Analog Filter Design

Part. 3: Time Continuous (TC) Filter Implementation

Sect. 3-a: Active Filters

Motivations

- Inductors are generally difficult to miniaturize
  L ~ (coil area) x (number of coils)^2 x (magnetic permeability)
  Integrated inductors limited to a few nH (max)
  Stray magnetic field cause unwanted coupling
- Resistors and capacitors can be easily integrated: feasible ranges are much wider than for inductors
- Active Filters Target: Synthesis of arbitrary transfer functions using only resistors, capacitors and active elements.

Design approaches for active TC filters

System-level architectures

- Cascade of <u>Biquadratic (Biquad) </u>and Bilinear cells
- State Variable Filters (MLF: Multiple Loop Feedback circuits)
- Simulation of LC filters with active RC networks

Circuit-level architectures

- ➢ Op-amp based
- ➢ OTA (Operational transconductance amplifier) – based

Cascade of Biquad (Bilinear) functions

- Biquad Transfer Function

$$H_{BQ}(f)=\frac{c_2 s^2+c_1 s+c_0}{s^2+d_1 s+d_0}=H_0\frac{b_2 s^2+b_1\frac{\omega_z}{Q_z}s+b_0\omega_z^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

b_2,b_0 : 0, 1
b_1: 0, ±1

- Bilinear Transfer Function

$$H_{BL}(f)=H_0\frac{b_1 s+b_0\omega_z}{s+\omega_p}$$

b_1 : 0, 1
b_0: 0, ±1

"Bits" b_2,b_1,b_0 determine which terms are present in the numerator

Poles vs. Biquad coefficients

For a 2nd order polynomial with complex roots:

$$(s - s_p)(s - s_p^*) = s^2 + (s_p + s_p^*)s + s_p s_p^* = s^2 + 2\,\mathrm{Re}(s_p)s + |s_p|^2$$

$$\omega_p = |s_p| \qquad Q_P = \frac{|s_p|}{2\,\mathrm{Re}(s_p)}$$

Biquads can be easily extracted from "zpk" output of python or Matlab filter synthesis functions

For the zeroes, identical rules apply, with the exception of :

- Zeros in the origin, s^2 or s term only (b_0=0)
- Zeros to infinity, only constant term is present (b_1,b_2=0)

Notable cases

$$\frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Low pass

$$\frac{s^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

High pass

$$\frac{\frac{\omega_p}{Q_p}s}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Band pass

$$\frac{s^2 + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

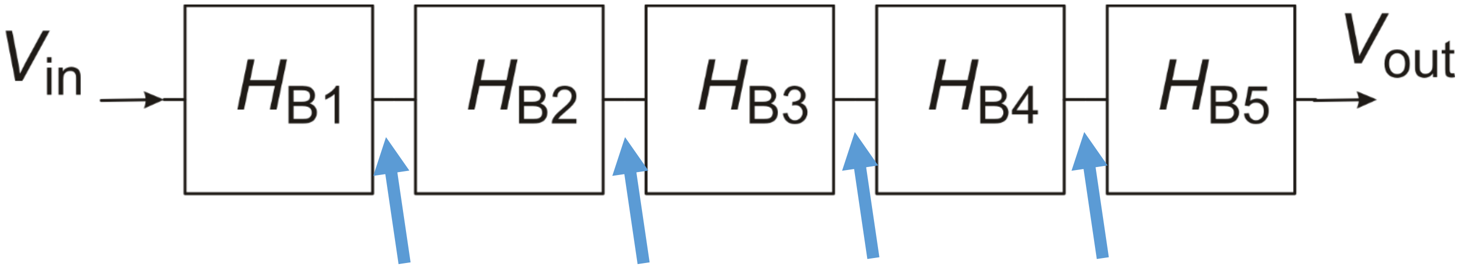
Band Stop

$$\frac{s^2 - \frac{\omega_p}{Q_p}s + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

All pass
(phase equalizer)

All these biquads have unity gain in their respective pass-bands

Sequencing criteria for biquad cascades



Requirement: $Z_{out} << Z_{in}$

Degrees of Freedom:

- Poles – Zeroes pairing (when zeroes are present)
- Physical position of each biquad in the cascade
- Pass-Band gain of each individual element of the cascade

Sequencing criteria: Targets and Rules of Thumb

Targets

- Maximize the Dynamic Range (DR)
- Minimize the transmission sensitivity (to component variations)
- Minimize the pass-band attenuation

Rules

- Pairing: couple together poles and zeroes which are closer in the s-plane (flatter response, less component spread)
- Position: Place the biquads with lower Q closer to the inputs
  Keep biquads with similar frequency of maximum as far away as possible
  If possible, place LP Biquads first and HP or BP Biquads last
- Gain distribution: balance the signal amplitude over the various biquads

Biquad implementations

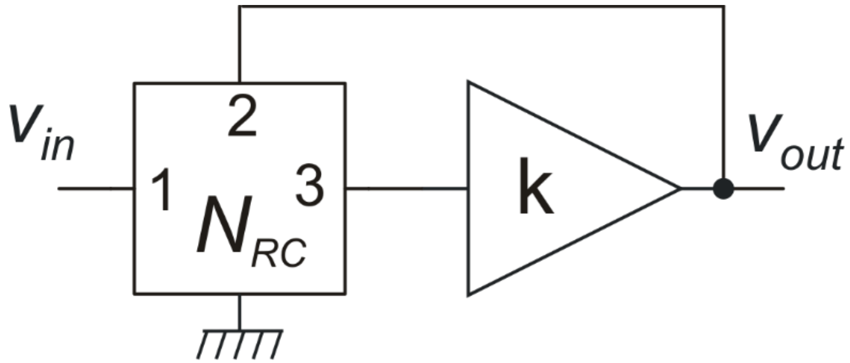
- Op-amp Based:

  - SAB (Single Opamp Biquad)
    - **Finite Gain SABs – positive feedback**
    - Finite Gain SABs – negative feedback
    - **Infinite Gain SABs**
  - Multiple op-amp Biquads (e.g. MFL)

- OTA based (Gm-C filters)

SABs

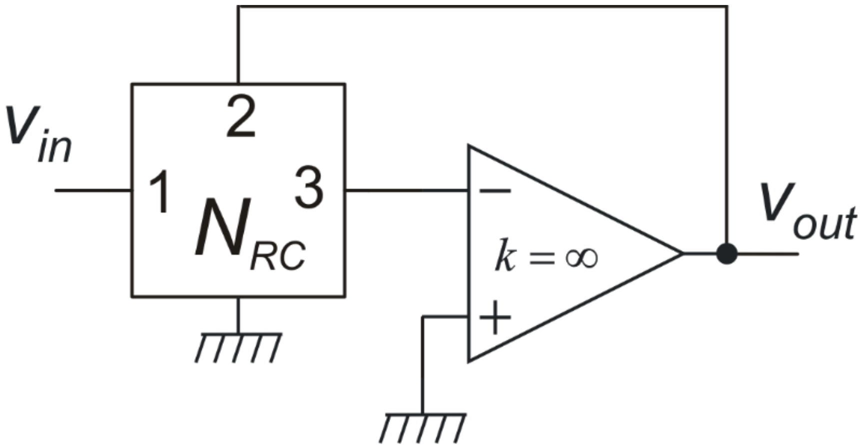
Finite gain



$$\begin{cases} I_3 = y_{13}V_1 + y_{23}V_2 + y_{33}V_3 = 0 \\ V_3 = V_2 / k \end{cases}$$

