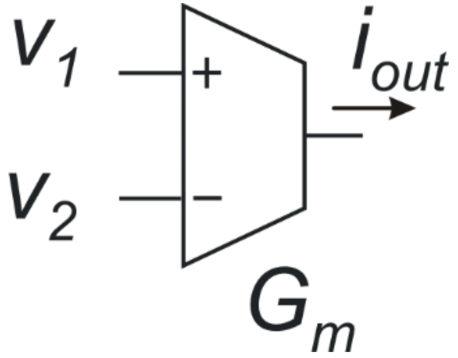


Kerwin-Huelsman-Newcomb (KHN) filter

(Produces LP, BP and HP outputs: Single Input – Multiple Output)

OTA: definitions and basic circuits

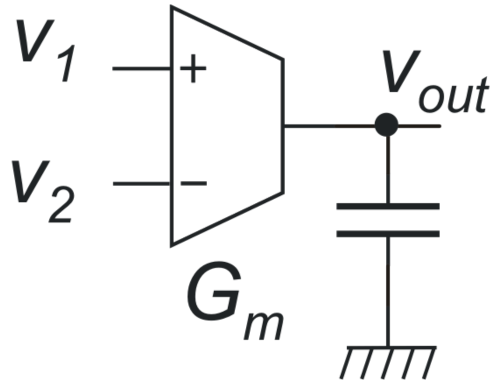
OTA (Transconductor)



Ideal operation

$$i_{out} = G_m(v_1 - v_2)$$

OTA-C (Gm-C) Integrator

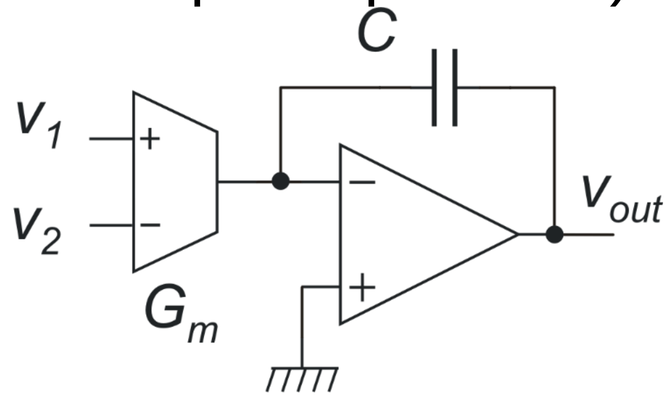


$$v_{out} = \frac{i_{out}}{sC} = \frac{G_m}{sC}(v_1 - v_2)$$

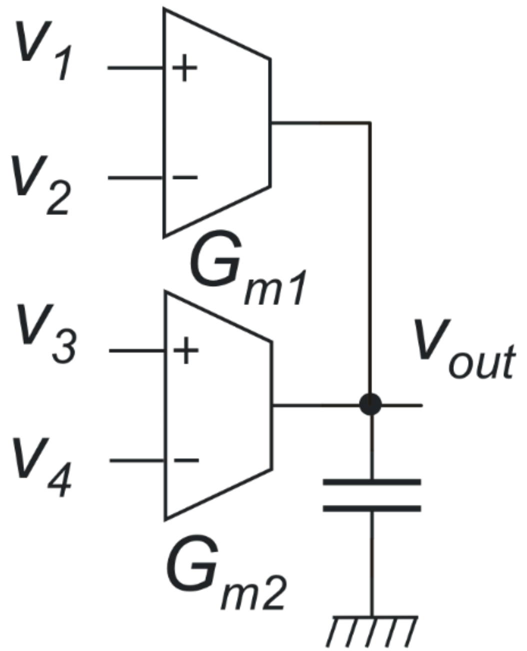
Typical non-idealities:

- Finite Rout
- Input Capacitance
- Frequency dependence of Gm
- Input/Output ranges

(Gm-Op-Amp) Integrator
(Lower output impedance)

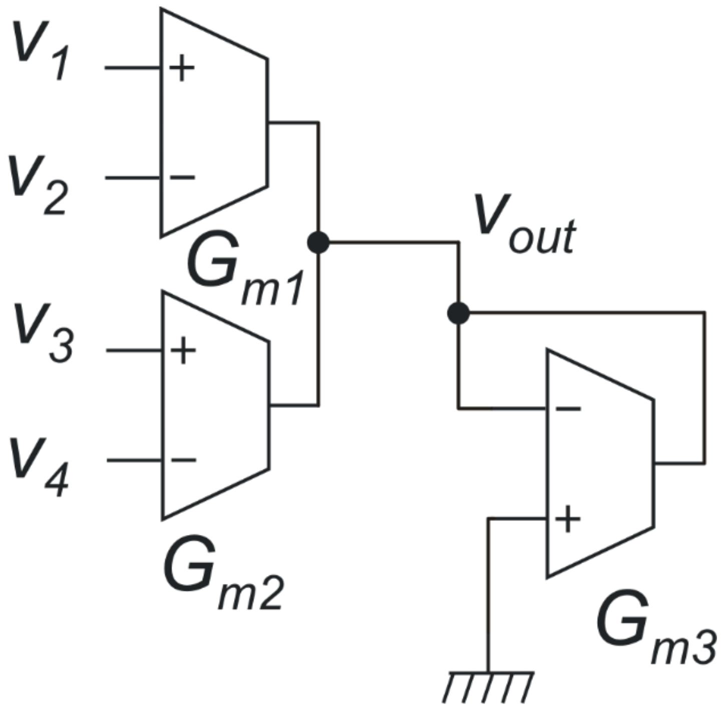


Ota-Based summing circuits



Summing Integrator
(inverting / non-inverting)

