Analog Filter Design

Part. 4: Discrete Time Filters

- Sect. 4-b: Switched Capacitors Filters

First SC circuits: simulations of resistances

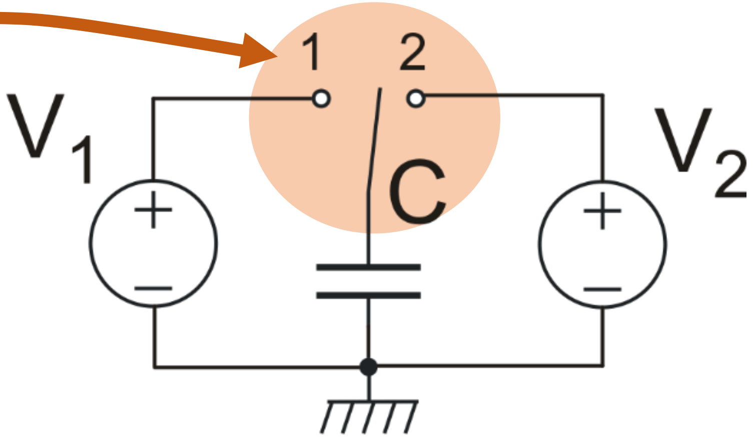
- In integrated RC active filters, the resistances may be the largest components, especially when low frequency singularities are required, as in audio analog processors.
- Singularities in RC filters are proportional to 1/RC factors. Since R and C are marked by non-correlated process variations, the spread in filter characteristic frequencies can be very large (up to 20 %).
- Resistances simulated with switches and capacitors are given by expressions like:

$$R_{eq} = \frac{1}{f_{ck} C}$$

where f_{ck} is the clock frequency. With the small capacitors available on chip it is possible to obtain very large resistors.

Switched Capacitor resistance: principle

Alternation between position 1 and 2 at the clock frequency f_{ck}

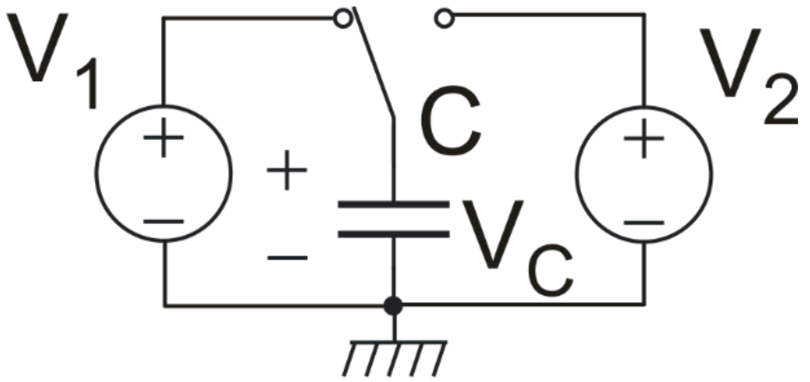


$$\Delta Q_C = C\left(V_C^{(final)} - V_C^{(initial)}\right)$$

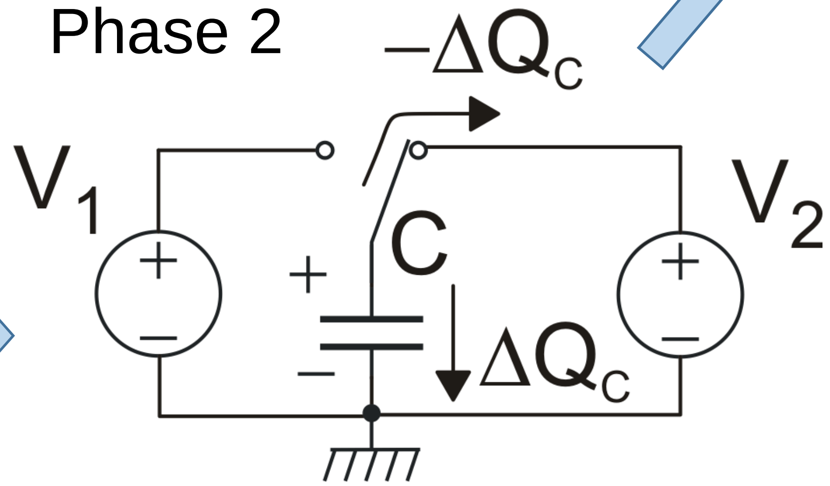
$$-\Delta Q_C = C\left(V_1 - V_2\right)$$