$$\frac{V_{out}}{V_{in}} = \frac{V_2}{V_1} = \frac{-y_{13}}{y_{23} + y_{33}/k}$$

Infinite gain

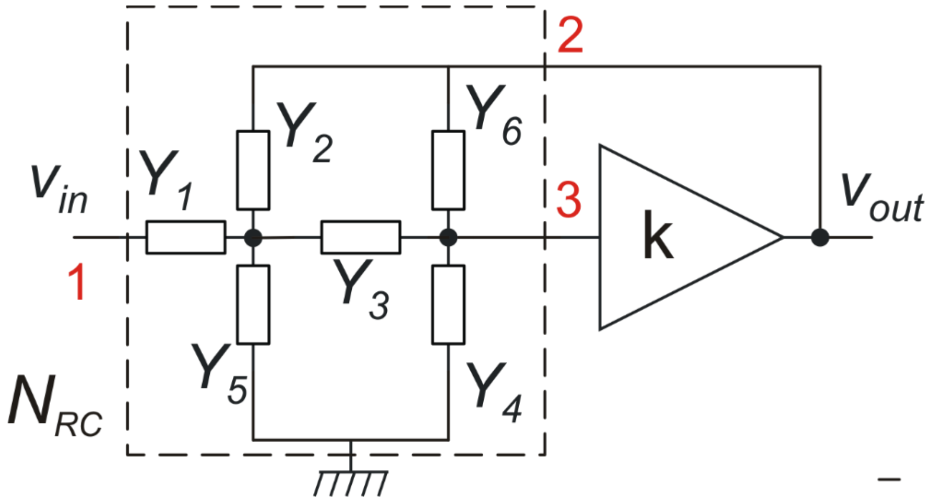


$$I_3 = y_{13}V_1 + y_{23}V_2 = 0$$

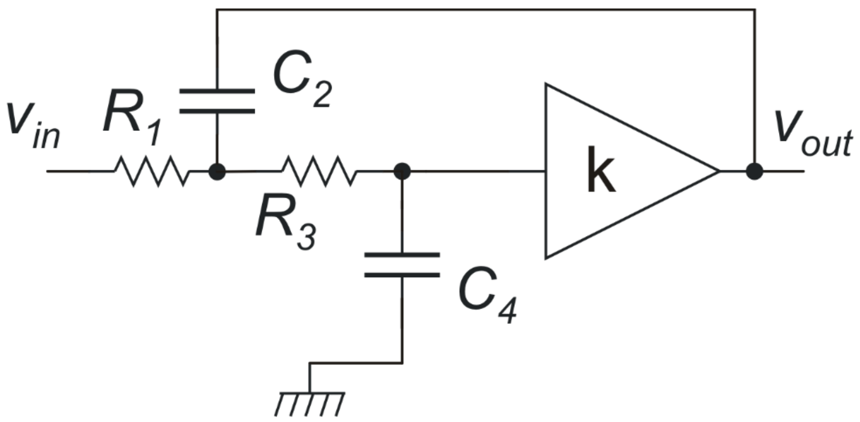
$$\frac{V_{out}}{V_{in}} = \frac{V_2}{V_1} = \frac{-y_{13}}{y_{23}}$$

Example: Sallen-Key Biquads

(R.P. Sallen, E.L. Key – MIT Labs, 1955)



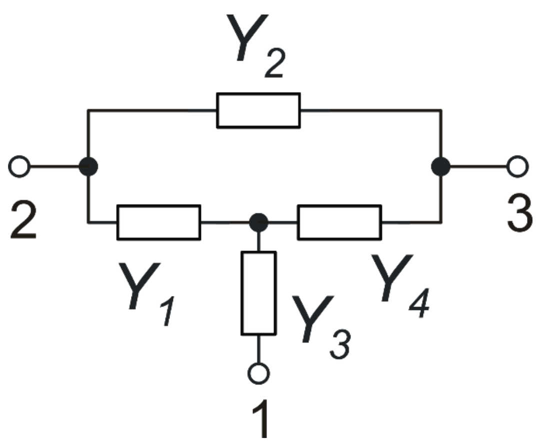
SK General configuration



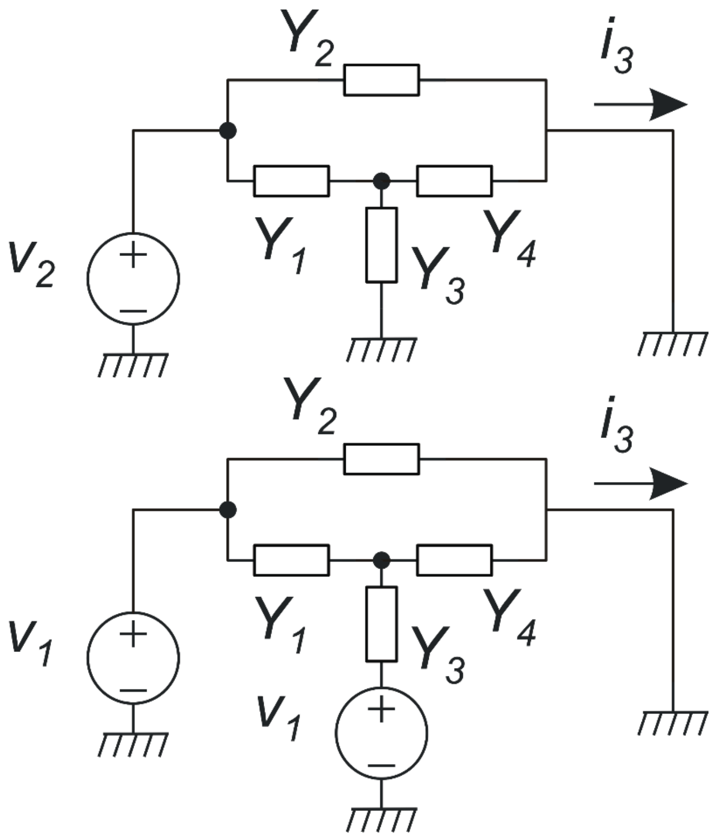
SK- Low pass filter

$$\frac{V_{out}}{V_{in}} = \frac{KY_1Y_3}{(Y_1+Y_2+Y_5)(Y_3+Y_4+Y_6)+Y_3(Y_4+Y_6)-K\{Y_6(Y_1+Y_2+Y_3+Y_5)+Y_2Y_3\}}$$

Example II: SAB with infinite amplifier gain and "bridged-T" network



Bridged –T network



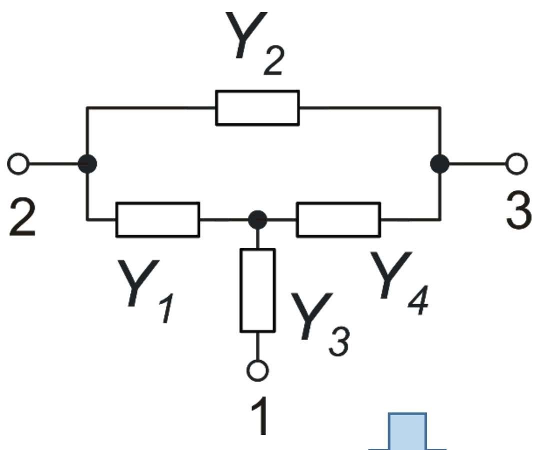
$$-y_{23} = \frac{i_3}{v_2} = \frac{Y_2\left(Y_1 + Y_3 + Y_4\right) + Y_1 Y_4}{\left(Y_1 + Y_3 + Y_4\right)}$$