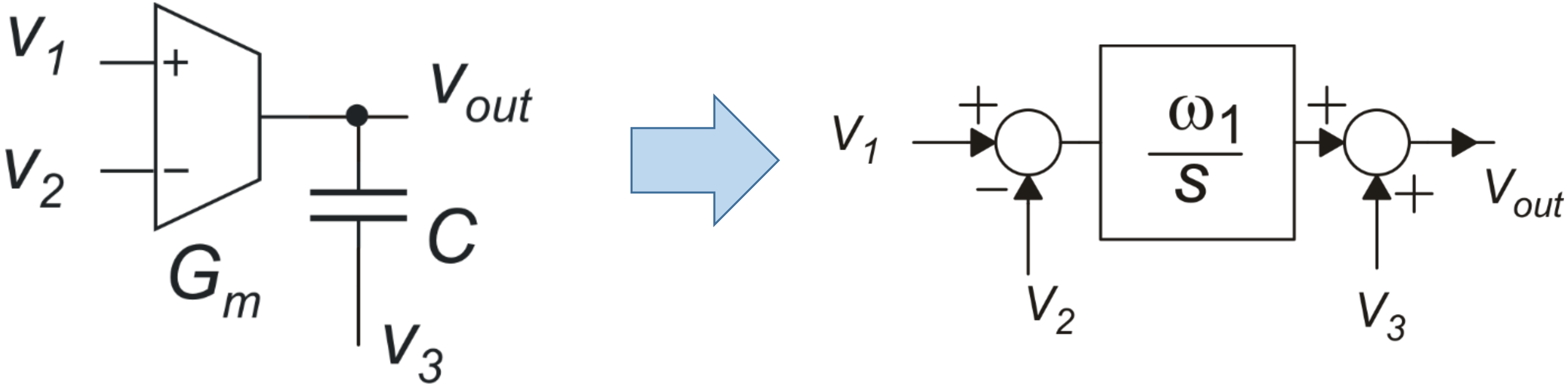
$$v_{out} = \frac{G_{m1}}{sC}\left[(v_1 - v_2) + \frac{G_{m2}}{G_{m1}}(v_3 - v_4)\right]$$



Summing amplifier
(inverting / non-inverting)

$$v_{out} = \frac{G_{m1}}{G_{m3}}(v_1 - v_2) + \frac{G_{m2}}{G_{m3}}(v_3 - v_4)$$

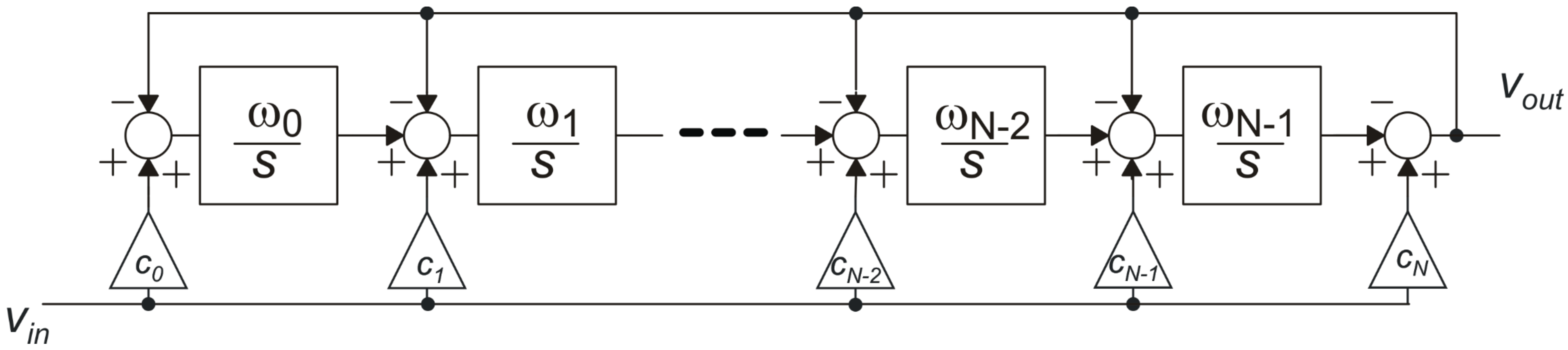
Gm-C integrator with feed-forward input



$$v_{out} = \frac{G_m}{sC}\left(v_1 - v_2\right) + v_3$$

$$\omega_1 = \frac{G_m}{C}$$

State variable filters – alternative solution



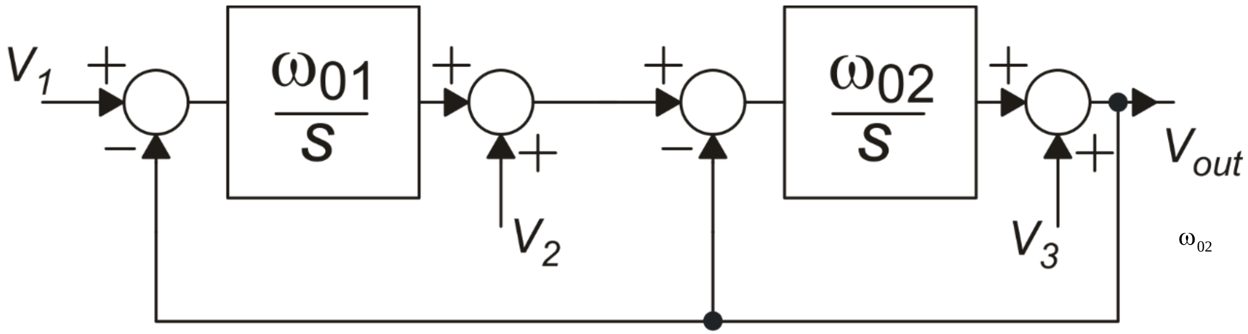
Differently from the FL filter (Follow the Leader) all the integrator outputs are fed back to the first integrator and fed forward to the summing node, here the output voltage is fed back to the input of each integrator where also input signal is fed forward. This architecture is more suitable for Gm-C implementations, where summing several inputs would require several OTAs

$$\frac{V_2}{V_1} = \frac{s^n + a_{n-1}s^{n-1} \ldots + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} \ldots + b_1 s + b_0}$$

$$b_{N-1} = \omega_{N-1} \quad b_{N-2} = \omega_{N-1}\omega_{N-2} \quad b_0 = \omega_{N-1}\omega_{N-2}\ldots\omega_0$$

$$a_N = c_N \quad a_{N-1} = c_{N-1}b_{N-1} \quad a_{N-2} = c_{N-2}b_{N-2} \quad a_0 = c_0 b_0$$

