Analog Filter Design

Part. 4: Discrete Time Filters

- Sect. 4-c: Optional Subjects

Summary of analog filters

Pure electronic / electric filters

- **RCL passive filters**
- **Active RC filters**
- **Active Gm-C filters**
- Current mode filters*
- **Switched-Capacitor filters**

Electro-mechanical filters

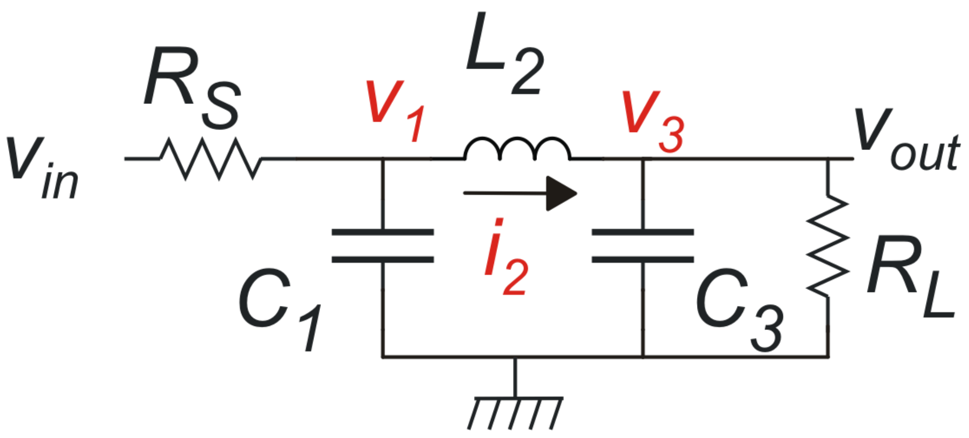
- SAW (surface acoustic wave) filters*
- BAW (Bulk acoustic wave) filters*
- MEMS resonators*

* Not covered in this course

Filter synthesis by means of LC ladder network simulation with SC integrators

- Advantage: low sensitivity with respect to component value variations

Example



$$v_1 = \frac{1}{sC_1}\left(\frac{v_{in} - v_1}{R_S} - i_2\right)$$

$$i_2 = \frac{1}{sL_2}(v_1 - v_3)$$

$$v_{out} \equiv v_3 = \frac{1}{sC_3}\left(i_2 - \frac{v_3}{R_L}\right)$$

In order to obtain an homogeneous variable set, it is convenient to define:

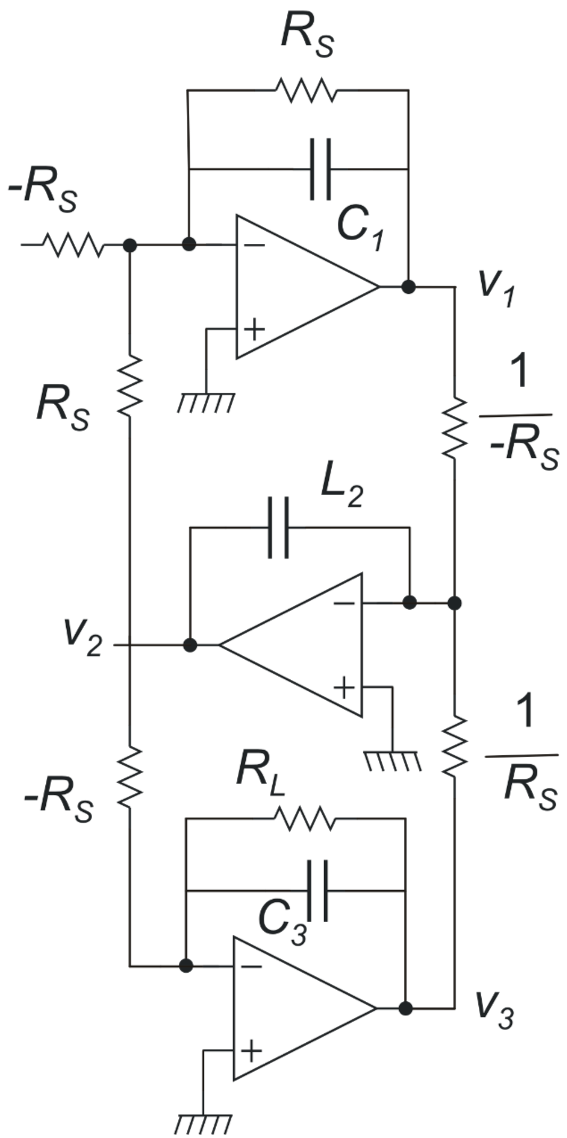
$$v_2 \equiv R_S i_2$$

Example: ladder LC network simulation

$$v_1 = \frac{1}{sC_1R_S}\left(v_{in} - v_1 - v_2\right)$$

$$v_2 = \frac{R_S}{sL_2}\left(v_1 - v_3\right)$$

$$v_{out} \equiv v_3 = \frac{1}{sC_3R_L}\left(v_2\frac{R_L}{R_S} - v_3\right)$$



-) Indicated values represent numerical identities (dimensions are not relevant)

-) Resistors are implemented with either positive or negative parasitic insensitive switched capacitors resistances.

SC Filters that do not require the equivalent resistance approximation

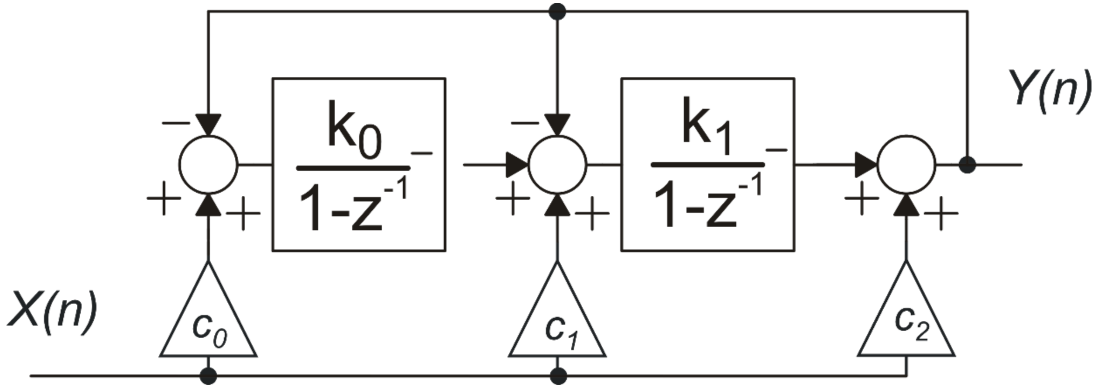
- These filters are obtained by direct implementation of the H(z) transfer function.

