Analog Filter Design

Part. 3: Time Continuous Filter Implementation

Design approaches

- Passive LC (R) ladder filters
- Cascade of Biquadratic (Biquad) and Bilinear cells
- State Variable Filters
- Simulation of LC filters with active RC networks

Filter Parameters

- For a given transfer function *H(s)*, a particular implementation is characterized by several FOMs (Figures Of Merit). The most frequently used are:

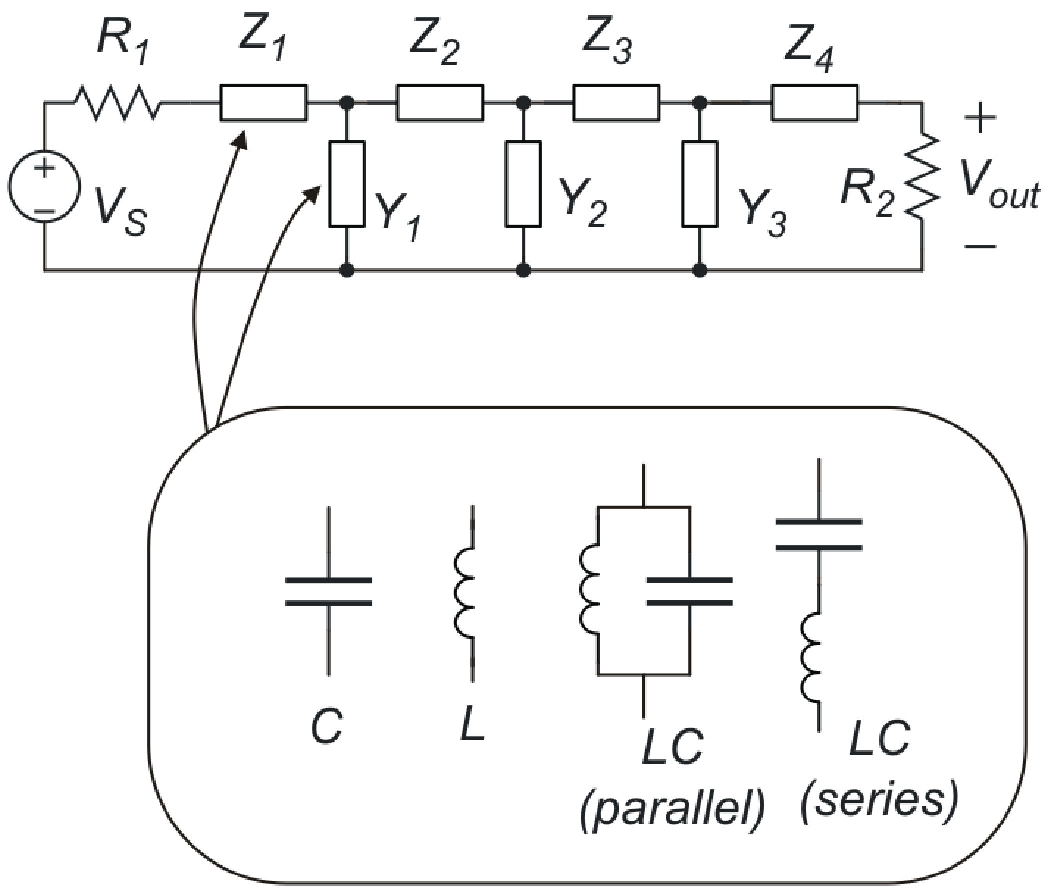
➢ Dynamic Range: $DR = \dfrac{\max(V_{out})}{v_{n-out}}$ v_{n-out}= output noise

➢ Sensitivity to component variations

$$\begin{cases} S_x^{\omega_0} = \dfrac{d\omega_0/\omega_0}{dx/x} = \dfrac{x}{\omega_0}\dfrac{d\omega_0}{dx} \\ S_x^{Q} = \dfrac{x}{Q}\dfrac{dQ}{dx} \end{cases}$$