Example: State variable Gm-C biquad



$$V_{out}(s) = \frac{v_3 s^2 + v_2 \frac{\omega_z}{Q_z} s + v_1 \omega_z^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

$$v_1 = B_0 v_{in}$$

$$v_2 = B_1 v_{in}$$

$$v_3 = B_2 v_{in}$$

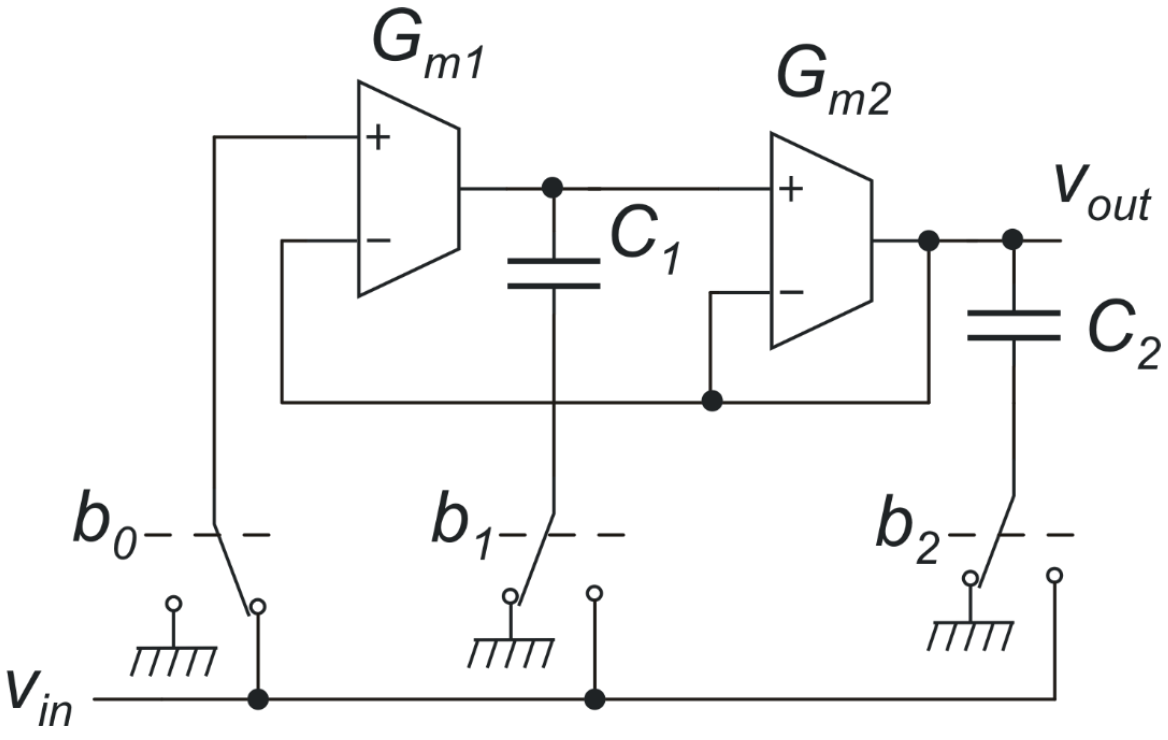
$$B_0, B_1, B_2 = \{0, 1\}$$

Flexible Biquad

$$\omega_p = \omega_z = \sqrt{\omega_{01}\omega_{02}}$$

$$Q_p = Q_z = \sqrt{\frac{\omega_{01}}{\omega_{02}}}$$

State variable Flexible Biquad – OTA implementation



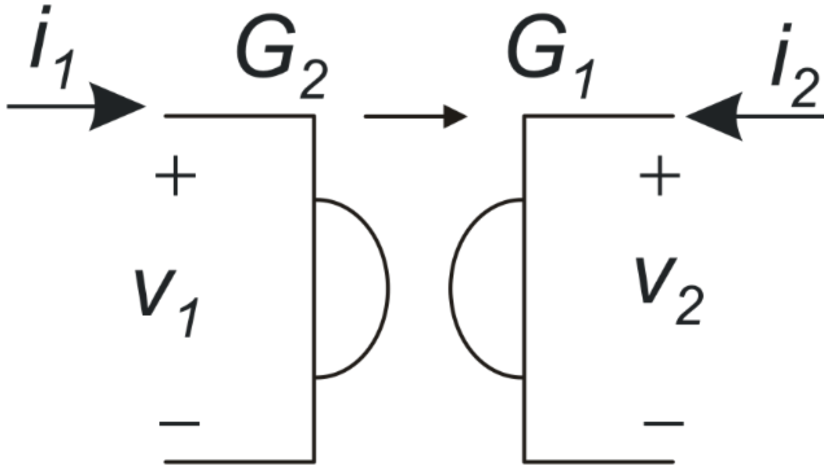
$$\omega_{01} = \frac{G_{m1}}{C_1} \qquad \omega_{02} = \frac{G_{m2}}{C_2}$$

Function	b0	b1	b2
Low pass	1	0	0
High pass	0	0	1
Band-Pass	0	1	0
Notch	1	0	1

Simulation of Ladder Filters with OTAs

- **Simulation of the inductor: application of the OTA based Gyrator**
- **Simulation of the nodal equations by means of OTAs (signal flow path)** May require inductor simulation, depending on the transfer function to synthesize and/or architecture