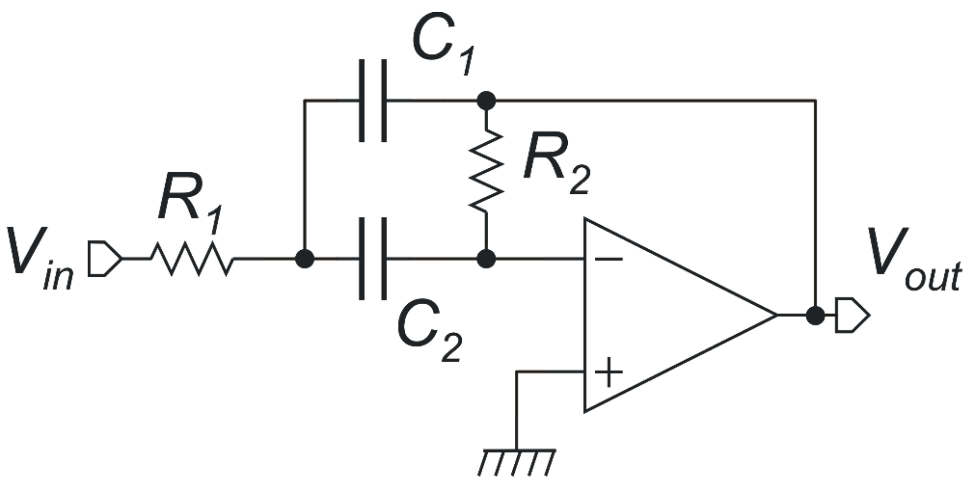
$$-y_{13} = \frac{i_1}{v_2} = \frac{Y_3 Y_4}{\left(Y_1 + Y_3 + Y_4\right)}$$

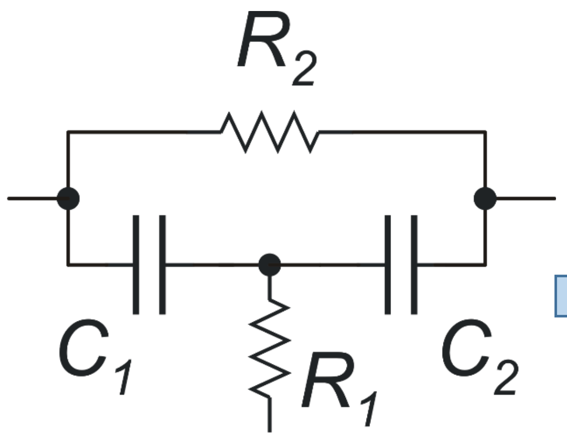
For infinite (negarive) amplifier gain:

$$H = \frac{-y_{13}}{y_{23}} = \frac{-Y_3 Y_4}{Y_2\left(Y_1 + Y_3 + Y_4\right) + Y_1 Y_4}$$

Band-pass Delyannis-Friend Biquad







$$Y_1 = sC_1$$

$$Y_2 = 1/R_2$$

$$Y_3 = 1/R_1$$

$$Y_4 = sC_2$$

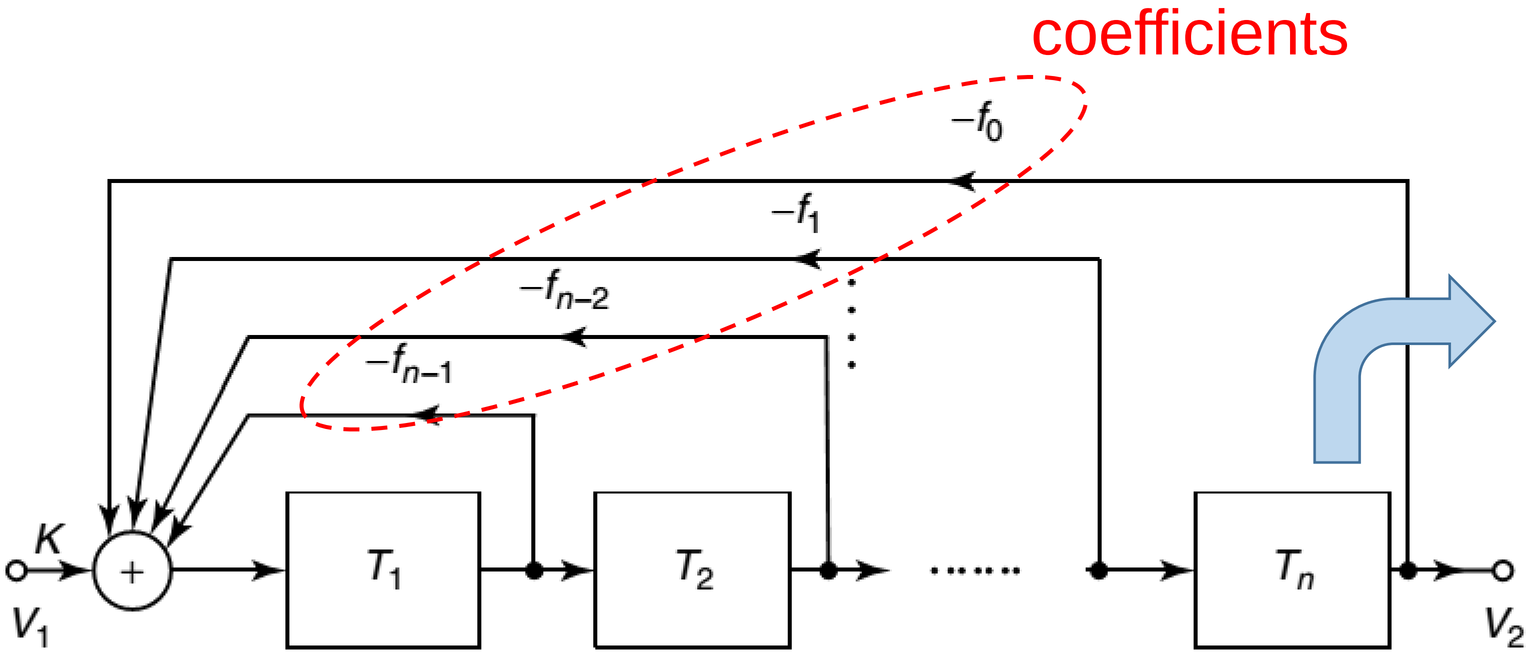
Band-pass Biquad

$$H(s) = \frac{\dfrac{s}{R_1 C_2}}{s^2 + \dfrac{1}{R_2 C_s} s + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

Multiple Feedback Loop Filters

- Cascaded Biquads: Feedback exist only inside blocks
- MFL Filters: Feedback involve all stages together

⇨ More Interaction: less sensitivity to component variations



$T_i(s)$ can be:

- Integrators
- Lossy Integrators
- Biquads

"Follow the Leader Filter" (FLF) architecture"

Integral representation of transfer functions

$$\frac{V_2}{V_1}=\frac{a_n s^n + a_{n-1}s^{n-1}....+a_1 s + a_0}{s^n + b_{n-1}s^{n-1}....+b_1 s + b_0}$$