➢ Component value spread, e.g. $\dfrac{C_{\max}}{C_{\min}}$

Passive Lossless Ladder Filters

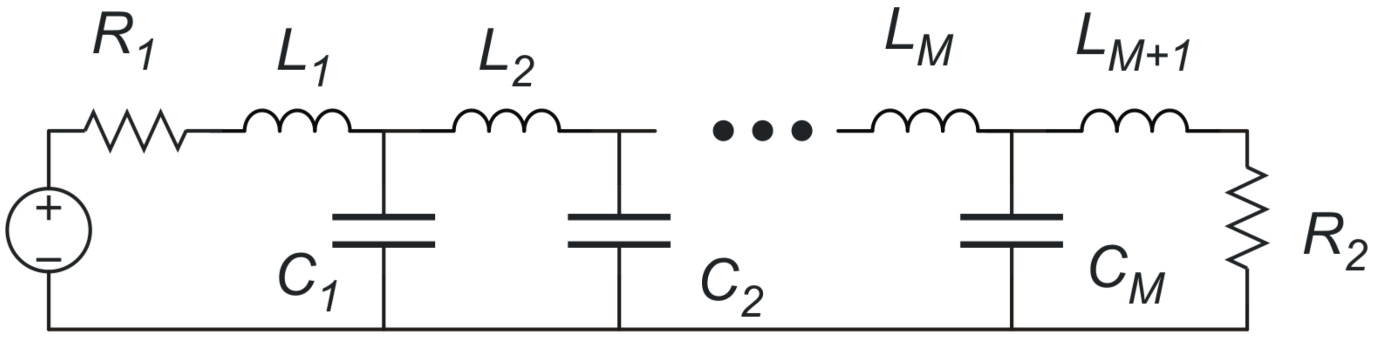


- Doubly terminated LC ladder network
- Advantages: minimum sensitivity to component variation in the pass-band
- The lowest sensitivity is achieved with equally terminated networks (R_1=R_2).
- Can be used as starting point for the synthesis of active RC filters
- Drawback: tuning requires change of al components.

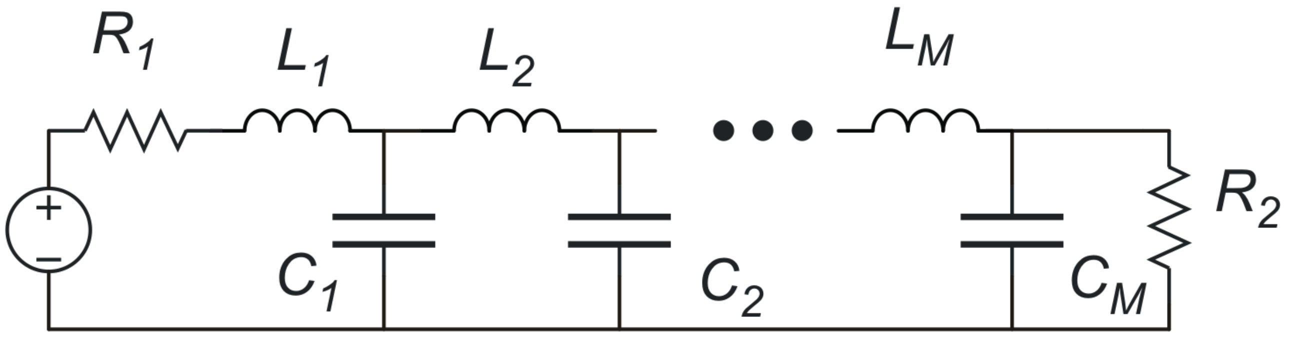
order (N) = number of capacitors + number of inductors

Prototype Filter Configurations (all poles)

N=2M+1 (odd order)



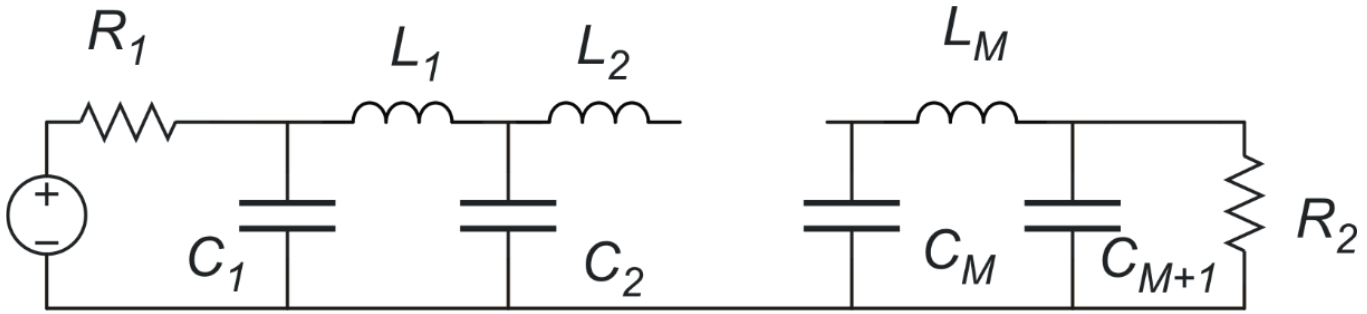
N=2M (even order)