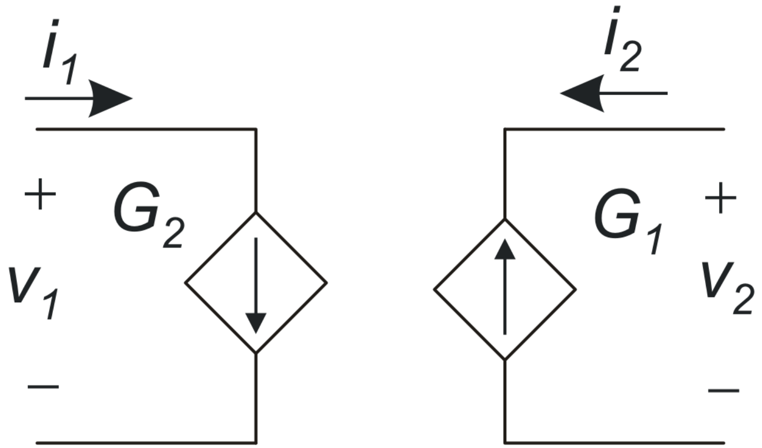
Gyrator



$$Y = \begin{pmatrix} 0 & G_2 \\ -G_1 & 0 \end{pmatrix}$$

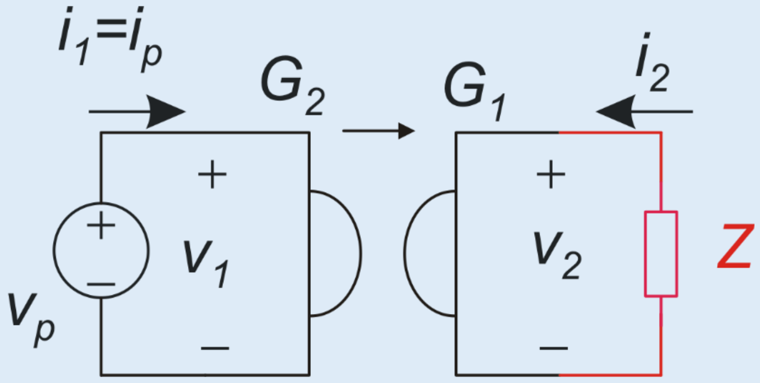
$$I_2 = -G_1 V_1$$

$$I_1 = G_2 V_2$$



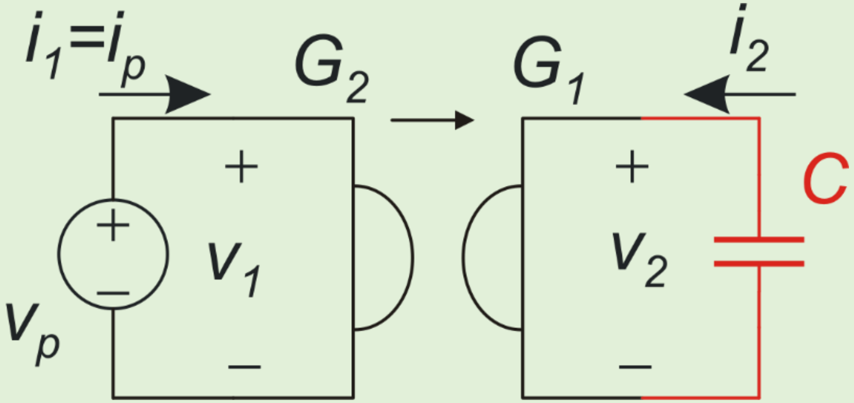
Y-parameters equivalent circuit

Inductance simulation by means of a gyrator and a capacitor.



$$Z_V = \frac{V_P}{I_P} = \frac{V_P}{G_2 G_1 V_P Z} = \frac{1}{G_1 G_2 Z}$$

Generic Impedance Inversion

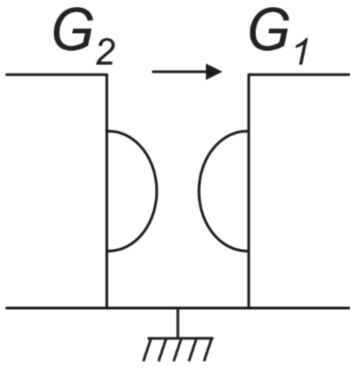


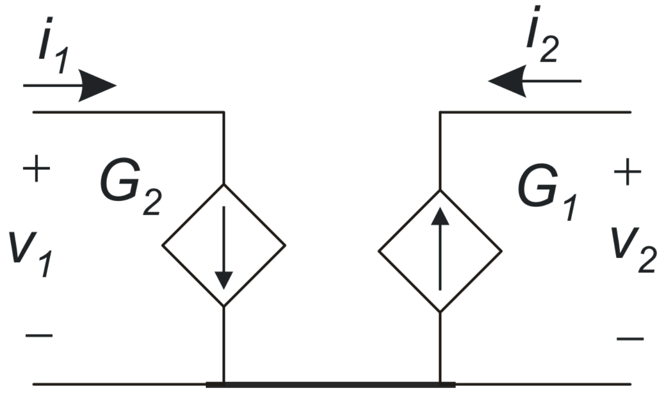
$$Z = \frac{1}{Cs} \Rightarrow Z_V = \frac{Cs}{G_1 G_2}$$