- The H(z) can be obtained by means of:
  -) conversion of a CT transfer function into the DT domain, by substituting *"s"* with a proper rational function of *"z"* (e.g. bilinear transformation);
  -) synthesis with the typical approaches of digital filters (e.g. FIR filters)

- Analog implementation of the H(z) can be obtained using:
  -) Conventional SC integrators cascades
  -) Bilinear integrators
  -) Direct implementation of the one-cycle delay function

Exact H(z) synthesis by means of SC Euler integrators

Example: biquad function



$$H(z) = -\frac{\dfrac{b_0}{\left(1-z^{-1}\right)^2} + \dfrac{b_1}{1-z^{-1}} + b_2}{\dfrac{a_0}{\left(1-z^{-1}\right)^2} + \dfrac{a_1}{1-z^{-1}} + 1}$$

Equivalent block diagram of an SC biquad. We consider that X is maintained constant across the whole ck period (an input Sample & Hold circuit is required.

$$a_0 = k_1 k_0 \quad a_1 = k_1$$

$$b_0 = c_0 a_0 \quad b_1 = c_1 a_1 \quad b_2 = c_2$$

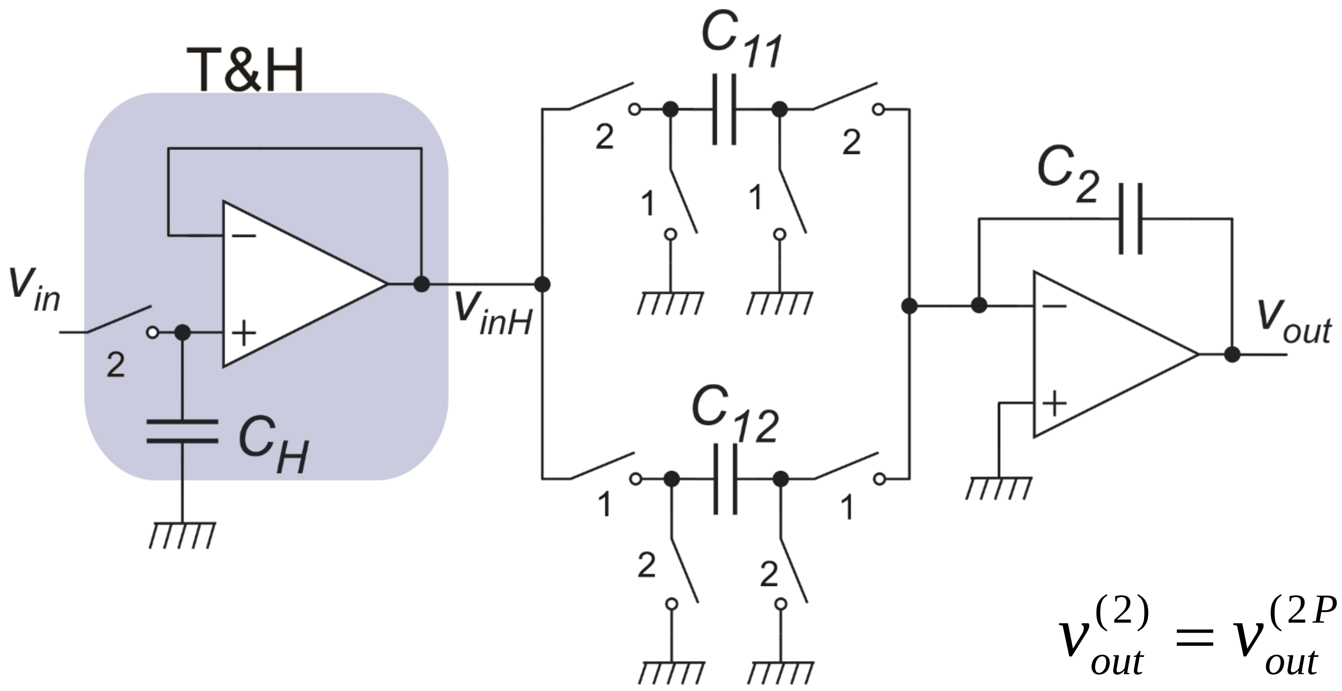
Exact H(z) synthesis by means of SC Euler integrators

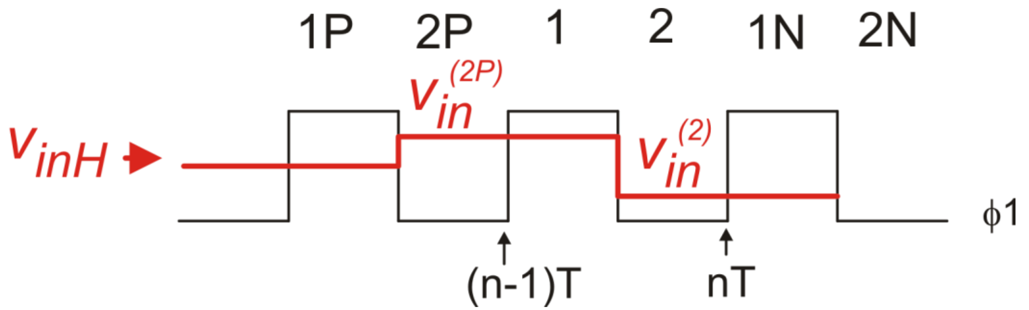
$$H(z) = -\frac{\dfrac{b_0}{\left(1-z^{-1}\right)^2} + \dfrac{b_1}{1-z^{-1}} + b_2}{\dfrac{a_0}{\left(1-z^{-1}\right)^2} + \dfrac{a_1}{1-z^{-1}} + 1} = \frac{b_0 + b_1\left(1-z^{-1}\right) + b_2\left(1-z^{-1}\right)^2}{a_0 + a_1\left(1-z^{-1}\right) + \left(1-z^{-1}\right)^2}$$

$$H(z) = \frac{b_2 z^{-2} + \left(-b_1 - 2b_2\right) z^{-1} + \left(b_0 + b_1 + b_2\right)}{z^{-2} + \left(-a_1 - 2\right) z^{-1} + \left(a_0 + a_1 + 1\right)}$$

The cascade of two simple SC integrators can implement an arbitrary 2nd order transfer function with a proper choice of the integrator coefficients (i.e. capacitance ratios).