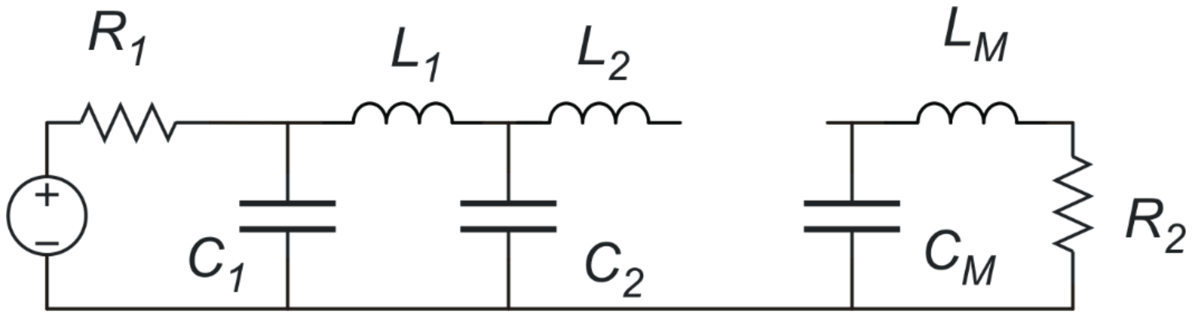
Pass-band gain: $$k = \frac{R_2}{R_1 + R_2} < 1$$ (0.5 for equally terminated networls)

Alternate solution (all poles)

N=2M+1 (odd order)

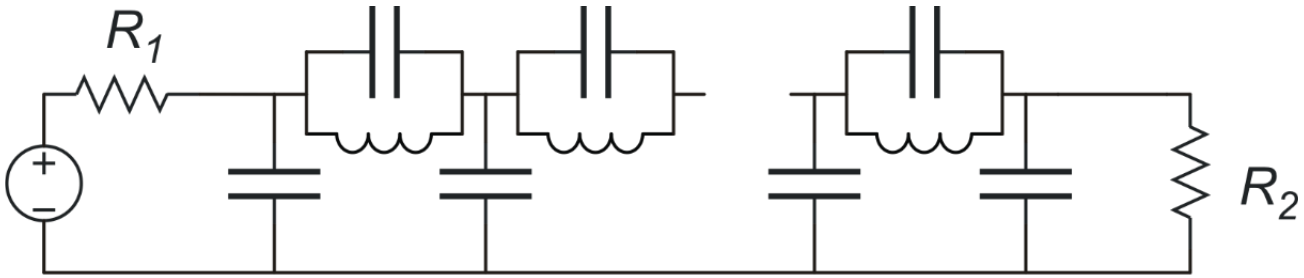


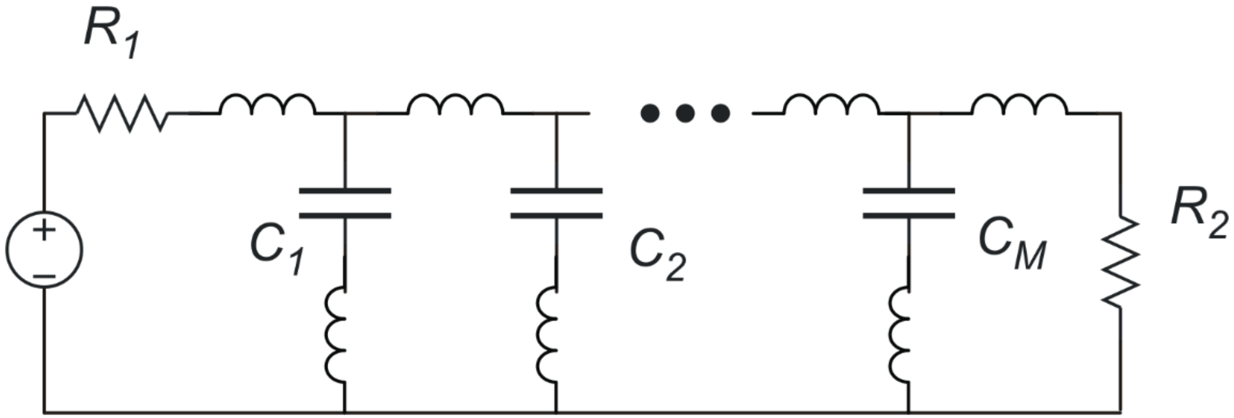
N=2M (even order)