$$I_{eq} = -\Delta Q \cdot f_{CK} = C f_{ck}\left(V_1 - V_2\right)$$

Phase 1

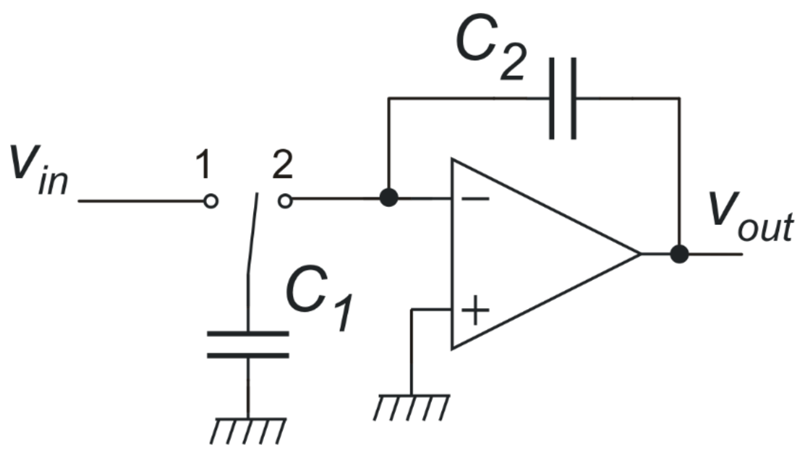


Phase 2

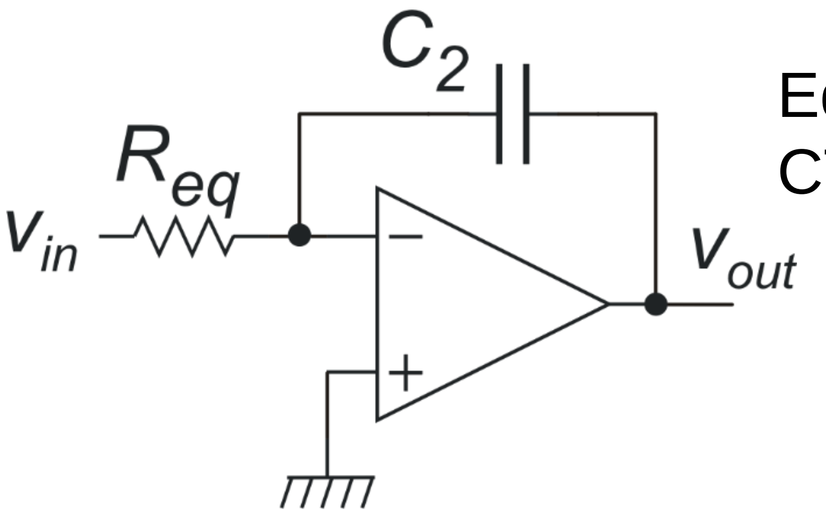


$$R_{eq} = \frac{1}{f_{ck} C}$$

Simple integrator based on SC resistor



Simple SC integrator

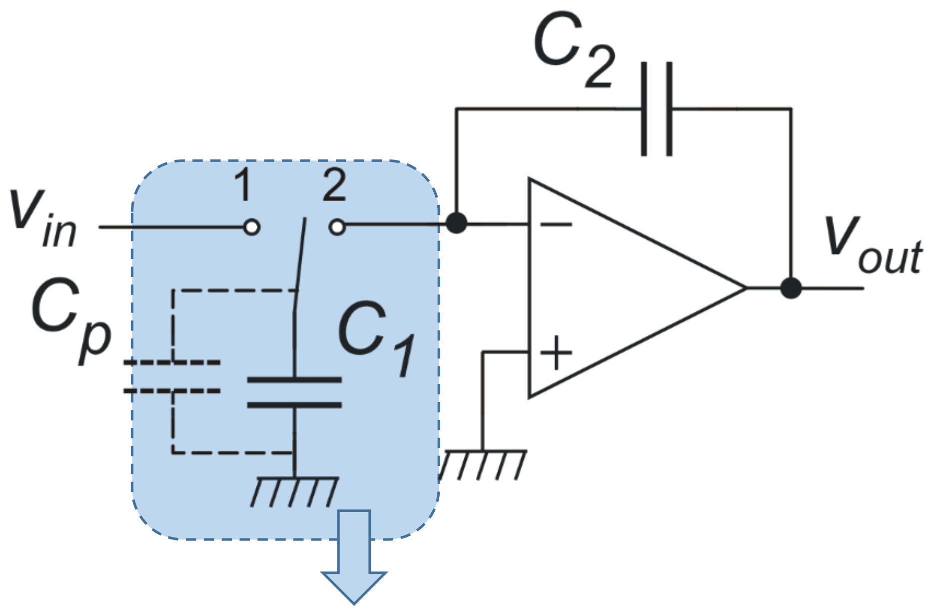


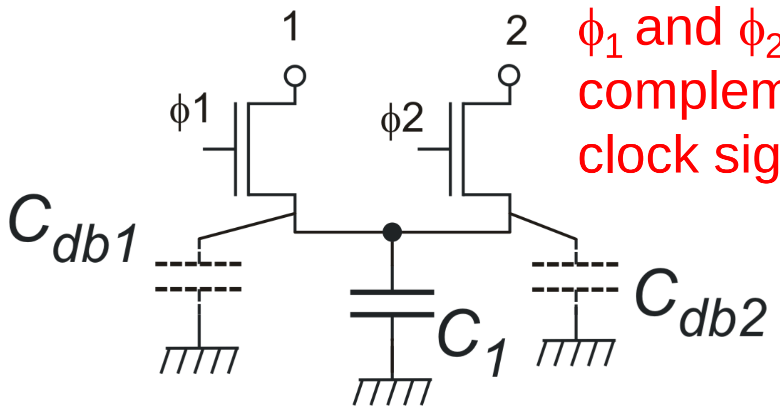
Equivalent CT Integrator

$$v_{out} = -\frac{\omega_0}{s} \qquad \omega_0 = \frac{1}{R_{eq}C_2} = \frac{C_1}{C_2} f_{ck}$$

- Important: The unity gain angular frequency of the integrator (ω_0) depends only on capacitance ratios and the clock frequency. Ratios can be fabricated with high precision and accurate frequencies can be obtained from crystal oscillators
- **Filter with precise corner frequencies can be obtained.**

Effect of parasitic capacitances

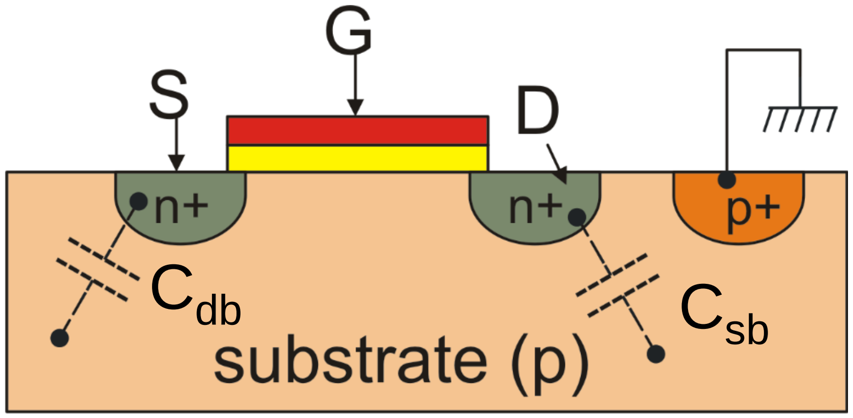




ϕ_1 and ϕ_2 are complementary clock signals

Parasitic capacitance

$$\omega'_0 = \frac{C_1 + C_p}{C_2} f_{ck}$$



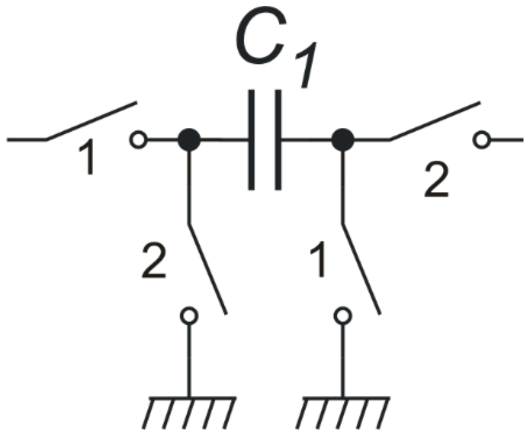
Mosfet cross-section

Parasitic capacitances are not well predictable and generally non linear

- Inaccuracy of the corner frequencies
- Distortion

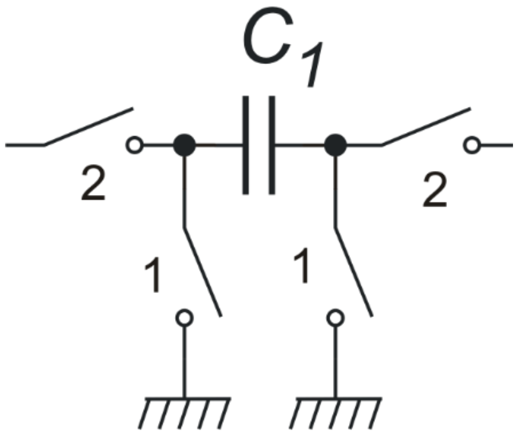
Parasitic Insensitive (PI) SC integrator