$$L_{EQ} = \frac{C}{G_1 G_2}$$

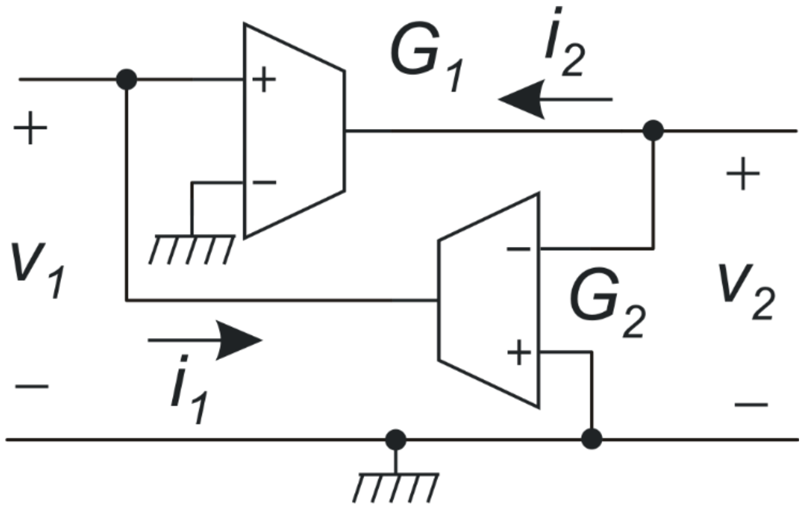
Inductor Synthesis

OTA Based Gyrators





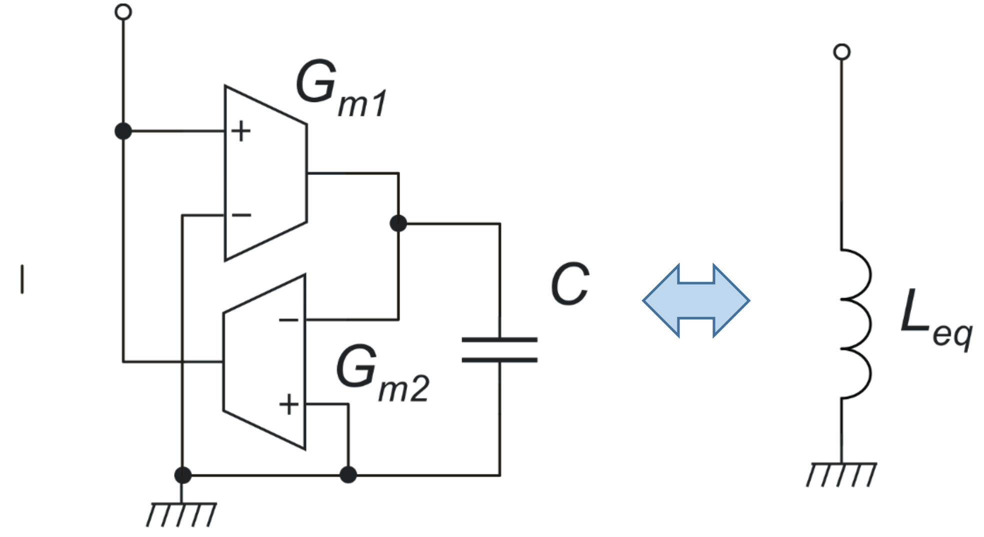
Grounded gyrator



$$I_2 = -G_1 V_1$$

$$I_1 = G_2 V_2$$

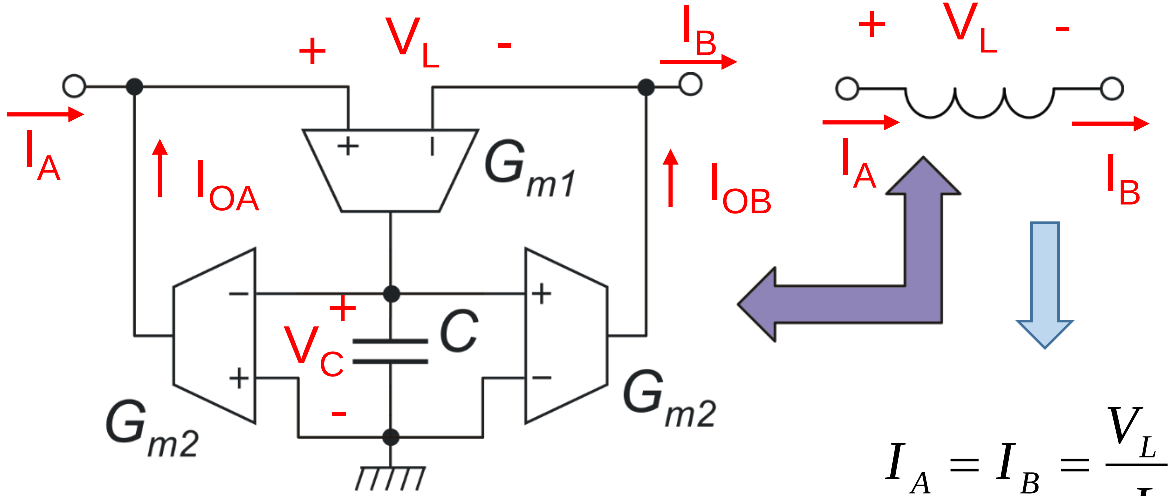
Inductor simulation with OTAs



Grounded Inductor

$$L_{EQ} = \frac{C}{G_{m1} G_{m2}}$$

Floating Inductor



$$I_A = I_B = \frac{V_L}{sL}$$

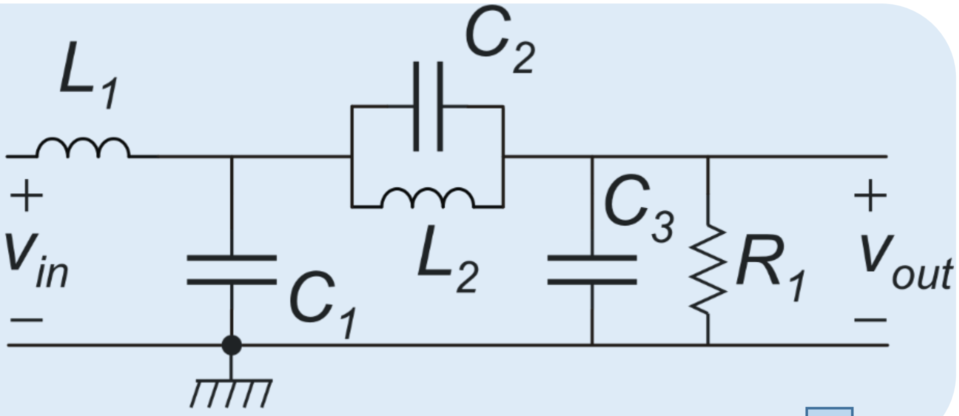
$$V_C = \frac{1}{sC} G_{m1} V_L \;;$$

$$I_A = -I_{OA} = G_{m2} V_C \;;\; I_B = I_{OB} = G_{m2} V_C$$

$$I_A = I_B = \frac{G_{m2} G_{m1}}{sC} V_L$$

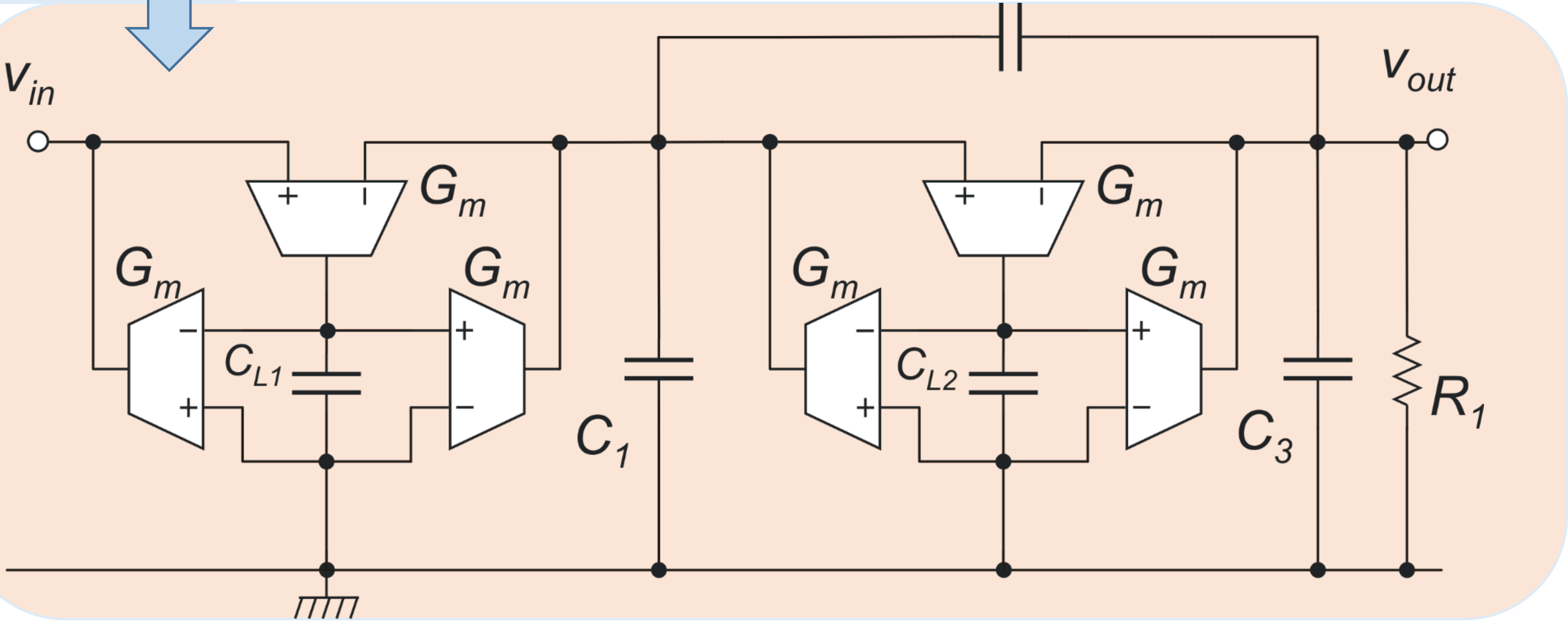