generic rational transfer function

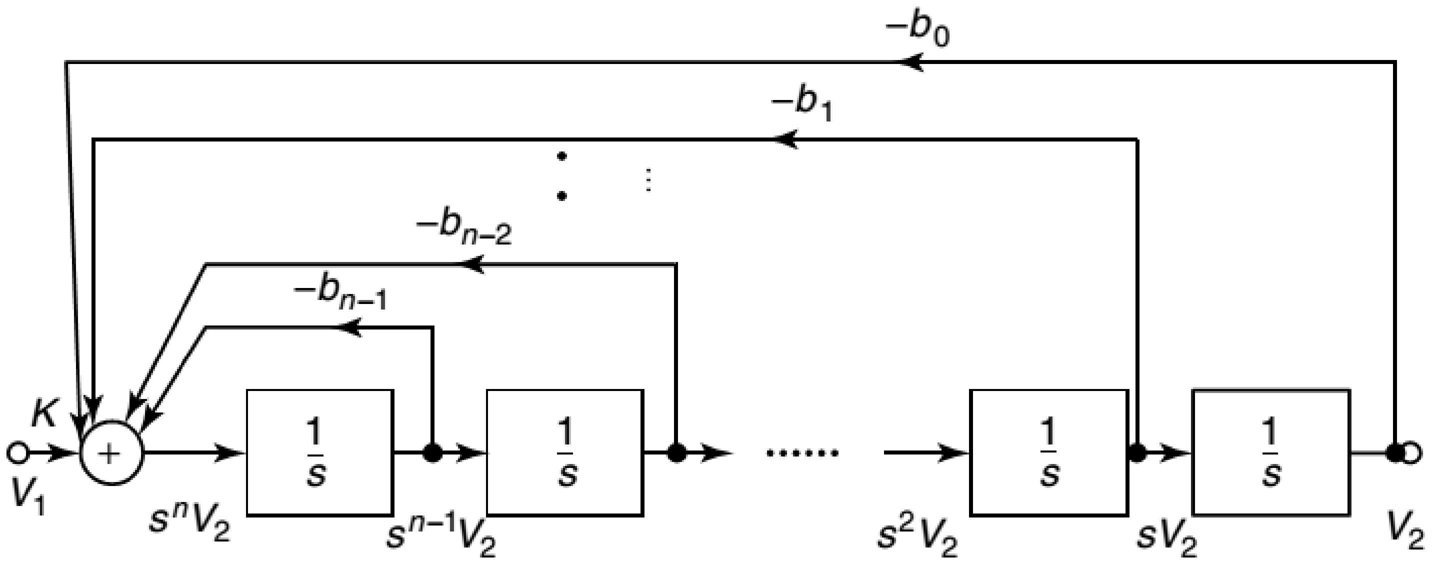
$$\left(s^n + b_{n-1}s^{n-1}....+b_1 s + b_0\right)V_2 = \left(a_n s^n + a_{n-1}s^{n-1}....+a_1 s + a_0\right)V_1$$

divide by s^n

$$\left(1+b_{n-1}\frac{1}{s},...,+b_1\frac{1}{s^{n-1}}+b_0\frac{1}{s^n}\right)V_2 = \left(a_n + a_{n-1}\frac{1}{s},...,+a_1\frac{1}{s^{n-1}}+a_0\frac{1}{s^n}\right)V_1$$

$$V_2 = \left(-b_{n-1}\frac{1}{s},...,-b_1\frac{1}{s^{n-1}}-b_0\frac{1}{s^n}\right)V_2 + \left(a_n + a_{n-1}\frac{1}{s},...,+a_1\frac{1}{s^{n-1}}+a_0\frac{1}{s^n}\right)V_1$$

State variable filters
(MFL filters based on Integrators)

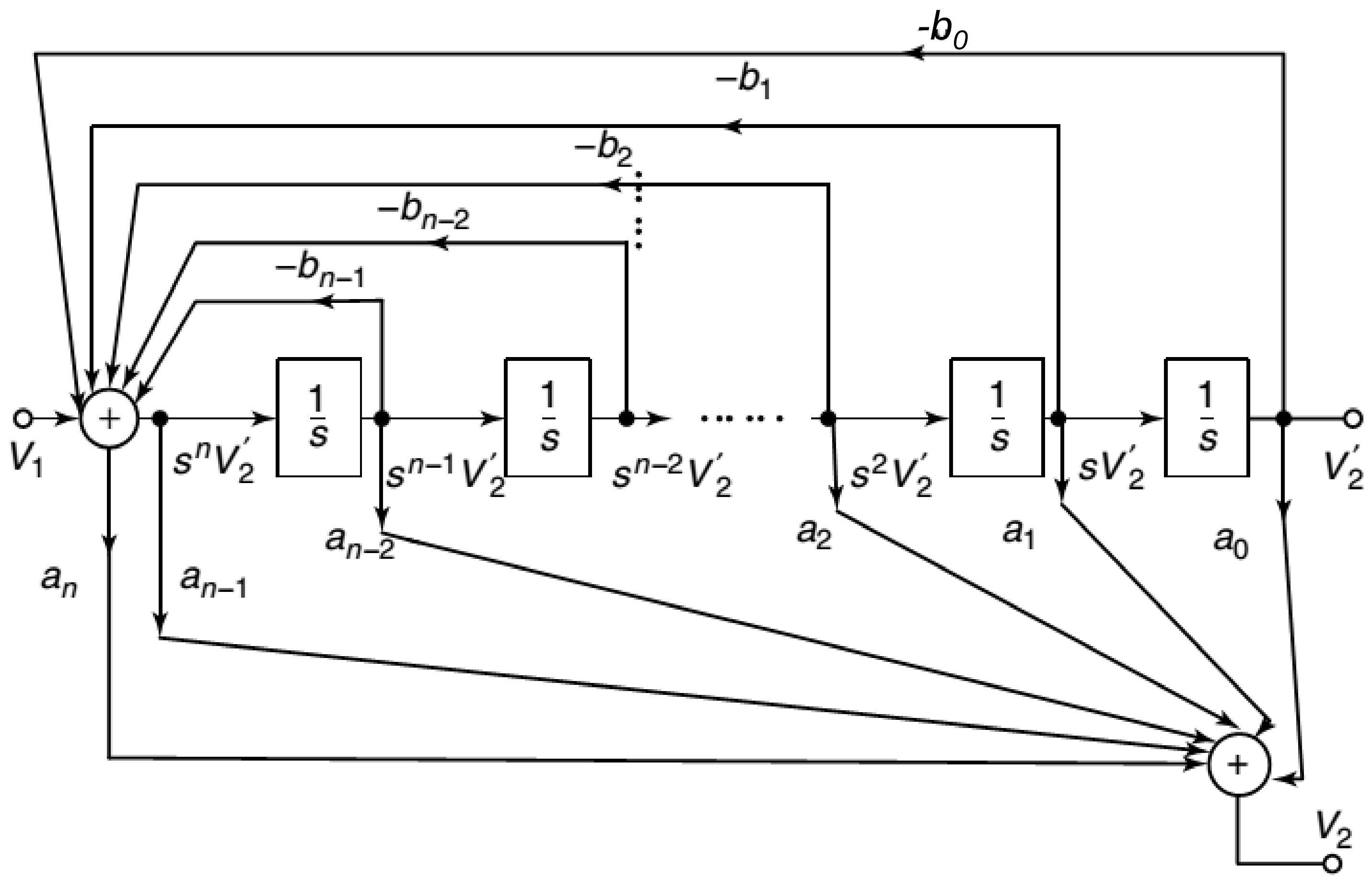


$$V_2 = V_1\left(\frac{1}{s}\right)^n - b_0\left(\frac{1}{s}\right)^n V_2 - b_1\left(\frac{1}{s}\right)^{n-1} V_2 \;\ldots.. - b_{n-1}\left(\frac{1}{s}\right)V_2$$

$$\frac{V_2}{V_1} = \frac{K}{s^n + b_{n-1}s^{n-1} \ldots. + b_1 s + b_0}$$

⇒ Low pass, all poles filters

Multi Feedback – Multi Feed Forward



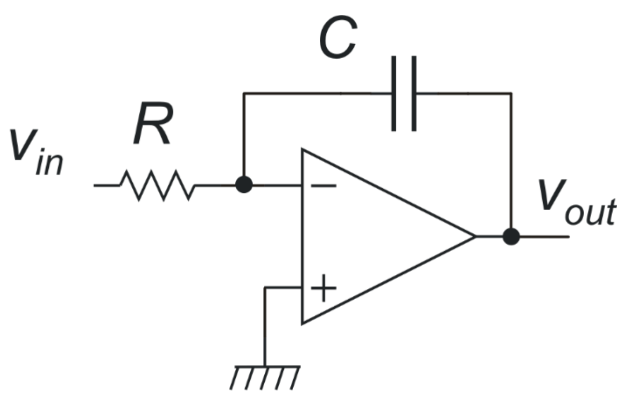
$$\frac{V_2'}{V_1} = \frac{1}{s^n + b_{n-1}s^{n-1} .... + b_1 s + b_0}$$

$$\frac{V_2}{V_1} = \frac{a_n s^n + a_{n-1}s^{n-1} .... + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} .... + b_1 s + b_0}$$