- Parasitic Insensitive SC resistors

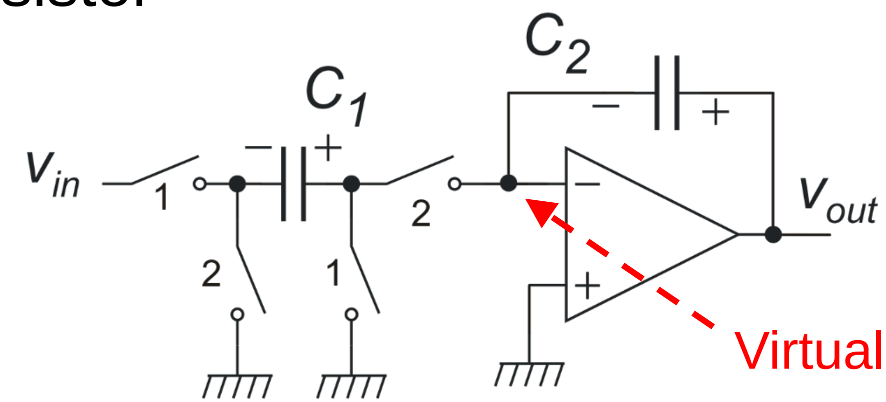


This side should be virtual gnd!

Negative SC resistor



Positive SC resistor

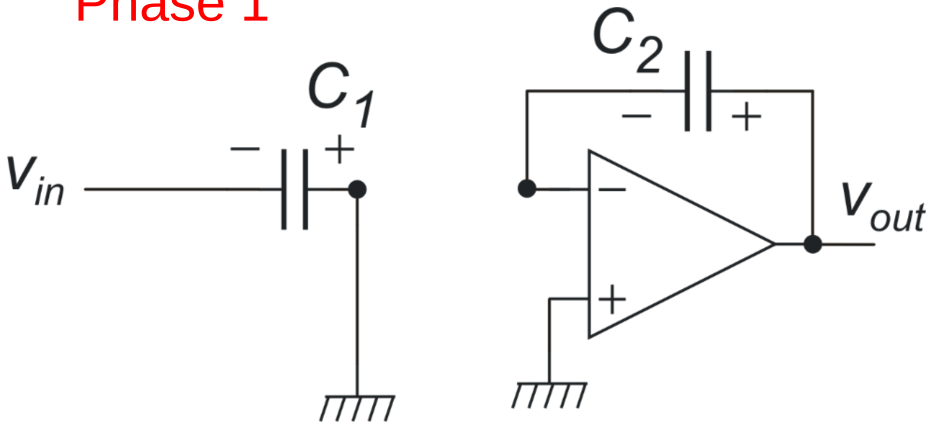


Virtual gnd

Example: Integrator with negative SC Res.

Parasitic Insensitive (PI) SC integrator: negative resistor

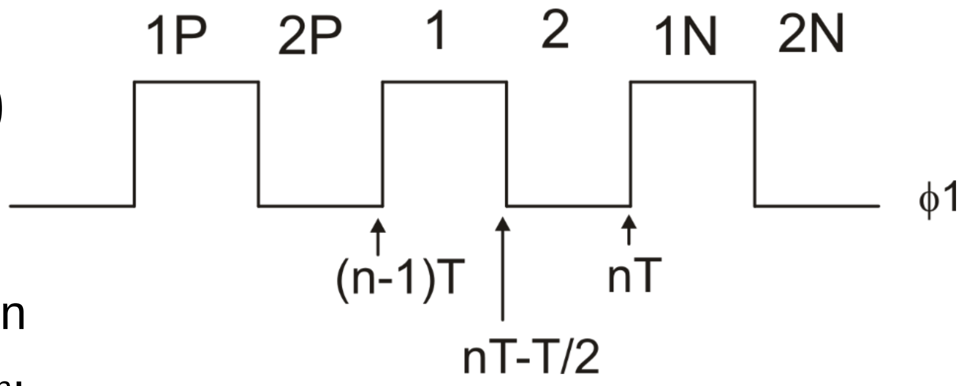
Phase 1



$$V_{C1}^{(1)} = -v_{in}^{(1)}$$

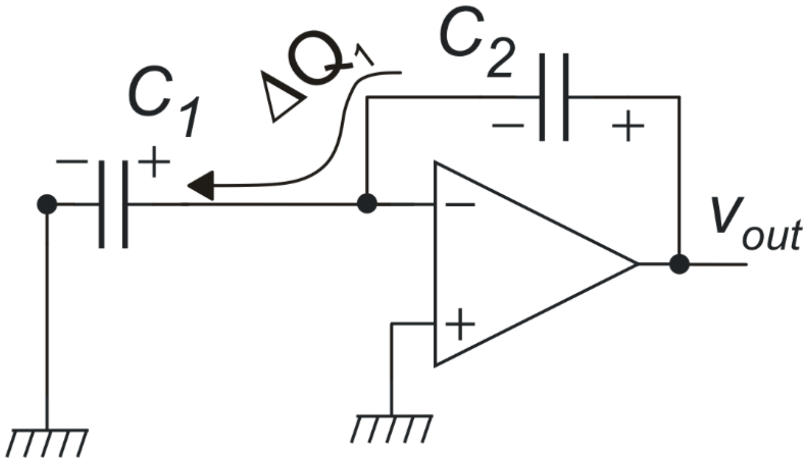
$$V_{C2}^{(1)} = V_{C2}^{(2P)} \quad \text{(hold)}$$

Clock at frequency f_{ck}



In the 1->2 phase transition C_1 is disconnected from v_{in}, so that v_{in} is sampled just across the transition.

Phase 2



$$V_{C1}^{(2)} = 0$$

$$V_{C2}^{(2)} = V_{C2}^{(2P)} + \frac{\Delta Q_{C1}}{C_2}$$

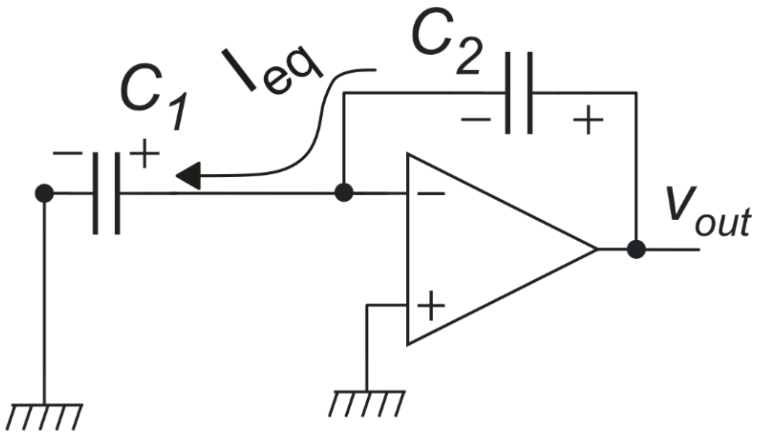
$$\Delta Q_{C1} = C_1\left(V_{C1}^{(2)} - V_{C1}^{(1)}\right) = v_{in}^{(1)} C_1$$