Example: synthesis of a bilinear integrator





$$v_{inH}^{(1)} = v_{in}^{(2P)}$$
$$v_{inH}^{(2)} = v_{in}^{(2)}$$

$$v_{out}^{(2)} = v_{out}^{(2P)} - \frac{C_{12}}{C_2} v_{inH}^{(1)} - \frac{C_{11}}{C_2} v_{inH}^{(2)}$$

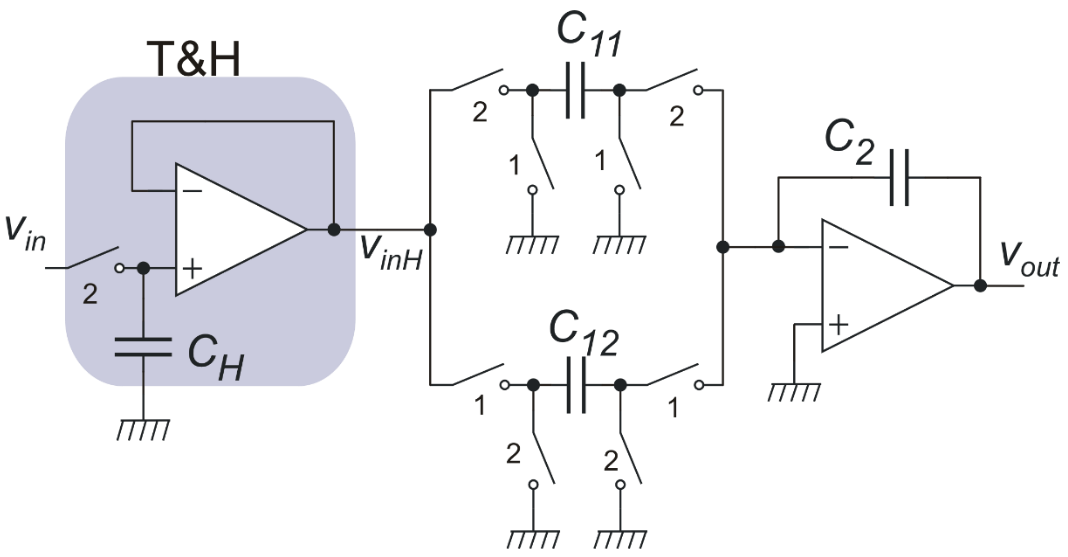
$$v_{out}^{(1)} = v_{out}^{(2P)} - \frac{C_{12}}{C_2} v_{inH}^{(1)}$$

$$v_{out}^{(2)} = v_{out}^{(1)} - \frac{C_{11}}{C_2} v_{inH}^{(2)}$$

$$v_{out}^{(2)} = v_{out}^{(2P)} - \left(\frac{C_{12}}{C_2} v_{in}^{(2P)} + \frac{C_{11}}{C_2} v_{inH}^{(2)} \right)$$

Example: synthesis of a bilinear integrator

$$v_{out}^{(2)} = v_{out}^{(2P)} - \left(\frac{C_{12}}{C_2} v_{in}^{(2P)} + \frac{C_{11}}{C_2} v_{in}^{(2)} \right)$$



$$H(z) = -\frac{\frac{C_{11}}{C_2} + \frac{C_{12}}{C_2} z^{-1}}{1 - z^{-1}}$$

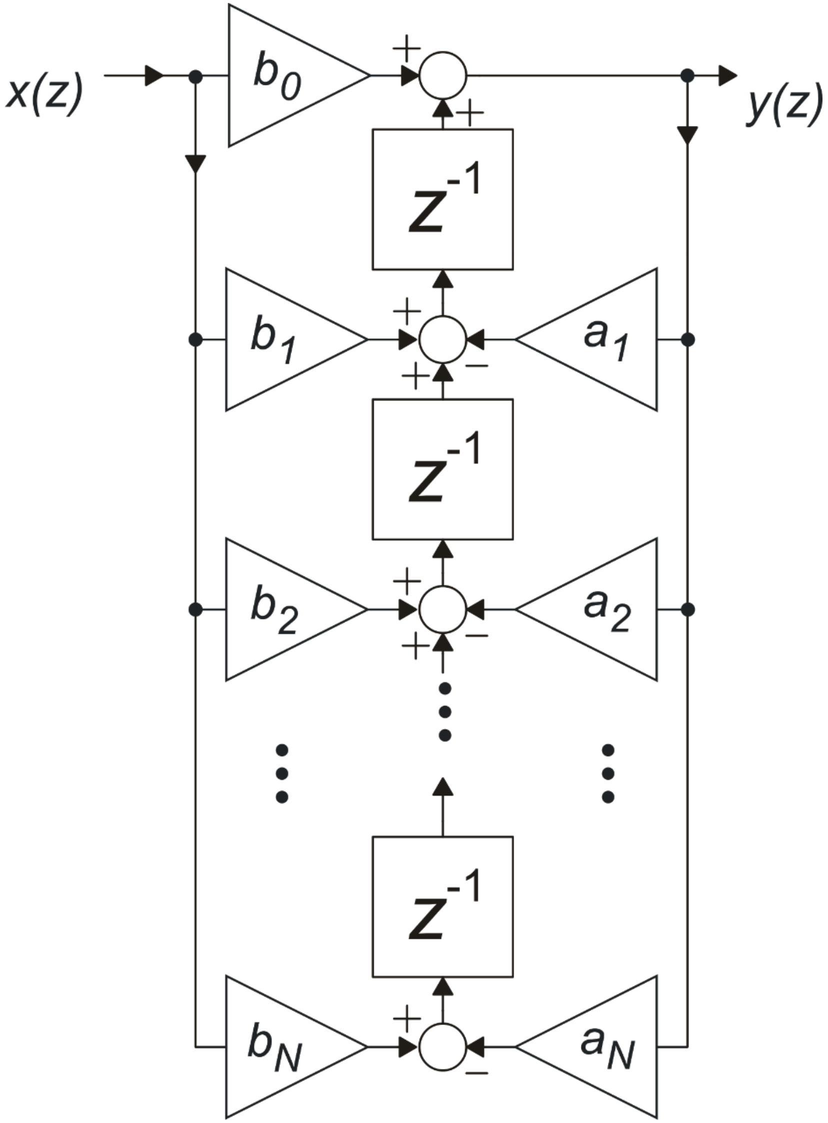
bilinear integrator (inverting)