Arbitrary Functions
(poles and zeros)

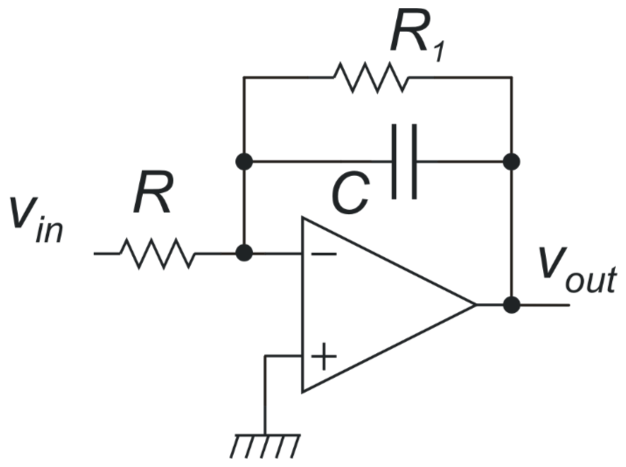
Integrators: Op-amp based solution

Integrator (inverting)



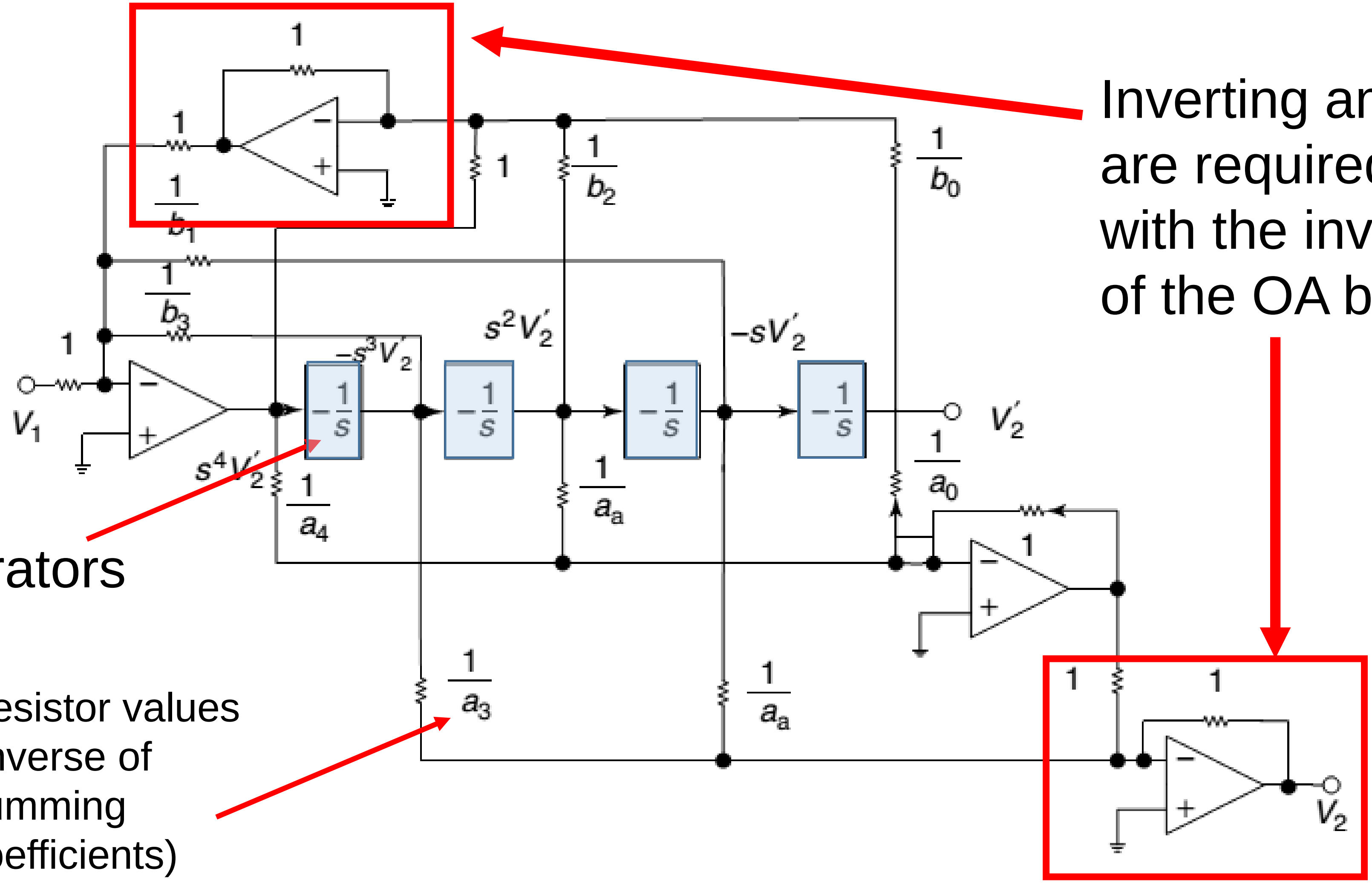
$$\frac{v_{out}}{v_{in}} = -\frac{1}{RC}\frac{1}{s}$$

Lossy Integrator (inverting)



$$\frac{v_{out}}{v_{in}} = -\frac{R_1}{R}\left(\frac{\frac{1}{R_1 C}}{s+\frac{1}{R_1 C}}\right)$$

Example: State variable Filter with Op-amp Integrators

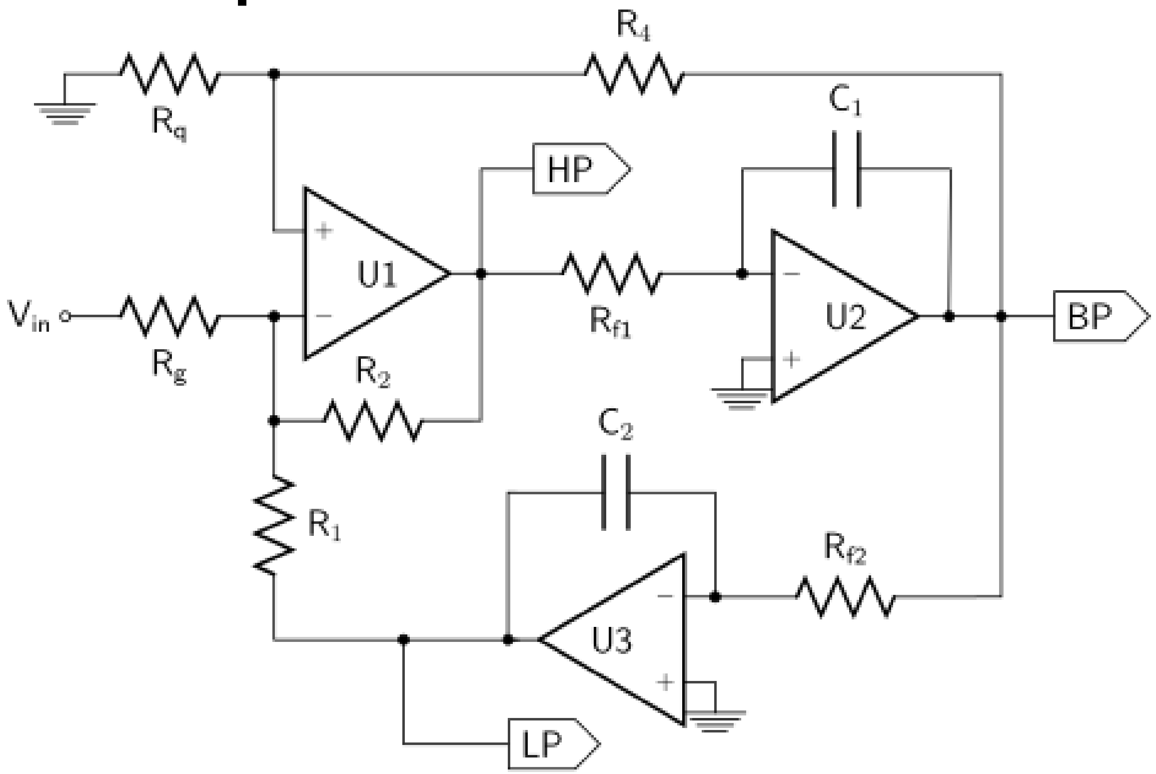


Inverting amplifiers are required to cope with the inverting function of the OA based Integrators