$$V_{out}^{(2)} = V_{out}^{(2P)} + \frac{C_1}{C_2} v_{in}^{(1)}$$

$$V_{out} = V_{C2}$$

Negative resistor: equivalent currents

Phase 2

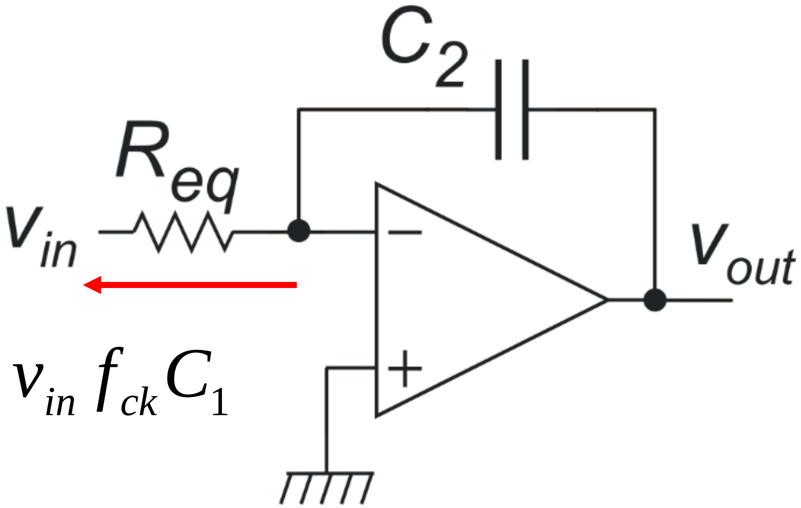


$$\Delta Q_{C1} = v_{in}^{(1)} C_1$$

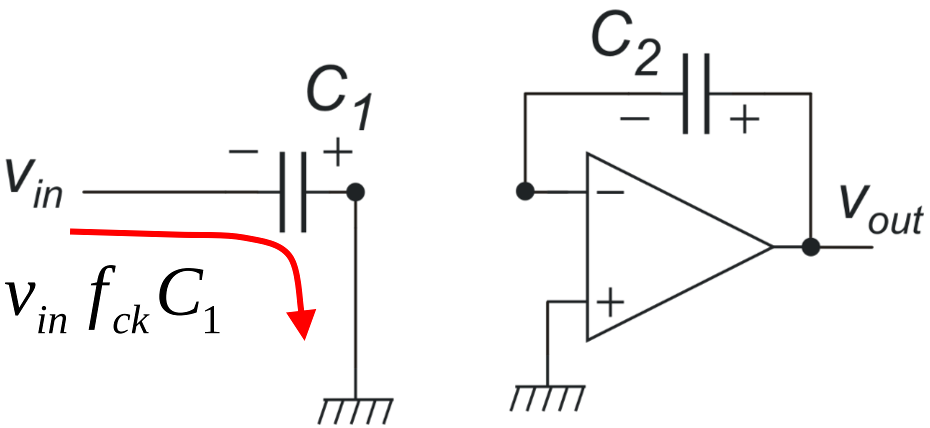
$$I_{eq} = f_{CK} \Delta Q_{C1} = v_{in} f_{ck} C_1$$

$$R_{eq} = -\frac{1}{f_{ck} C_1} < 0$$

In terms of charge delivered to the amplifier



$$I_{eq} = v_{in} f_{ck} C_1$$



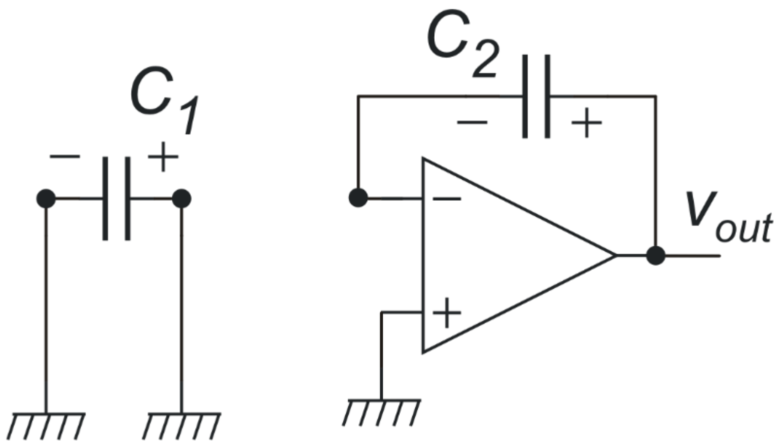
$$I_{eq} = v_{in} f_{ck} C_1$$

$$R_{eq} = \frac{1}{f_{ck} C_1} > 0$$

Seen by the source v_{in}

Parasitic Insensitive (PI) SC integrator: positive resistor

Phase 1



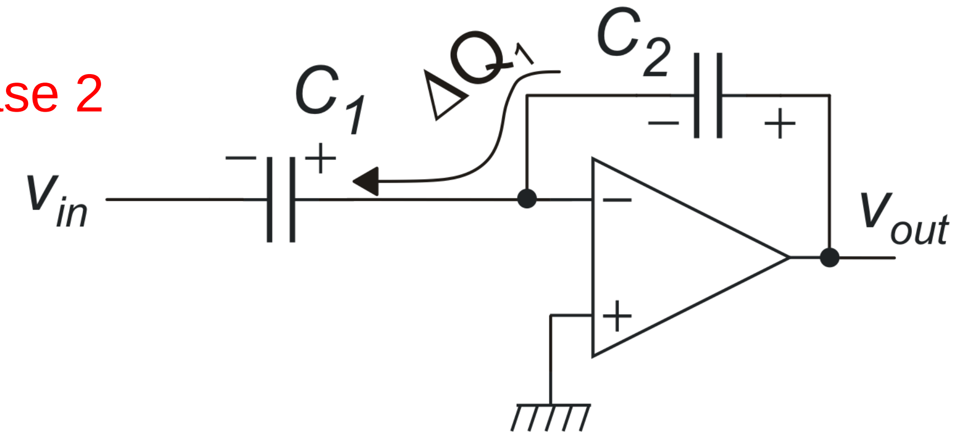
$$V_{C1}^{(1)} = 0$$

$$V_{C2}^{(1)} = V_{C2}^{(2P)} \quad \text{(hold)}$$

Note: V_{out} is sampled at the end of phase 2, when C_1 is disconnected from v_{in}. In this same instant, together with V_{out}, we sample also v_{in}