$$L_{EQ} = \frac{C}{G_{m1} G_{m2}}$$

Inductor simulation with OTAs - Example



Initial passive filter

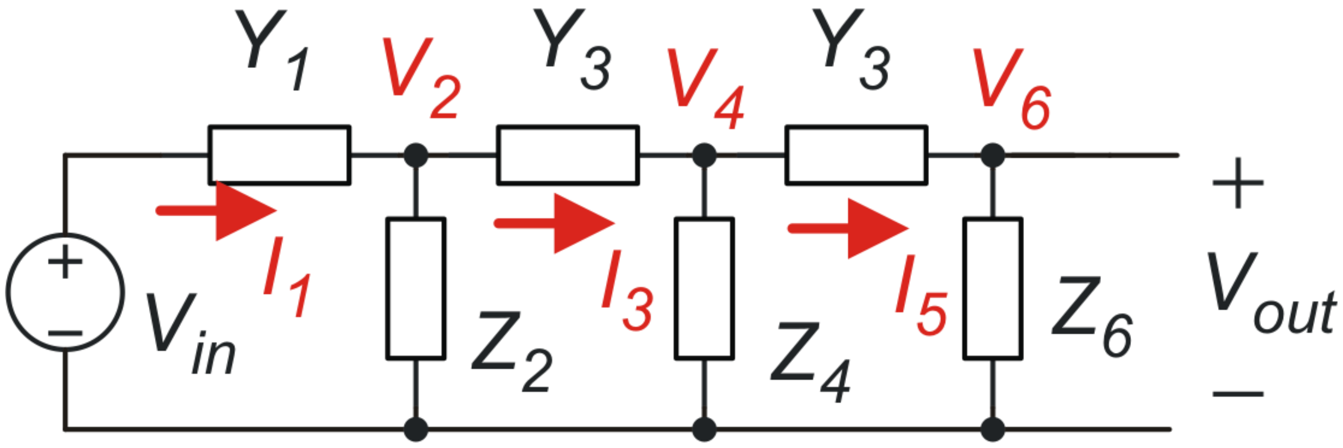
Same filter, with simulated inductors



$$C_{L1} = L_1 G_m^2$$

$$C_{L2} = L_2 G_m^2$$

Signal flow simulation of ladder (LC) networks with OTAs



Network Equations

$$I_1 = Y_1(V_{in} - V_2)$$
$$V_2 = Z_2(I_1 - I_3)$$
$$I_3 = Y_3(V_2 - V_4)$$
$$V_4 = Z_4(I_3 - I_5)$$
$$I_5 = Y_5(V_4 - V_6)$$
$$V_6 = Z_6 I_5$$

Variable transformations

Target: Transform current variables (I_1,I_3,I_5)
ùinto voltage variables

$$V_1 = \frac{1}{g} I_1 \quad V_3 = \frac{1}{g} I_3 \quad V_5 = \frac{1}{g} I_5$$