for : $C_{11} = C_{12}$

$$H(z) = -\frac{C_{11}}{C_2} \frac{1 + z^{-1}}{1 - z^{-1}}$$

$$= -\omega_0 \frac{T}{2} \frac{1 + z^{-1}}{1 - z^{-1}} \Leftrightarrow -\frac{\omega_0}{s}$$

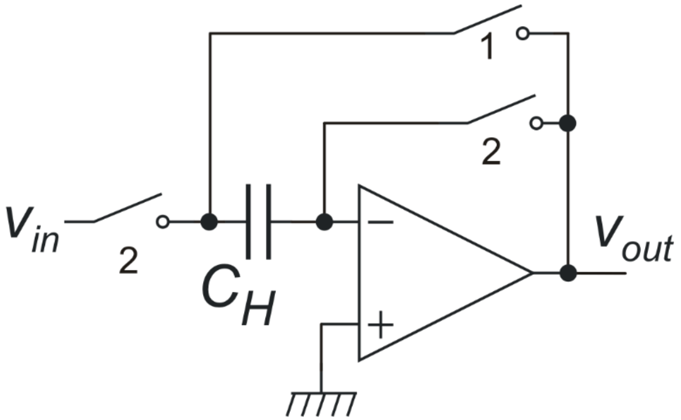
Direct synthesis



$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots. + a_N z^{N}}$$

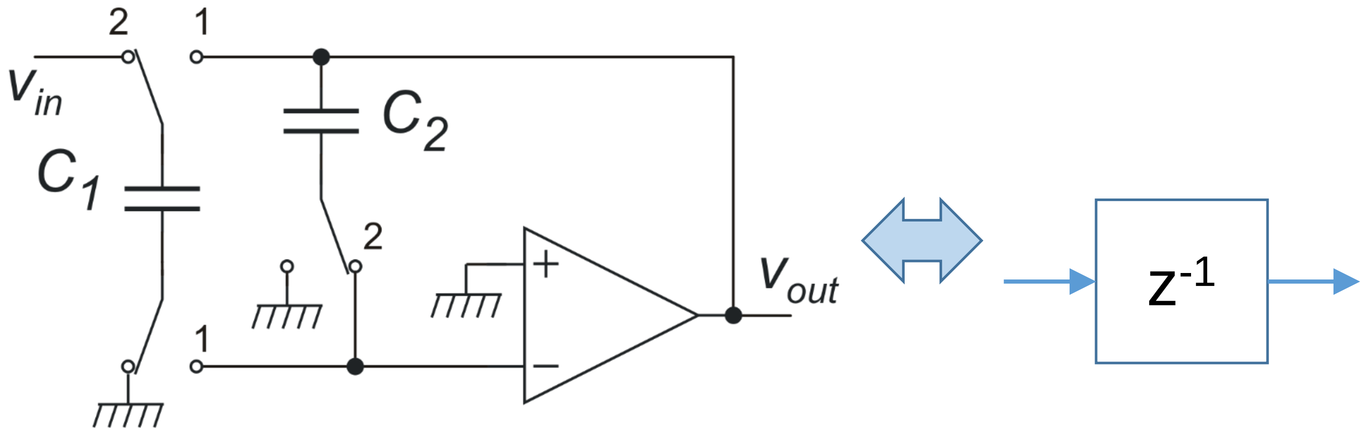
Delay lines and adders (summing amplifiers) are necessary