$$V_{out}^{(2)} = V_{out}^{(2P)} - \frac{C_1}{C_2} v_{in}^{(2)}$$

Phase 2



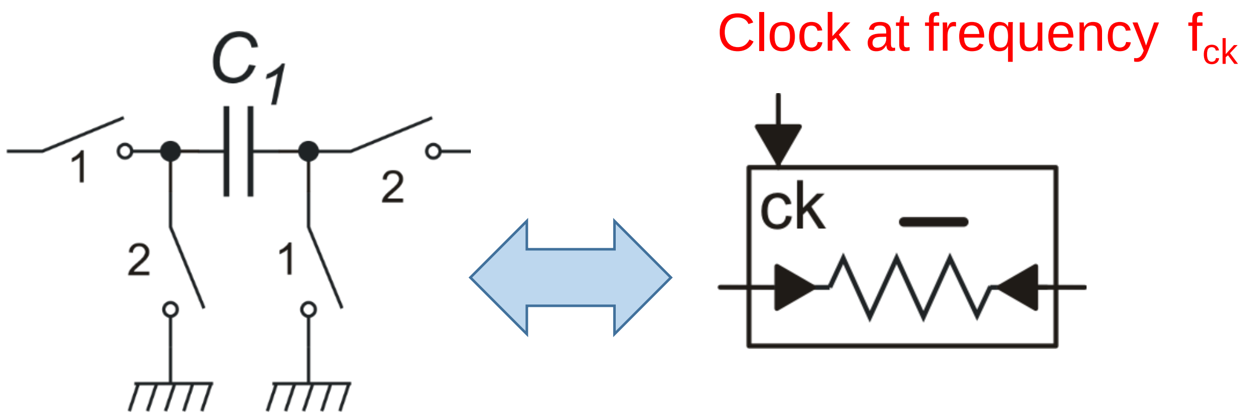
$$V_{C1}^{(2)} = -v_{in}^{(2)}$$

$$V_{C2}^{(2)} = V_{C2}^{(2P)} + \frac{\Delta Q_{C1}}{C_2}$$

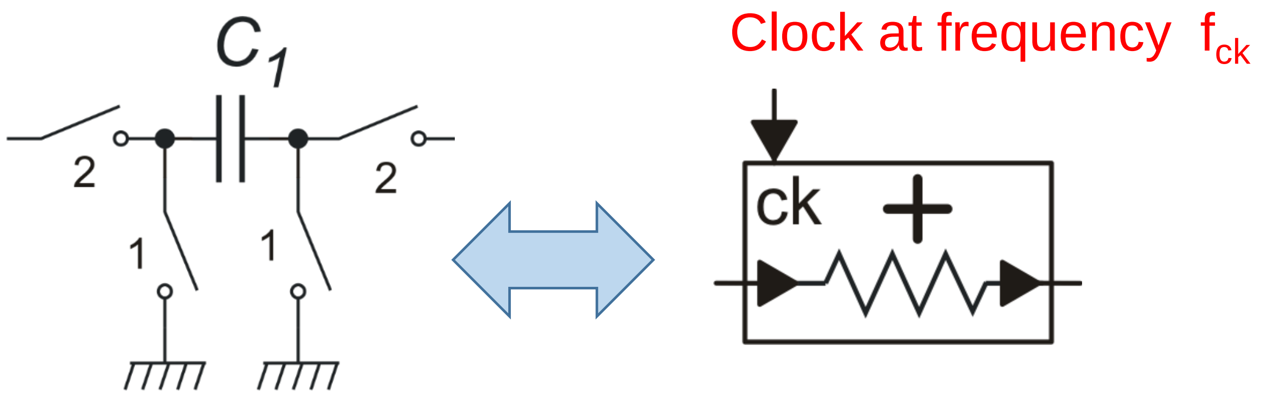
$$\Delta Q_{C1} = C_1 \left(V_{C1}^{(2)} - V_{C1}^{(1)} \right) = -v_{in}^{(2)} C_1$$