Homogeneous equivalent equations

$$V_1 = \frac{Y_1}{g}\left(V_{in} - V_2\right)$$

$$V_2 = g Z_2 \left(V_1 - V_3\right)$$

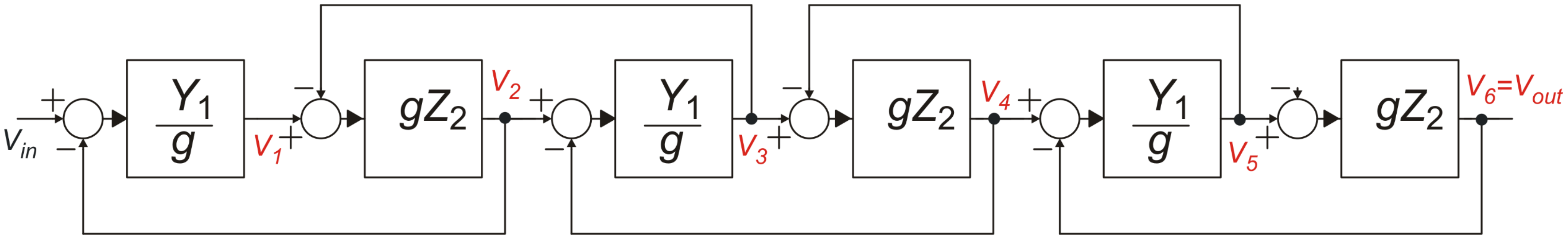
$$V_3 = \frac{Y_3}{g}\left(V_2 - V_4\right)$$

$$V_4 = Z_4 \left(V_3 - V_5\right)$$

$$V_5 = \frac{Y_5}{g}\left(V_4 - V_6\right)$$

$$V_6 = Z_6 I_5$$

Leap-Frog architecture



Homogeneous equivalent equations

$$V_1 = \frac{Y_1}{g}\left(V_{in} - V_2\right)$$

$$V_2 = gZ_2\left(V_1 - V_3\right)$$

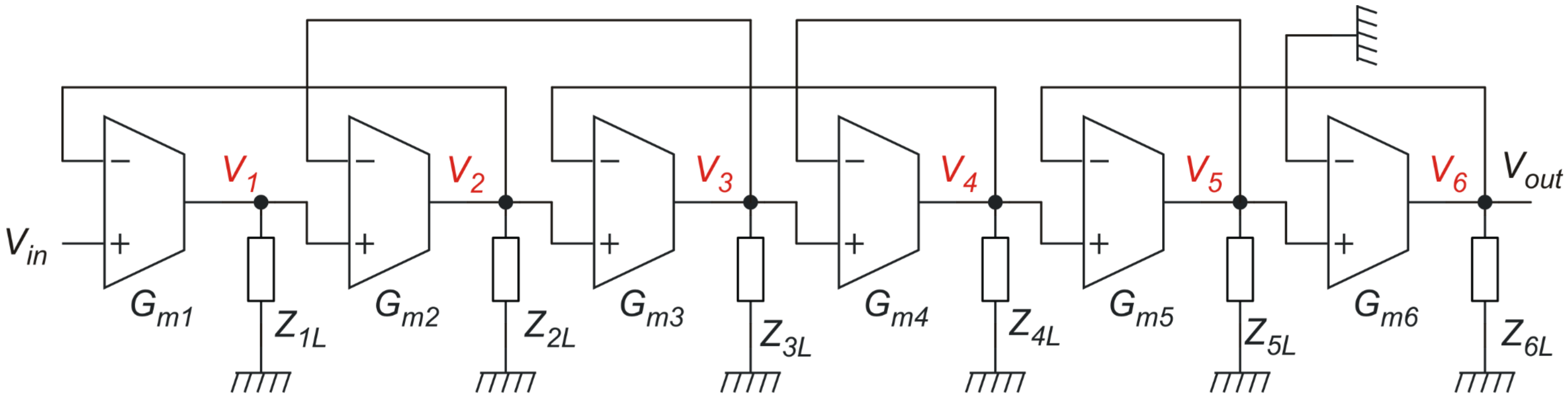
$$V_3 = \frac{Y_3}{g}\left(V_2 - V_4\right)$$

$$V_4 = Z_4\left(V_3 - V_5\right)$$

$$V_5 = \frac{Y_5}{g}\left(V_4 - V_6\right)$$

$$V_6 = Z_6 I_5$$

OTA implementation of the Leap-Frog structure



$$V_1 = \frac{Y_1}{g}\left(V_{in} - V_2\right)$$

$$V_2 = gZ_2\left(V_1 - V_3\right)$$

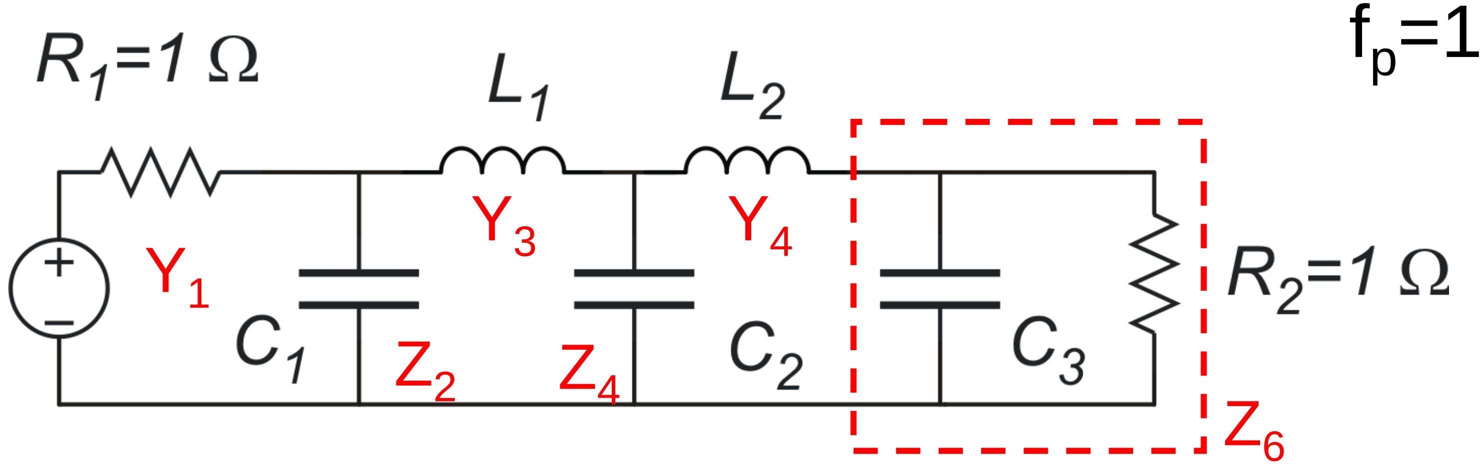
$$V_1 = Z_{1L}G_{m1}\left(V_{in} - V_2\right)$$