Direct synthesis: analog DT delay lines

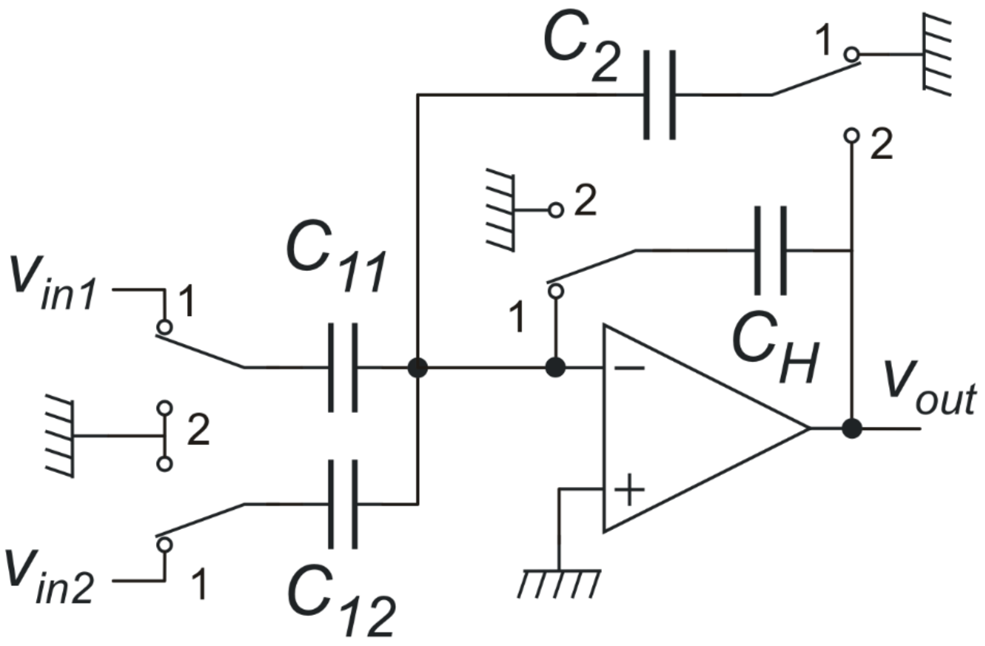


Half-period delay line
with return to zero in phase 2

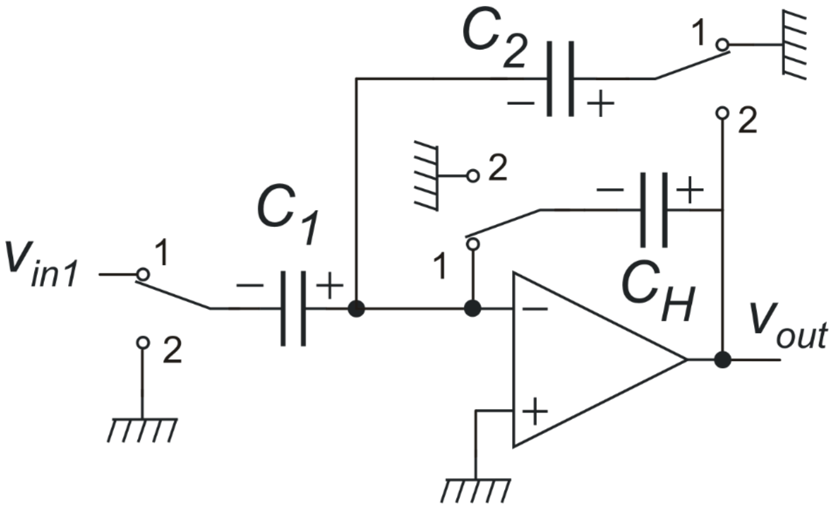
Sample and Hold
(zero tracking time)