Pass-band gain: $$k = \frac{R_2}{R_1 + R_2} < 1$$ (0.5 for equally terminated networls)

LC ladder network for TF with imaginary zeros (e.g. Inverse Chebyshev and Cauer Elliptic filters)





Frequency scaling rules

- LC ladder filters are synthetized in normalized (1 rad/s, 1 Ω) and low-pass form
- Frequency scaling allows to change the normalization frequency, allowing transformation of the characteristic frequencies of the filter

$$s_n \rightarrow \frac{s}{\omega_N}$$

$$\frac{1}{s_n C} \rightarrow \frac{\omega_N}{sC} \Rightarrow C \rightarrow \frac{C}{\omega_N}$$

$$s_n L \rightarrow s_n \frac{L}{\omega_N} \Rightarrow L \rightarrow \frac{L}{\omega_N}$$

$$R \rightarrow R$$

Impedance Scaling Rule

- Impedance scaling is used to change component values leaving the transfer function unaltered. The target is finding feasible component values for the chosen technology

If the network includes only:

- Two terminal impedances (L,R,C components)
- Voltage Controlled Voltage Sources (VCVS) i.e Ideal voltage amplifiers.
- Current Controlled Current Sources (CCCS) i.e. ideal current amplifiers

Then: the Vout/VS transfer function is unchanged when <u>all</u> the impedances are multiplied by the same function f(s)

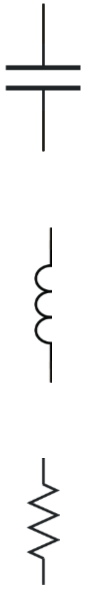
Impedance scaling: component transformation

An important case is when the function f(s) is a constant factor K:

$$\frac{1}{s_n C} \rightarrow K\frac{1}{sC} \Rightarrow \quad C \rightarrow \frac{C}{K}$$

$$sL \rightarrow KsL \Rightarrow \quad L \rightarrow KL$$

$$R \rightarrow KR$$



Element transformations

- **Goal:** to change the filter response from low-pass to the other three possibilities (high-pass, etc.) and perform frequency scaling at the same time.

Let us recall the following transformations:

From Low-Pass to:

High-Pass

$$s_n \to \frac{\omega_N}{s}$$

Band-Pass

$$s_n \to \frac{\omega_0}{B}\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)$$

Band -Stop

$$s_n \to \left[\frac{\omega_0}{B}\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)\right]^{-1}$$

Element Transformation

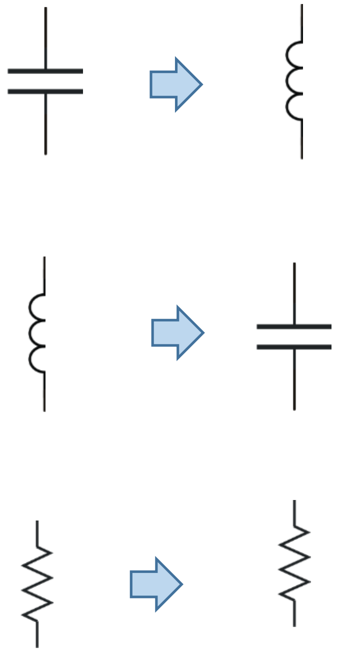
Low-pass to High-pass

$$s_n \to \frac{\omega_N}{s}$$

$$\frac{1}{s_n C} \to \frac{s}{\omega_N C} \quad \Rightarrow \quad C \to L = \frac{1}{\omega_N C}$$

$$s_n L \to \frac{\omega_N}{s} L \Rightarrow \quad L \to C = \frac{1}{\omega_N L}$$

$$R \to R$$



Element Transformation

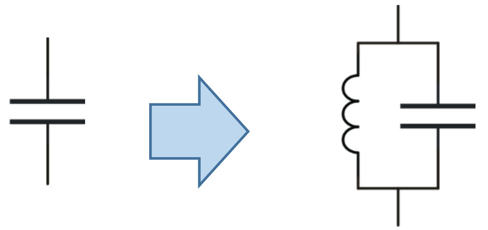
Low-pass to Band-pass

$$s_n \rightarrow \frac{\omega_0}{B}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)$$