$$V_2 = Z_{2L}G_{m2}\left(V_1 - V_3\right)$$

$$Z_{1L} = \frac{Y_1}{gG_{m1}}$$ same for all odd indexes

$$Z_{2L} = \frac{gZ_2}{G_{m2}}$$ same for all even indexes

Example: 5th Order Chebyshev Filter



f_p=10 kHz

C1=35.1 μF
L1=18.0 μH
C2=49.4 μF
L2=18.0 μH
C3=35.1 μF

$$Z_{1L} = \frac{1}{R_1}\frac{1}{gG_{m1}} \qquad g = 1\ \text{S} \qquad \text{R}_1 = 1\,\Omega \qquad Z_{1L} = \frac{1}{G_{m1}} \qquad Z_{1L} = 1\ \text{k}\Omega \Rightarrow G_{m1} = 1\ \text{mS}$$

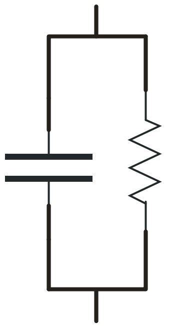
$$Z_{2L} = \frac{gZ_2}{G_{m2}} = \frac{g}{sC_1G_{m2}} \Rightarrow Z_{2L} = \frac{1}{sC_{2L}} \qquad C_{2L} = \frac{G_{m2}}{g}C_1 \qquad g = 1\ \text{S} \qquad G_{m1} = 10\ \mu\text{S} \qquad C_{2L} = 351\ \text{pF}$$

Example: 5th Order Chebyshev Filter

$$Z_{3L}=\frac{Y_3}{gG_{m1}} \qquad Y_3=\frac{1}{sL_1} \Rightarrow Z_{3L}=\frac{1}{sL_1 gG_{m1}}=\frac{1}{sC_{3L}} \Rightarrow C_{3L}=L_1 gG_{m1}$$

$$g=1\ \text{S}$$
$$G_{m1}=10\ \mu\text{S} \Rightarrow C_{3L}=180\ \text{pF}$$

$$\frac{1}{G_{m6}C_3}$$

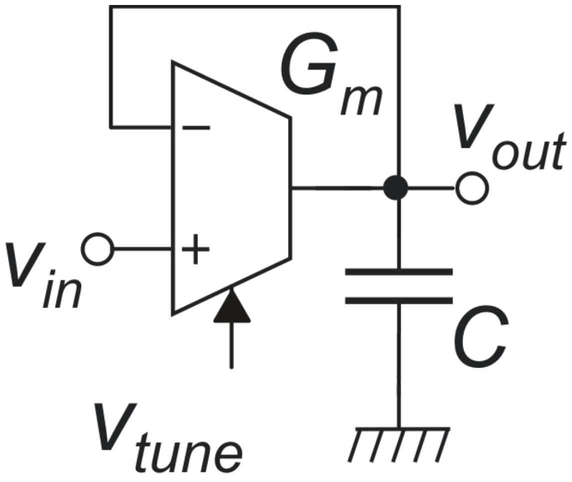


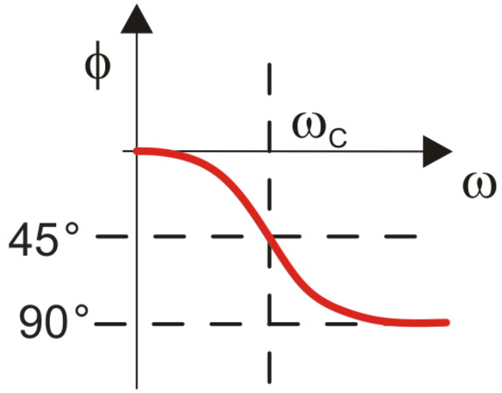
$$\frac{1}{G_{m6}}$$

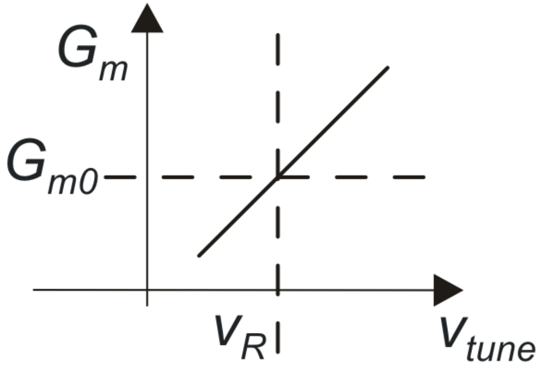
$$Z_{6L}=\frac{gZ_6}{G_{m6}}=\frac{g}{G_{m6}}\frac{1}{\frac{1}{R_2}+sC_3}=g\frac{1}{\frac{G_{m6}}{R_2}+sG_{m6}C_3}$$

$$R_{1L}=1\ \text{k}\Omega$$
$$C_{2L}=351\ \text{pF}$$
$$C_{3L}=180\ \text{pF}$$
$$C_{4L}=494\ \text{pF}$$
$$C_{5L}=180\ \text{pF}$$
$$Z_6=100\ \text{k}\Omega \| 351\ \text{pF}$$

Self-Tuning of OTA Filters







Self-Tuning of OTA Filters