Summing amplifier

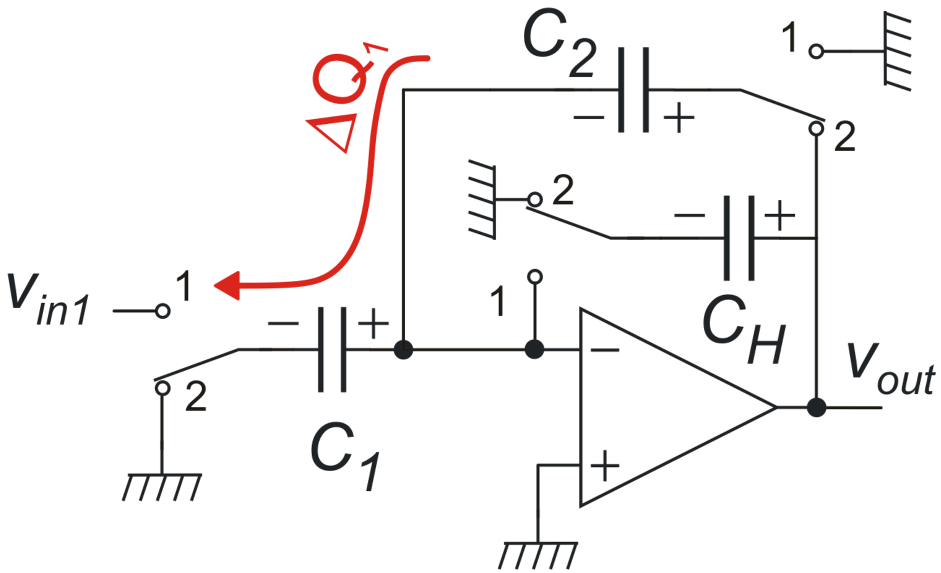


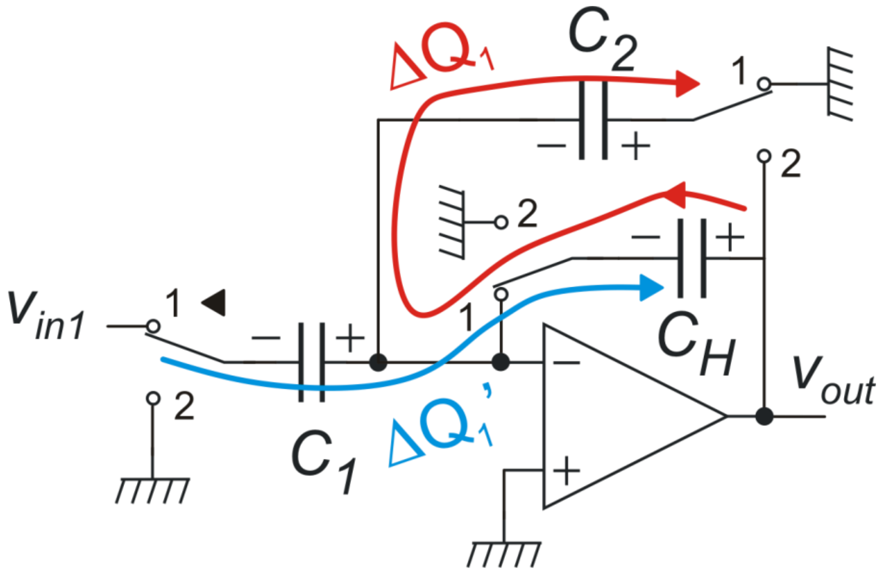
Simplified case: single input



Phase 1

Summing amplifier: analysis





$$\Delta Q_1 = C_1 v_{in}^{(1)}$$

$$v_{out}^{(2)} = \frac{C_1}{C_2} v_{in}^{(1)}$$

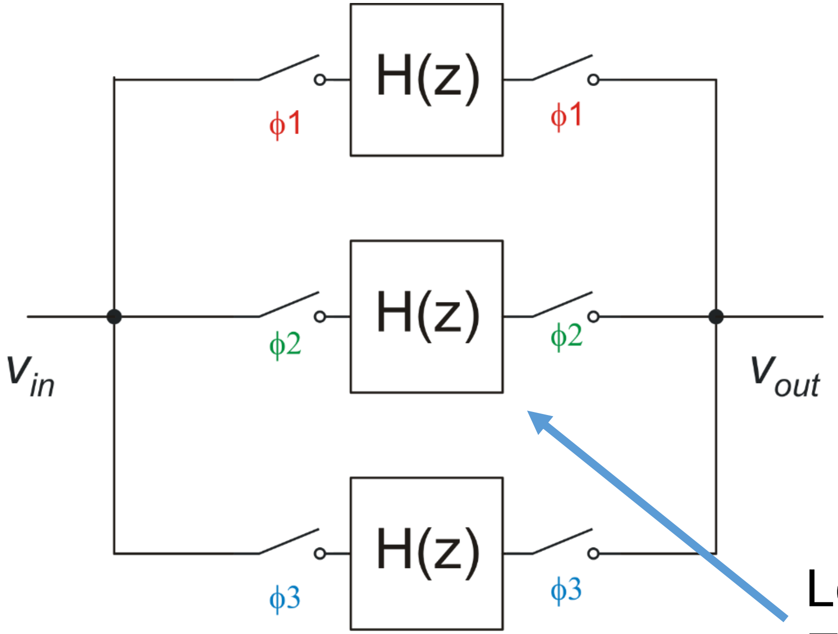
$$\Delta Q_1' = C_1 v_{in}^{(1N)}$$

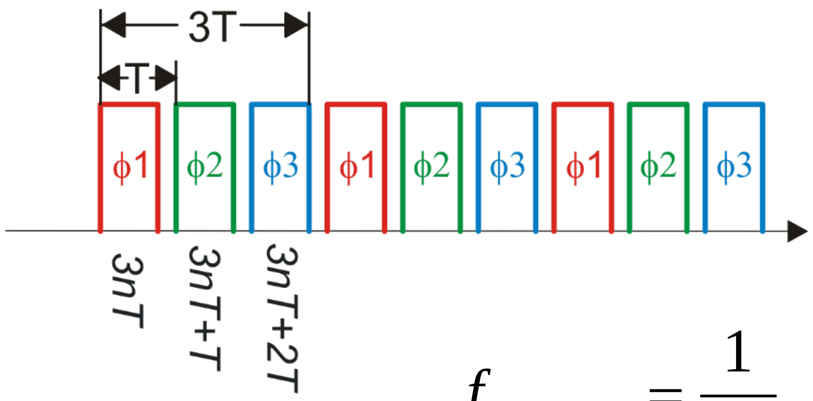
$$v_{out}^{(1N)} = v_{out}^{(2)} + \frac{C_1}{C_H}\left(v_{in}^{(1)} - v_{in}^{(1N)}\right)$$

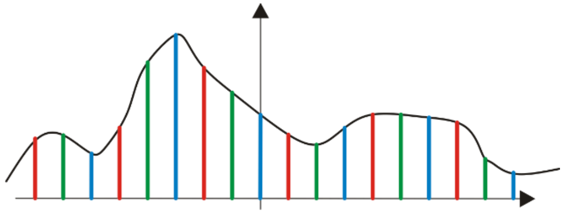
If v_{in} does not change much across a period, the output voltage is maintained in phase 1

Multipath filters

- The target is obtaining a band-pass filter with a very narrow band (i.e. an high $Q=B/f_0$) i.e. a very selective filters.
- Synthesis of very selective Band-Pass filters by means of traditional techniques is very difficult due to component inaccuracy and active element non-idealities (e.g. amplifier gain)
- Multipath filters uses N low pass filters (in this example N=3) fed with decimated sample sequences, in order to explicitly produce aliasing.



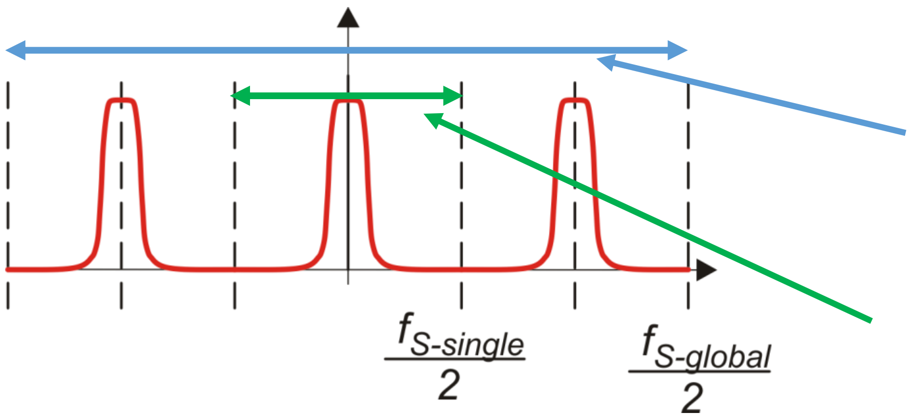




$$f_{S-single} = \frac{1}{3T} \qquad f_{S-global} = \frac{1}{T} = 3 f_{S-single}$$