PI-SC resistors: symbols used in this course

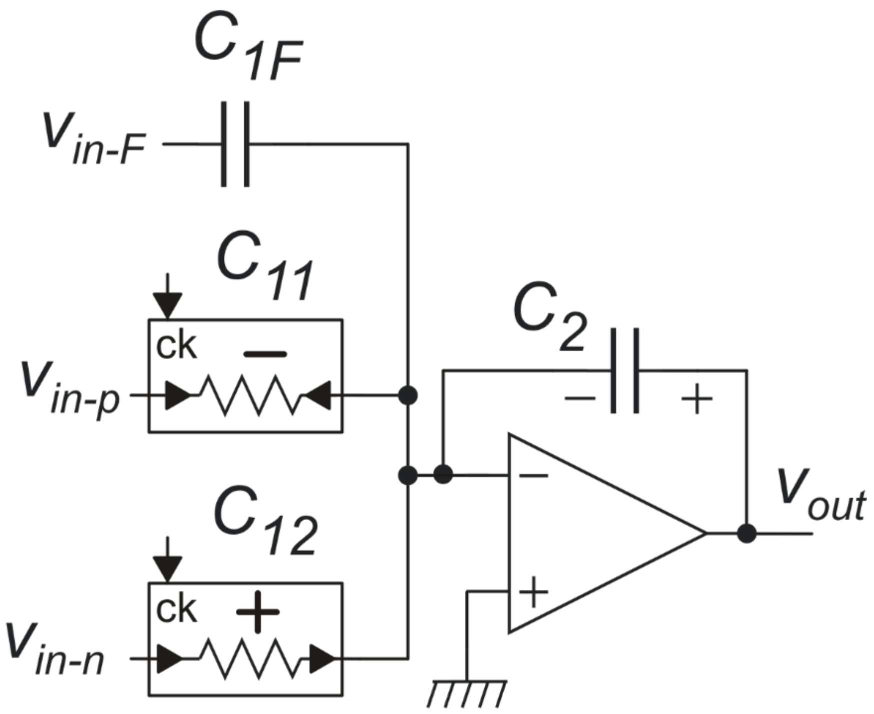
Negative resistor

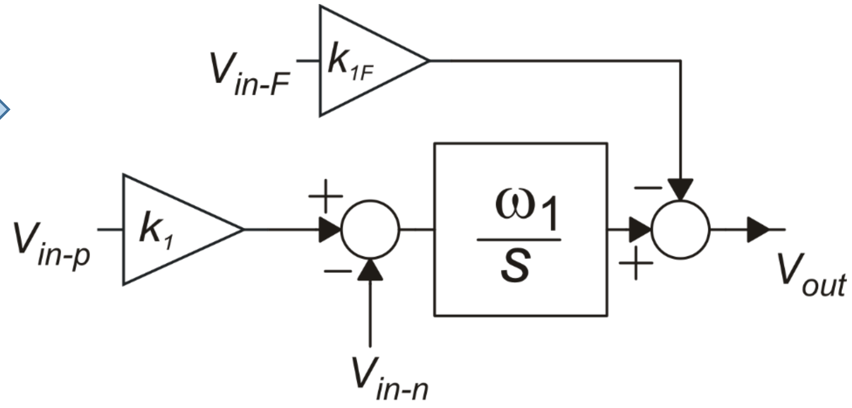


Positive resistor



Example 1: versatile integrator

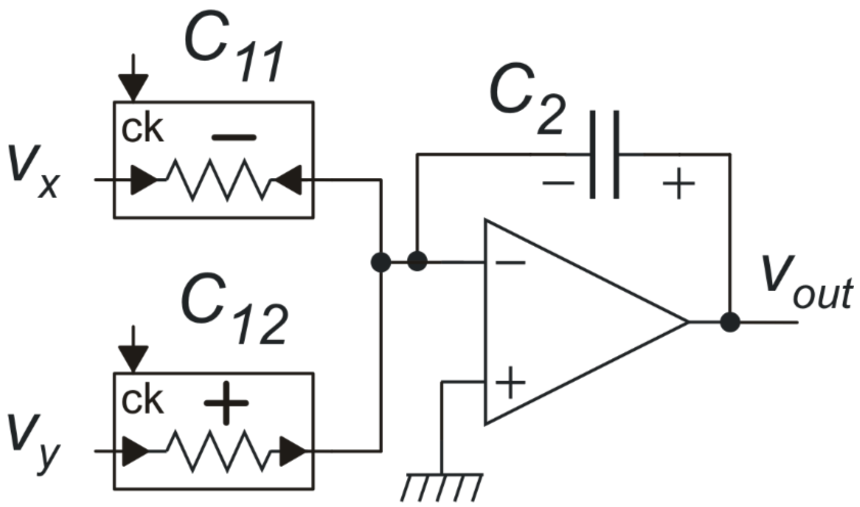




$$\omega_1 = f\frac{C_{12}}{C_2} \qquad k_1 = \frac{C_{11}}{C_{12}} \qquad k_{1F} = \frac{C_{1F}}{C_2}$$

Note this approximation is valid just when the equivalent resistance model is valid i.e. for $f \ll f_{ck}=f_s$

Discrete time nature of SC filters: Integrator



$$V_{out}^{(2)} = V_{out}^{(2P)} + \frac{C_1}{C_2} v_{in}^{(1)}$$

negative res.
non-inverting

$$V_{out}^{(2)} = V_{out}^{(2P)} - \frac{C_1}{C_2} v_{in}^{(2)}$$

positive res.
inverting

$$V_{out}(nT) = V_{out}[(n-1)T] - \frac{C_{12}}{C_2} v_y(nT) + \frac{C_{11}}{C_2} v_x\left(nT - \frac{T}{2}\right)$$

defining

$$v_{xD}(t) \equiv v_x\left(t - \frac{T}{2}\right)$$

$$V_{out}(n) = V_{out}(n-1) - \frac{C_{12}}{C_2} v_y(n) + \frac{C_{11}}{C_2} v_{xD}(n)$$

Discrete time nature of SC filters: Integrator

$$V_{out}(n) = V_{out}(n-1) - \frac{C_{12}}{C_2} v_y(n) + \frac{C_{11}}{C_2} v_{xD}(n)$$

$$V_{out}(z) = z^{-1} V_{out}(z) - \frac{C_{12}}{C_2} V_y(z) + \frac{C_{11}}{C_2} V_{xD}(z)$$

$$V_{out}(z) = \frac{1}{1-z^{-1}} \frac{C_{12}}{C_2} V_y(z) + \frac{1}{1-z^{-1}} \frac{C_{11}}{C_2} V_{xD}(z)$$

$$H(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

Backward Euler DT integrator
(non-delayed integrator)

Compare with:

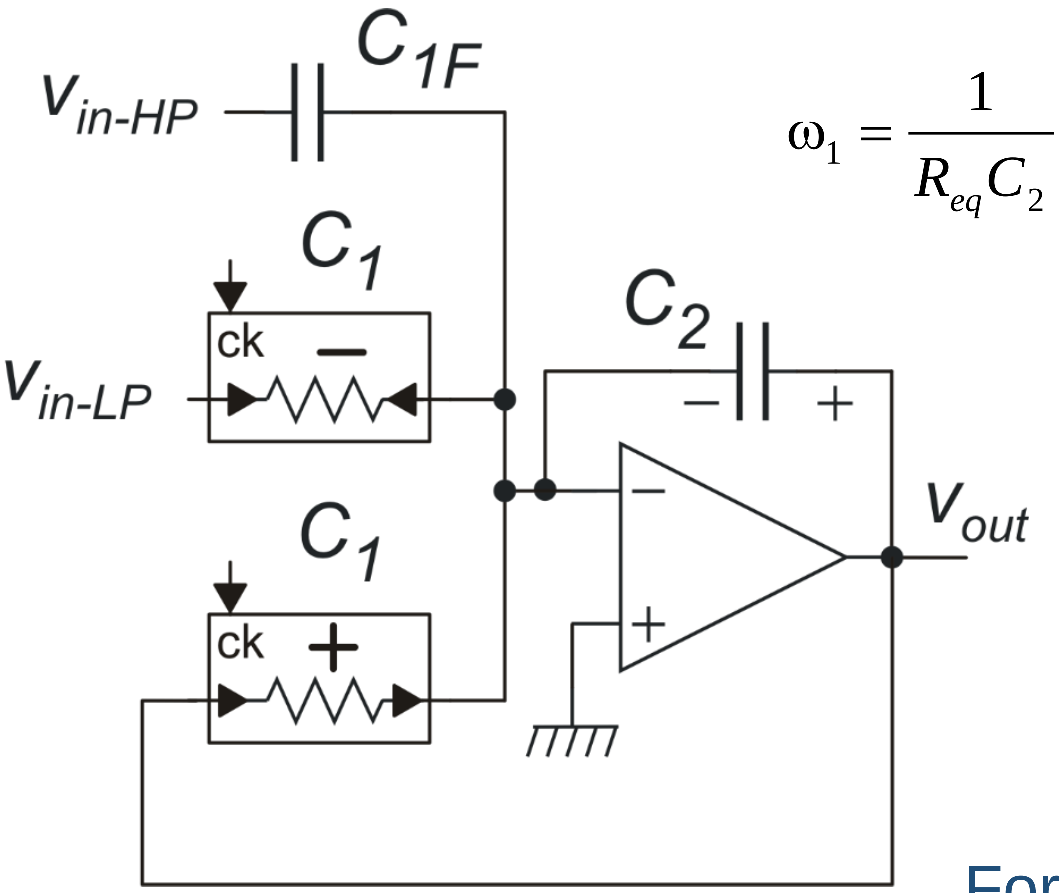
$$H(z) = \frac{z^{-1}}{1-z^{-1}} = \frac{1}{z-1}$$

Standard (forward) Euler DT integrator
(delayed form)