Opamp integrators (inverting)

Resistor values (inverse of summing coefficients)

MLF: Example Universal 2nd order Filter

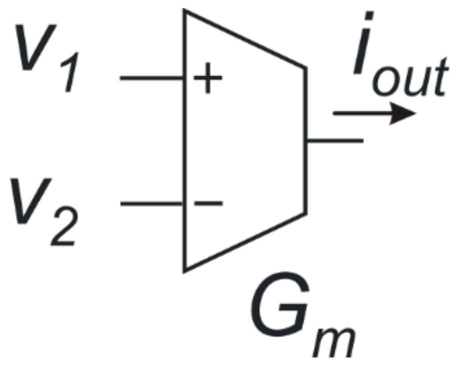


Kerwin-Huelsman-Newcomb (KHN) filter

(Produces LP, BP and HP outputs: Single Input – Multiple Output)

OTA: definitions and basic circuits

OTA (Transconductor)



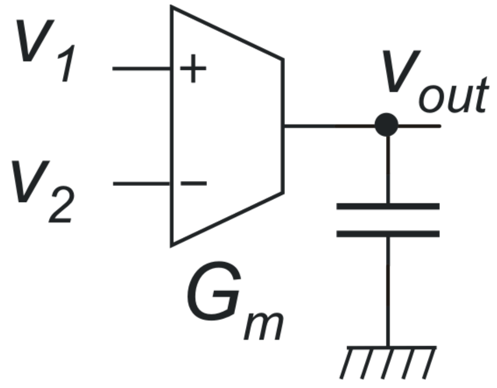
Ideal operation

$$i_{out} = G_m (v_1 - v_2)$$

Typical non-idealities:

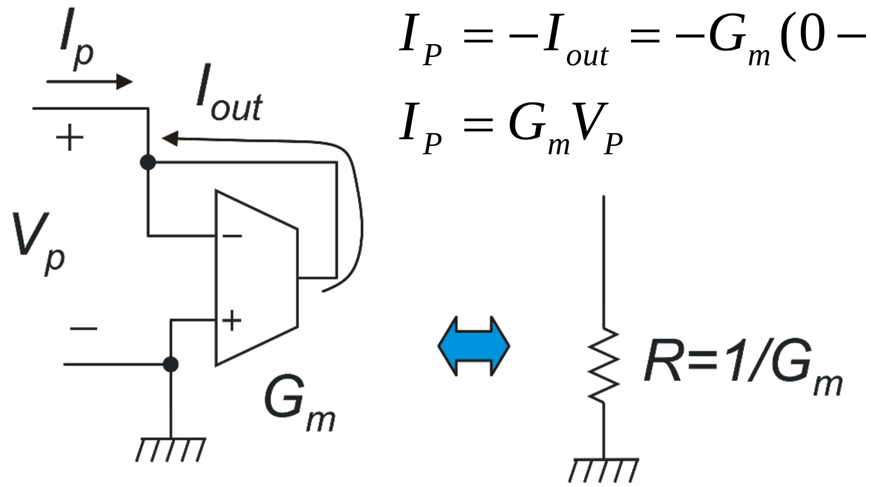
- Finite Rout
- Input Capacitance
- Frequency dependence of Gm
- Input/Output ranges