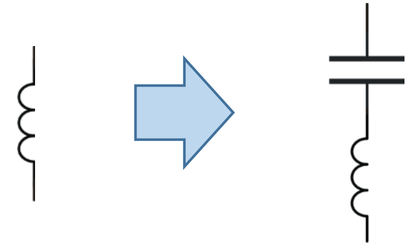
$$s_n \rightarrow \frac{s}{B}+\frac{\omega_0^2}{sB}$$

$$\frac{1}{s_n C} \rightarrow \frac{1}{\dfrac{sC}{B}+\dfrac{\omega_0^2 C}{sB}} = \frac{1}{sC_P+\dfrac{1}{sL_P}}$$

$$s_n L \rightarrow L\frac{s}{B}+L\frac{\omega_0^2}{sB} = sL_S+\frac{1}{sC_S}$$



$$C_P = \frac{C}{B} \qquad L_P = \frac{B}{\omega_0^2 C}$$

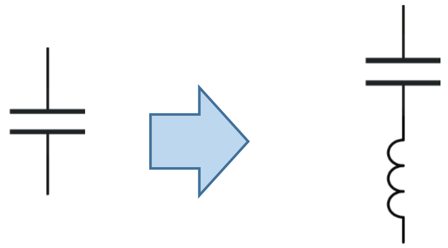


$$C_S = \frac{B}{\omega_0^2 L} \qquad L_S = \frac{L}{B}$$

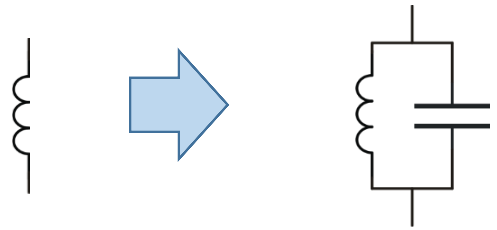
Element Transformation

Low-pass to Band-stop

$$s_n \rightarrow \left[\frac{\omega_0}{B} \left(\frac{s}{\omega_0} + \frac{\omega_0}{s} \right) \right]^{-1}$$



$$C_S = \frac{CB}{\omega_0^2} \qquad L_S = \frac{1}{BC}$$

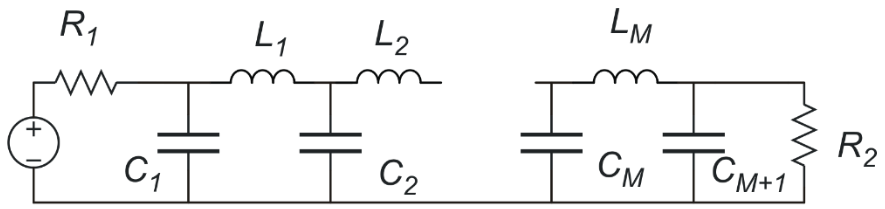


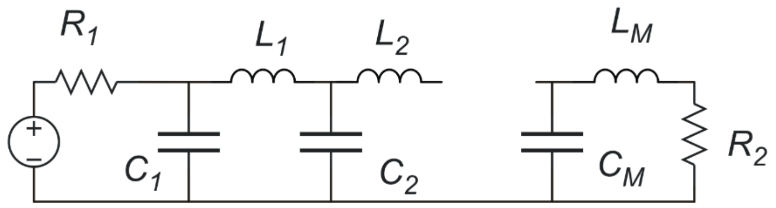
$$C_P = \frac{1}{BL} \qquad L_P = \frac{BL}{\omega_0^2}$$

Design of LC ladder passive filters

- A procedure that allows designing an arbitrary transfer function with a ladder structure does not exist.
- All-pole functions (e.g. Butterworth, Chebyshev I, Bessel) can be designed with a standard approach, where the branches of the ladder (Z and Y elemets) are pure capacitors or inductors. Given a class of networks, not all functions are feasible.
- The rigorous design of Cauer (elliptic) filters is less straightforward.
- Tables are available for the most frequently used ladder topologies and transfer functions. Several CAD design tools are also available.