Example 2 First order filter

$$v_{out} = \frac{\omega_1}{s+\omega_1} v_{in-LP} - \frac{s k_{1F}}{s+\omega_1} v_{in-HP}$$

$$\omega_1 = \frac{1}{R_{eq} C_2} = f_{ck} \frac{C_1}{C_2}$$



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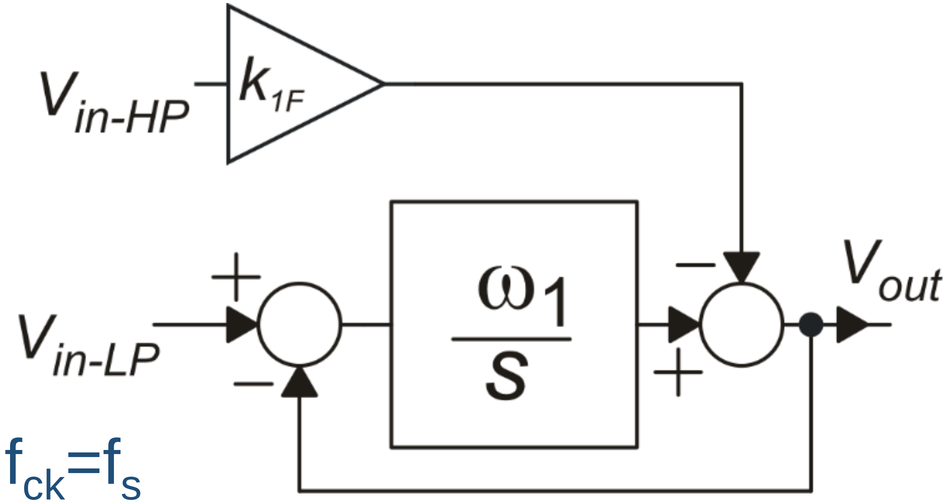
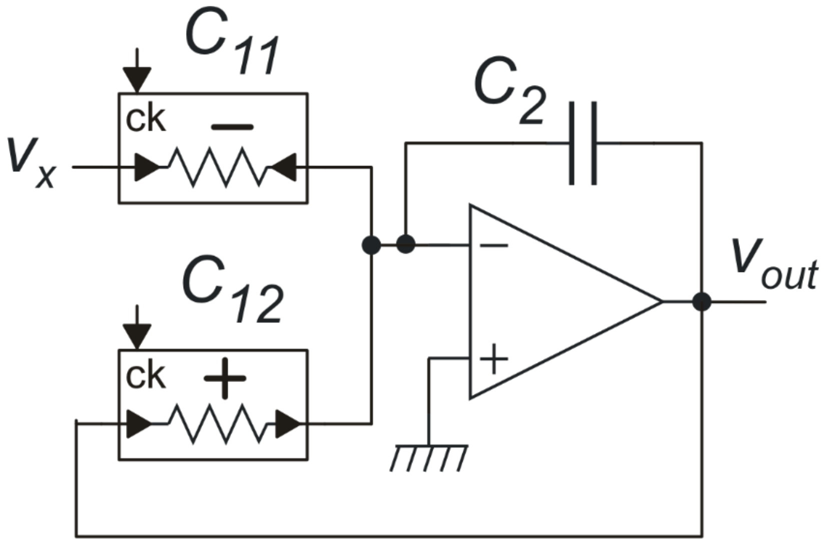


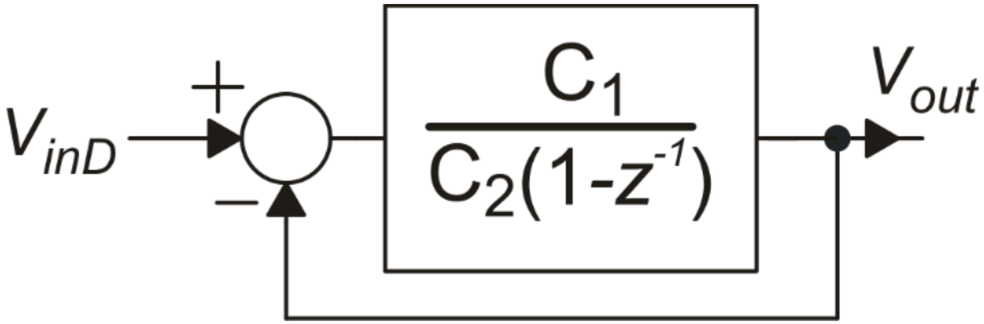
For $f \ll f_{ck}=f_s$

First order filter: discrete time nature



$$V_{out}(z) = \frac{k_C}{1-z^{-1}} v_{inD}(z) - \frac{k_C}{1-z^{-1}} V_{out}(z)$$

$$V_{out}(z) = \frac{k_C}{(1+k_C) - z^{-1}} v_{inD}(z)$$



$$z_p = \frac{1}{1+k_C} \qquad 0 < z_p < 1$$

always stable !

Step response $V_{out}(n) = \left(1 - z_p^n\right) u(n)$

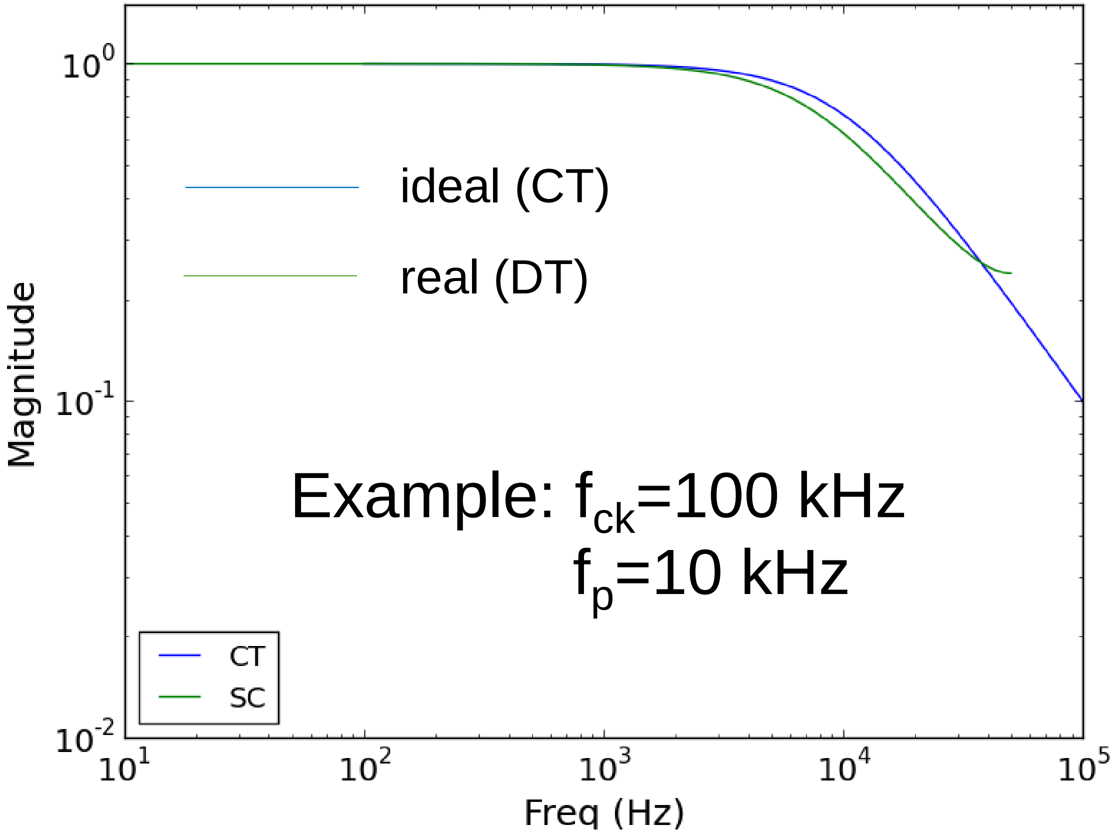
always monotonic !