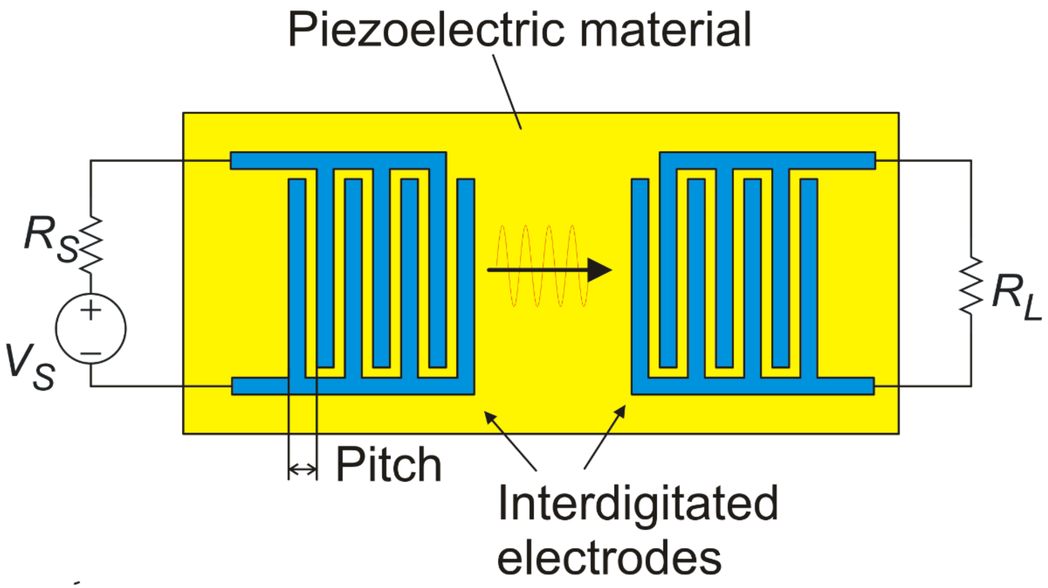
Low pass filters (nominally identical).
Each filter receive samples at frequemcy $f_{S\text{-}single}=f_{S\text{-}global}/3$

Multipath Filters



- Due to aliasing, the low pass response is duplicated around $f_{S\text{-single}}$.
- This would be meaningless for a single filter, since signals around $f_{S\text{-single}}$ are beyond the Nyquist limit
- Using all the three filters together with delayed phases is equivalent to sampling at $f_{S\text{-global}}$. Now, signals at $f_{S\text{-single}}$ are within the Nyquist limit
- The replica of the response around $f_{S\text{-single}}$ can be made very narrow, by simply reducing the bandwidth of the individual low pass filters.