OTA-C (Gm-C) Integrator



$$v_{out} = \frac{i_{out}}{sC} = \frac{G_m}{sC}(v_1 - v_2)$$

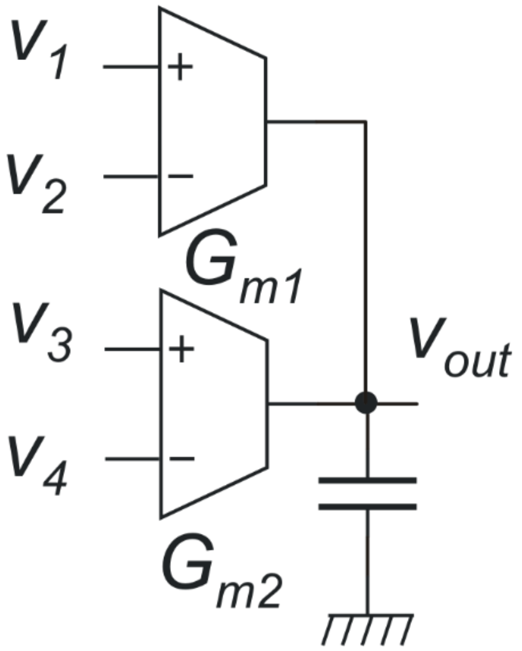
OTA-C (Gm-C) Eq. Resistor



$$I_P = -I_{out} = -G_m(0 - V_P)$$

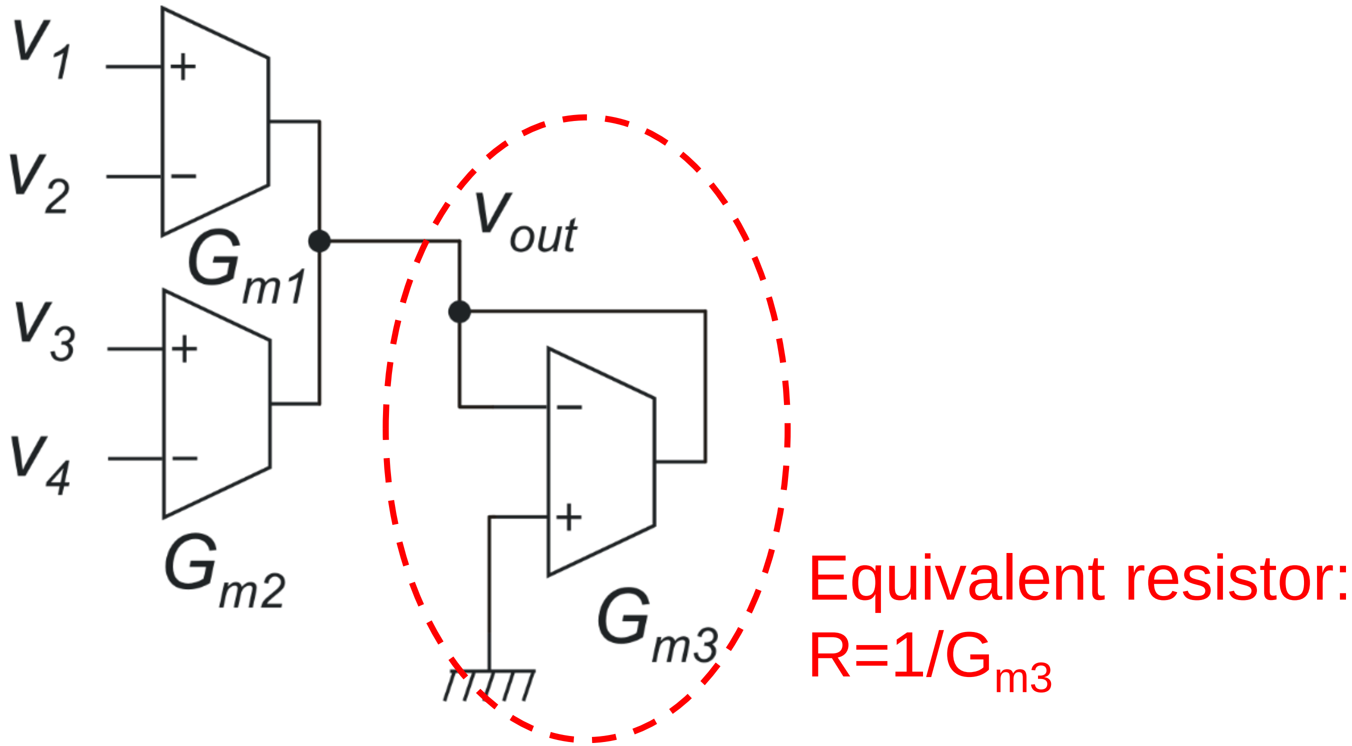
$$I_P = G_m V_P$$

Ota-Based summing circuits



Summing Integrator
(inverting / non-inverting)

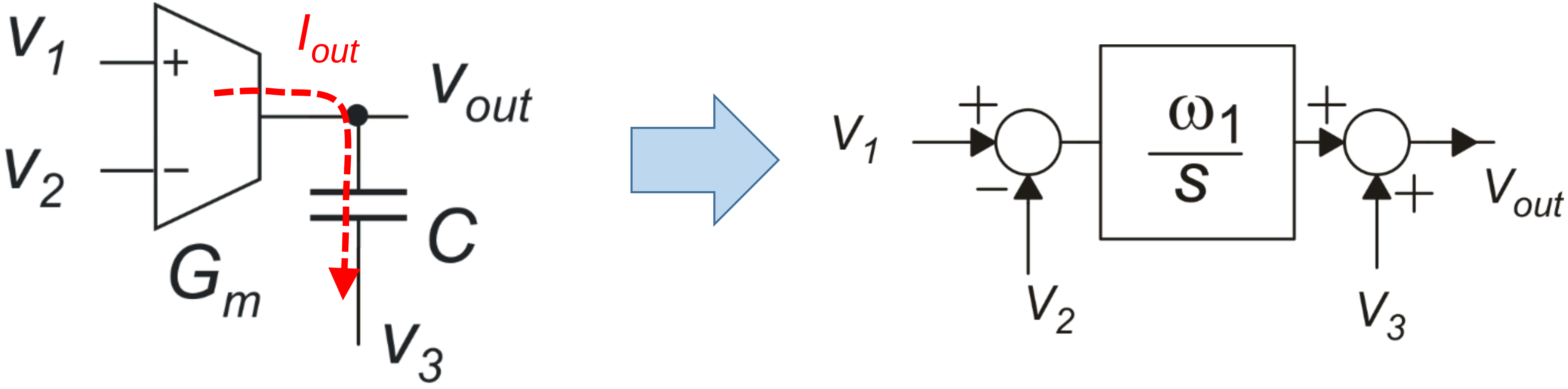
$$v_{out} = \frac{G_{m1}}{sC}\left[(v_1 - v_2) + \frac{G_{m2}}{G_{m1}}(v_3 - v_4)\right]$$



Summing amplifier
(inverting / non-inverting)

$$v_{out} = \frac{G_{m1}}{G_{m3}}(v_1 - v_2) + \frac{G_{m2}}{G_{m3}}(v_3 - v_4)$$

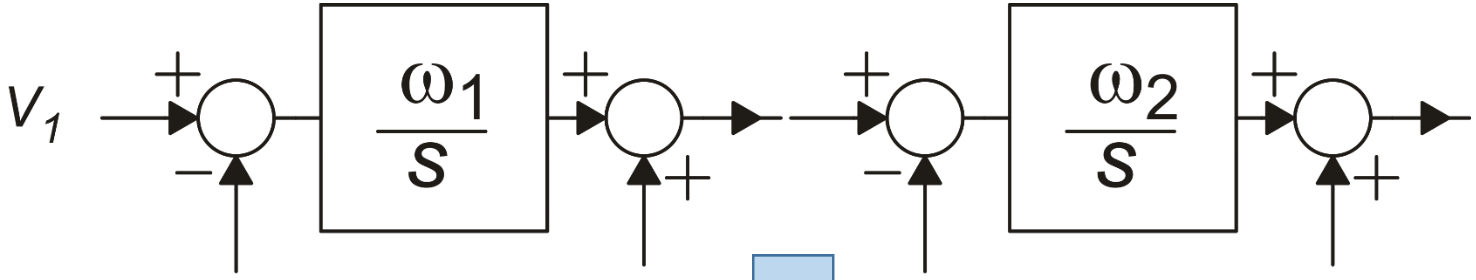
Gm-C integrator with feed-forward input



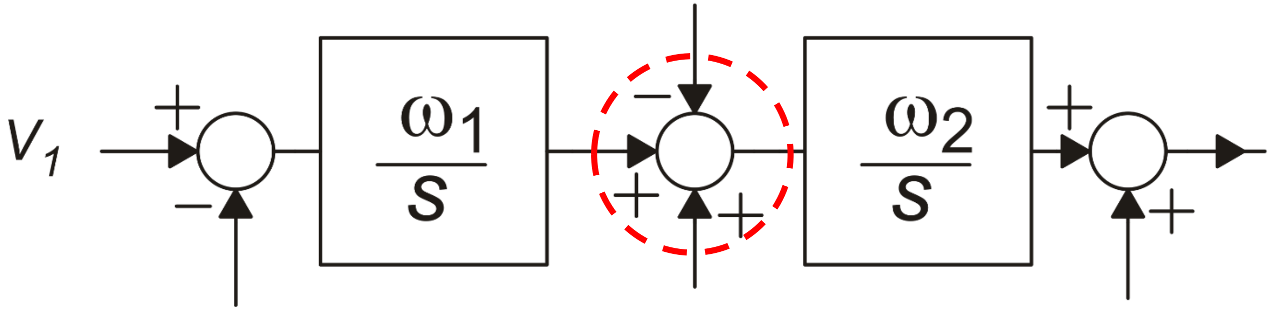
$$v_{out} = \frac{G_m}{sC}(v_1 - v_2) + v_3$$

$$\omega_1 = \frac{G_m}{C}$$

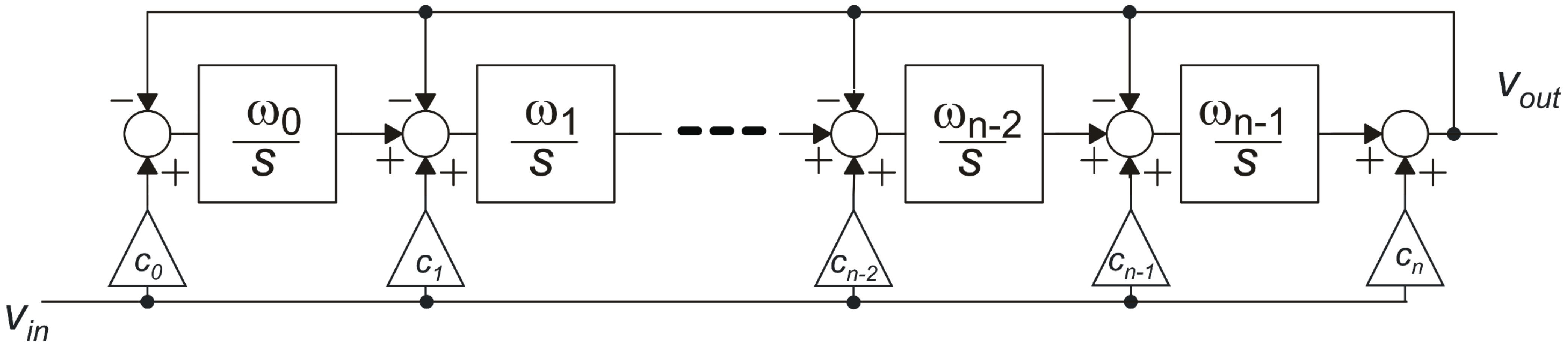
Gm-C integrator cascade



Two signals with opposite signs can be added at each internal node of the cascade