First order LP filter: frequency response

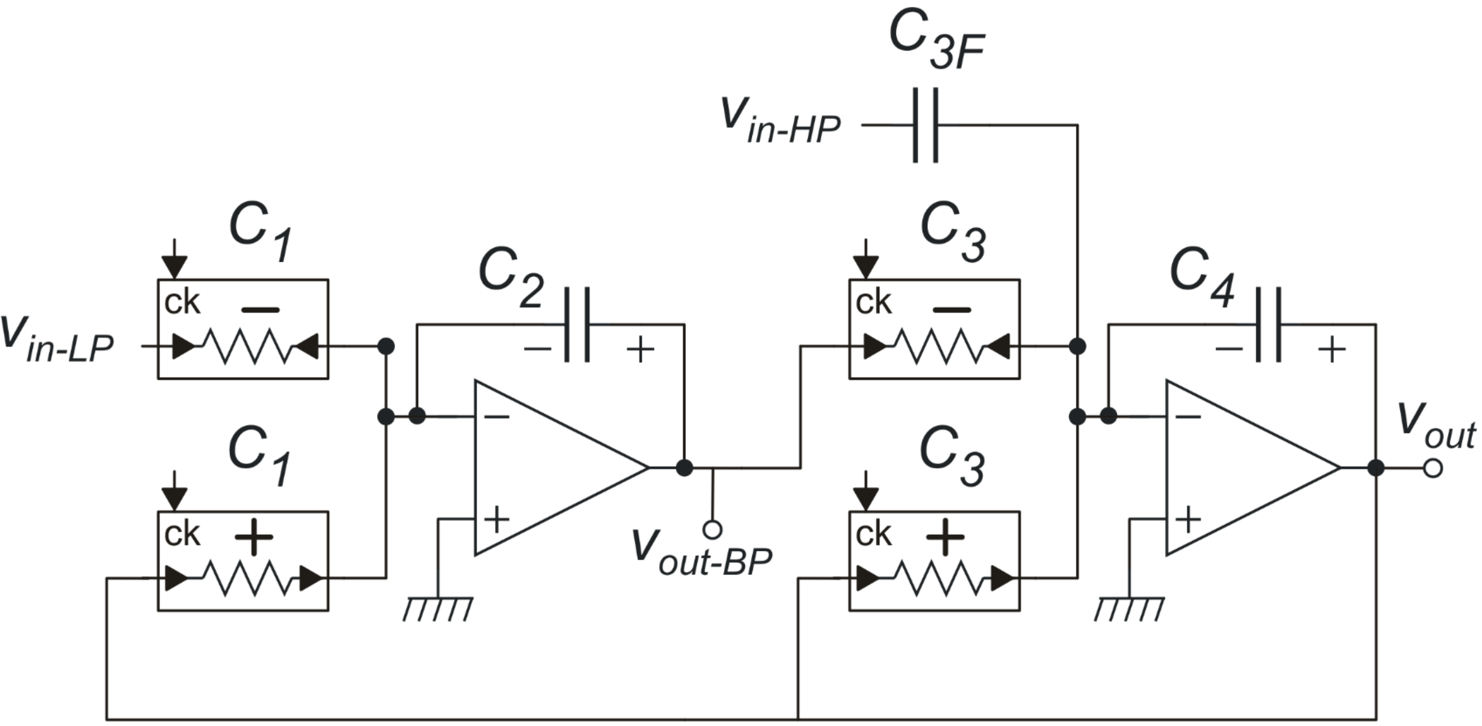
$$z^{-1} = e^{-j\omega T} \cong 1 - j\omega T$$

$$H(e^{j\omega T}) \cong \frac{k_C}{k_C + j\omega T} e^{-j\omega T/2}$$

$$\omega_p = \frac{k_C}{T} = f_{ck}\frac{C_1}{C_2}$$

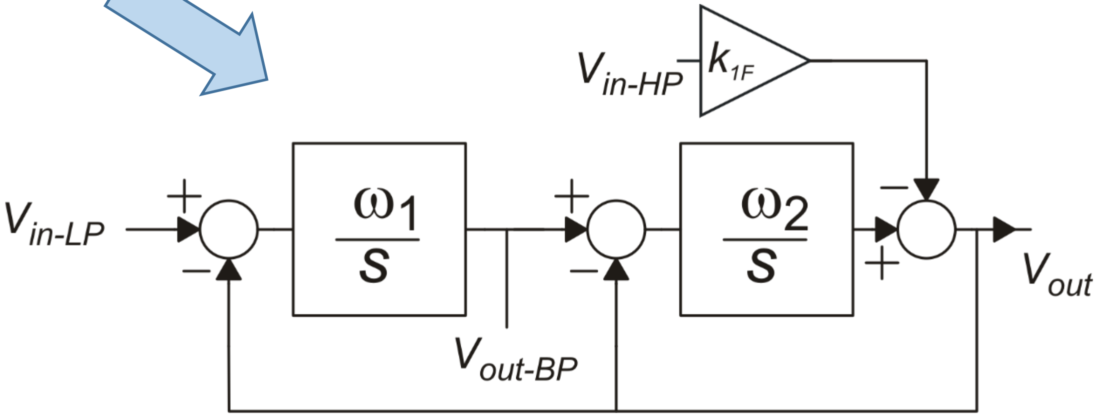


Example 3: Universal SC Biquad Filter



$$\omega_P = \sqrt{\omega_1 \omega_2} \qquad Q_P = \sqrt{\frac{\omega_1}{\omega_2}}$$

Approximation for
$f << f_{ck} = f_s$



$$\omega_1 = f_{ck} \frac{C_1}{C_2} \qquad \omega_2 = f_{ck} \frac{C_3}{C_4} \qquad k_{1F} = \frac{C_{3F}}{C_4}$$