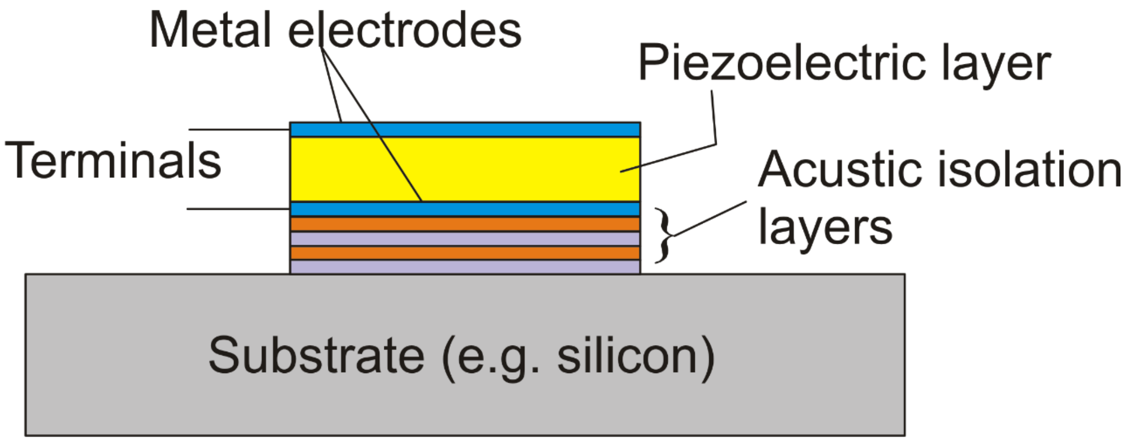
SAW and BAW filters



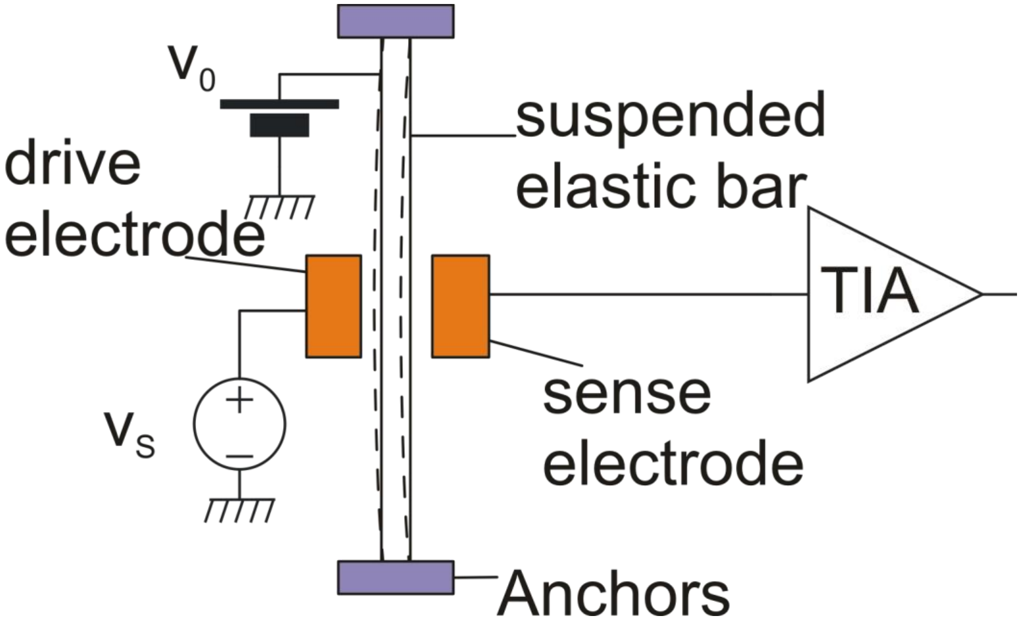
SAW basic device

$$f_{BP} = \frac{c}{pitch}$$

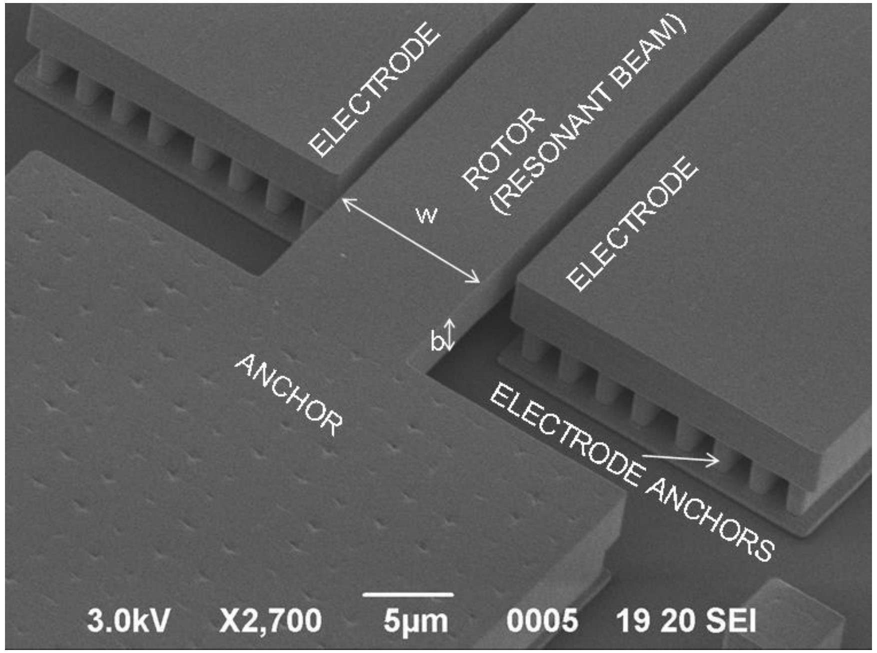
BAW device with Bragg reflector
FBAR (Film Bulk Acoustic Resonator)



MEMS resonators: examples



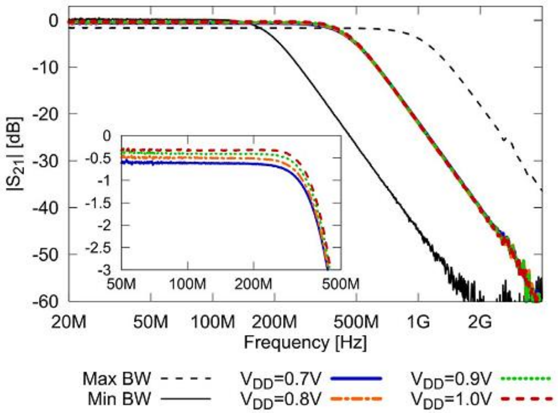
MEMS with capacitive actuation: principle of operation

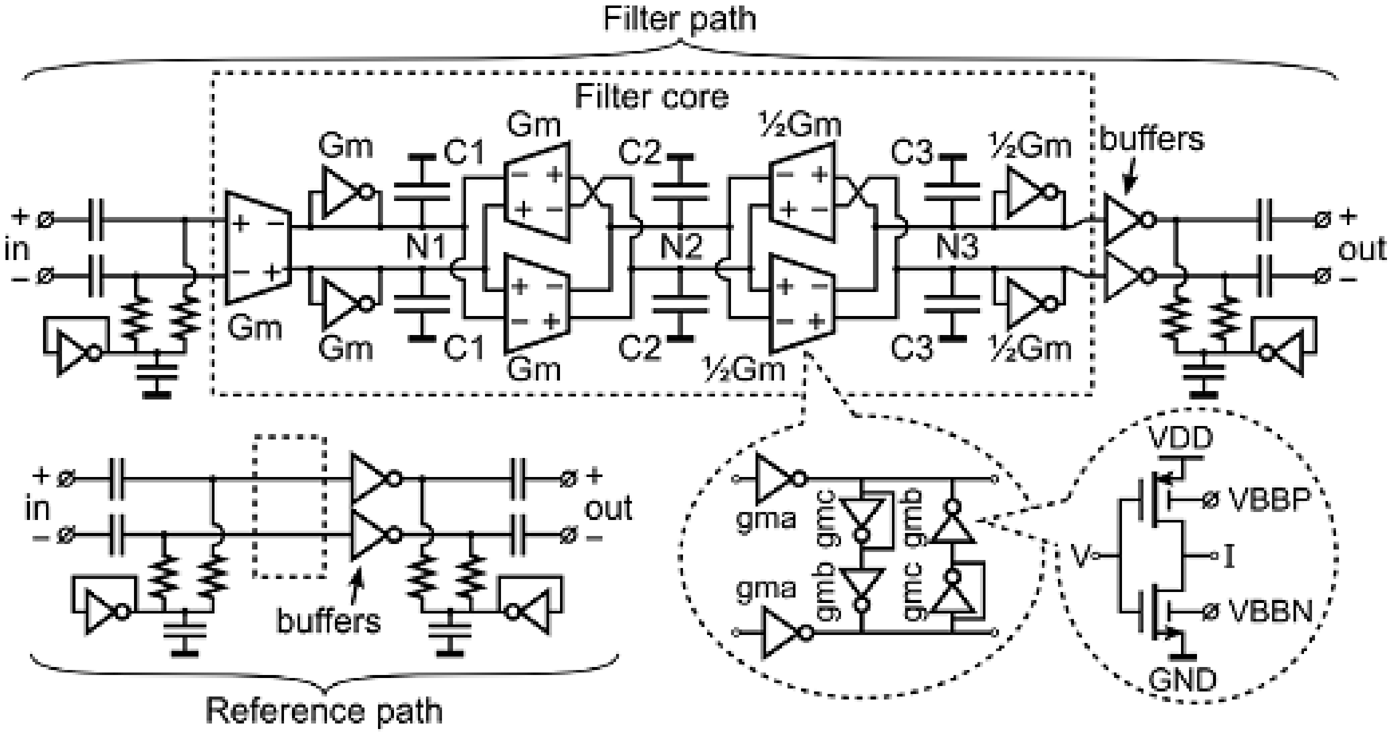


A resonator designed at the DII – Unipi (F. Pieri)

Examples of recently proposed integrated filters

The LP filter topology shown in Fig. 5.5.2 is derived from a 3rd-order, doubly terminated Butterworth LC ladder prototype using gyrator synthesis [1].





J. Lechevallier et. al, “A Forward-Body-Bias Tuned 450MHz GmC 3 rd-Order Low-Pass Filter in 28nm UTBB FD-SOI with >1dBVp IIP3 over a 0.7-to-1V Supply” ISSCC 2015