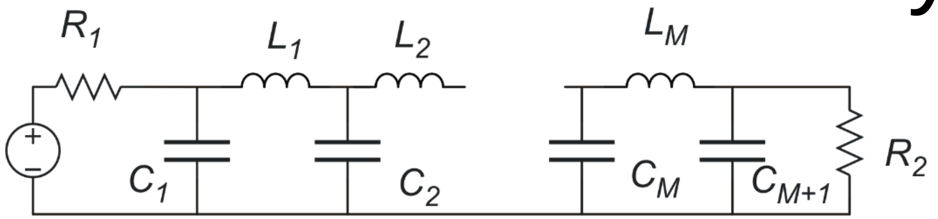
Example: Butterworth Prototype Filter

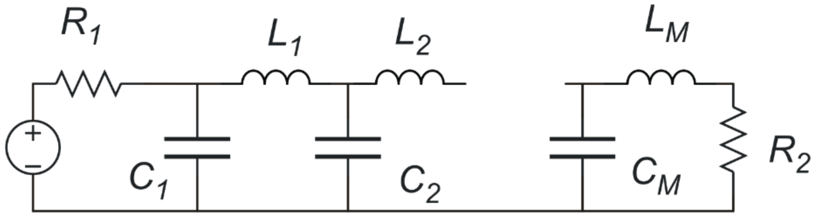




N	C1	L1	C2	L2	C3	L3	C4	L4	C5	L5
2	1.4142	1.4142								
3	1.0000	2.0000	1.0000			Butterworth (1 rps passband)				
4	0.7654	1.8478	1.8478	0.7654						
5	0.6180	1.6180	2.0000	1.6180	0.6180					
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2740	0.4450			
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129

Chebyshev 1 dB ripple





n	C1	L1	C2	L2	C3	L3	C7
2	0.572	3.132					
3	2.216	1.088	2.216				
4	0.653	4.411	0.814	2.535			
5	2.207	1.128	3.103	1.128	2.207		
6	0.679	3.873	0.771	4.711	0.969	2.406	
7	2.204	1.131	3.147	1.194	3.147	1.131	2.204

Example

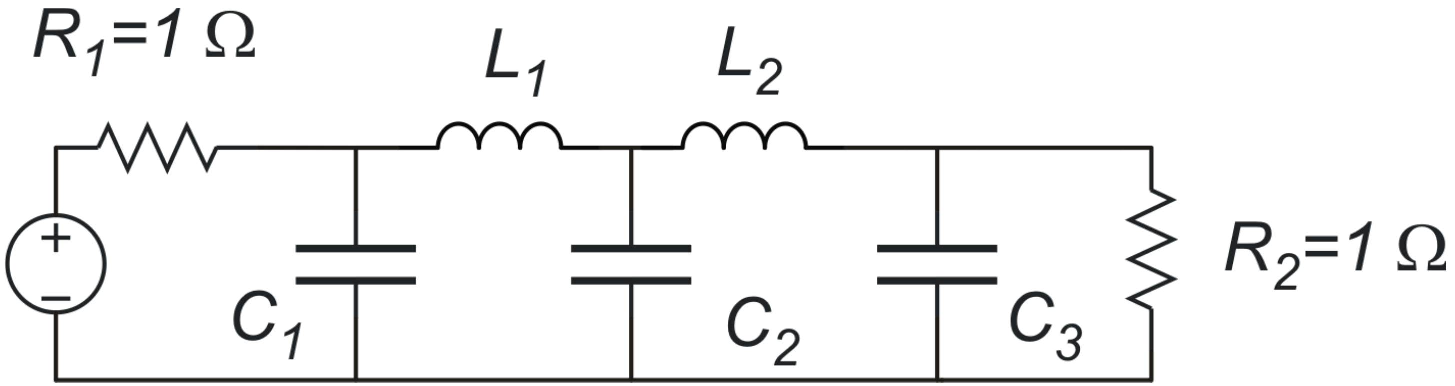
- Design a LC ladder Chebyshev filter with the following characteristics:

f_pass =10 kHz, Maximum Pass-band attenuation 1 dB

f_stop = 20 kHz Minimum Stop-Band Attenuation: 40 dB

Python: cheb1ord: Order=5, $\omega_N=\omega_P=62.8$ krad/s

LC Filter design using Tables



$\omega_N=\omega_P=1$ rad/s

C1=2.207 F
L1=1.128 H
C2=3.103 F
L2=1.128 H
C3=2.207 F

$$C \rightarrow \frac{C}{\omega_N}$$

$$L \rightarrow \frac{L}{\omega_N}$$

$\omega_N=\omega_P=62.8$ krad/s

C1=35.1 μF
L1=18.0 μH
C2=49.4 μF
L2=18.0 μH
C3=35.1 μF