State variable filters – alternative solution



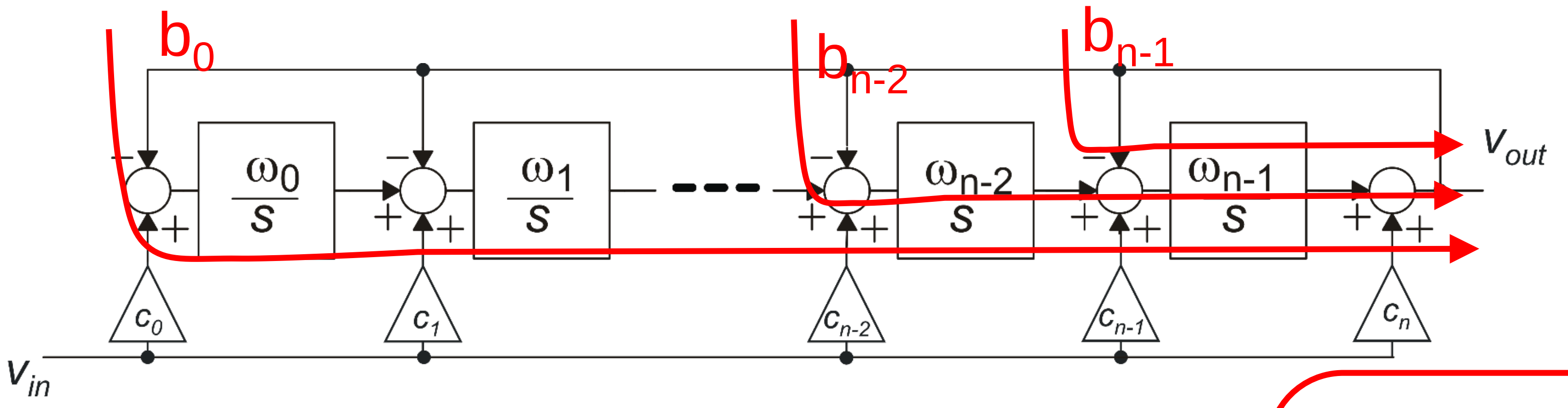
Differently from the FL filter (Follow the Leader), where all the integrator outputs are fed back to the first integrator and fed forward to the summing node, here the output voltage is fed back to the input of each integrator where also the input signal is fed forward. This architecture is more suitable for Gm-C implementations, where summing several inputs would require several OTAs

Target f.d.t.

$$\frac{V_{out}}{V_{in}} = \frac{s^n + a_{n-1}s^{n-1} .... + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} .... + b_1 s + b_0}$$

Gm-C state variable filter: coefficients

$$V_{out} = \left(-b_{n-1}\frac{1}{s},\ldots,-b_1\frac{1}{s^{n-1}}-b_0\frac{1}{s^n}\right)V_{out}+\left(a_n+a_{n-1}\frac{1}{s},\ldots,+a_1\frac{1}{s^{n-1}}+a_0\frac{1}{s^n}\right)V_{in}$$



$$b_{n-1} = \omega_{n-1}$$

$$b_{n-2} = \omega_{n-1}\omega_{n-2}$$

$$\vdots$$

$$b_0 = \omega_{n-1}\omega_{n-2}\ldots\omega_0$$

$$a_n = c_n$$

$$a_{n-1} = c_{n-1}b_{n-1}$$

$$a_{n-2} = c_{n-2}b_{n-2}$$

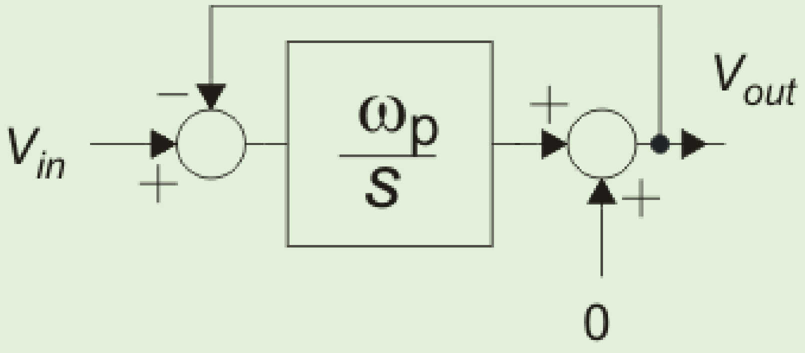
$$\vdots$$

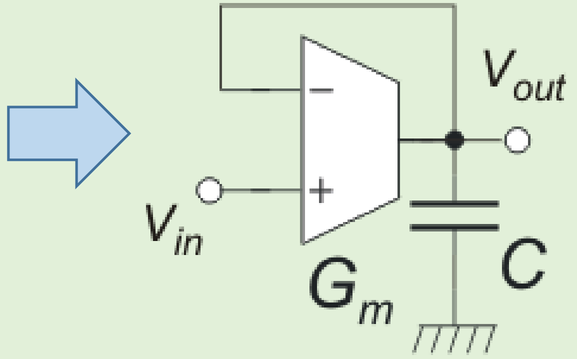
$$a_0 = c_0 b_0$$

Example: First order high pass / low pass filters

$$H(s) = \frac{\omega_p}{s + \omega_p}$$

low pass

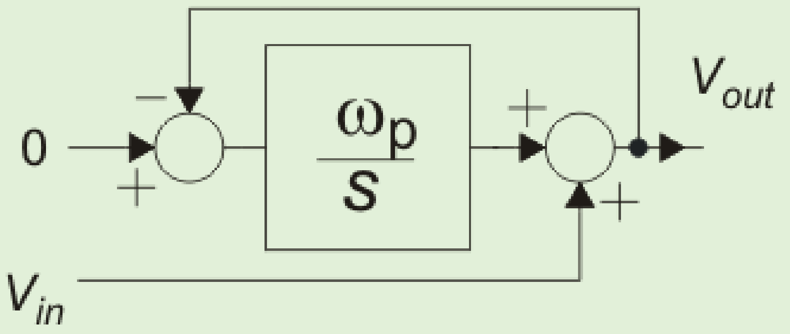


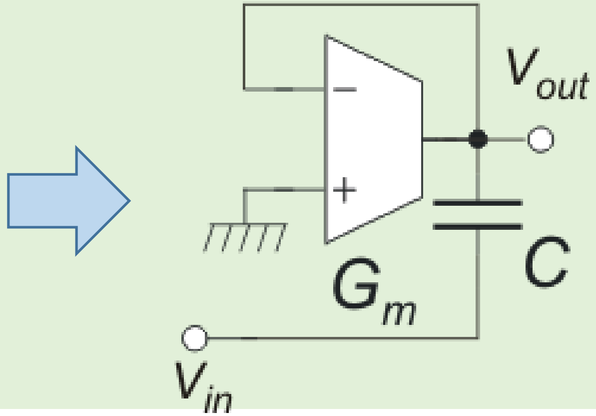


$$\omega_p = \frac{G_m}{C}$$

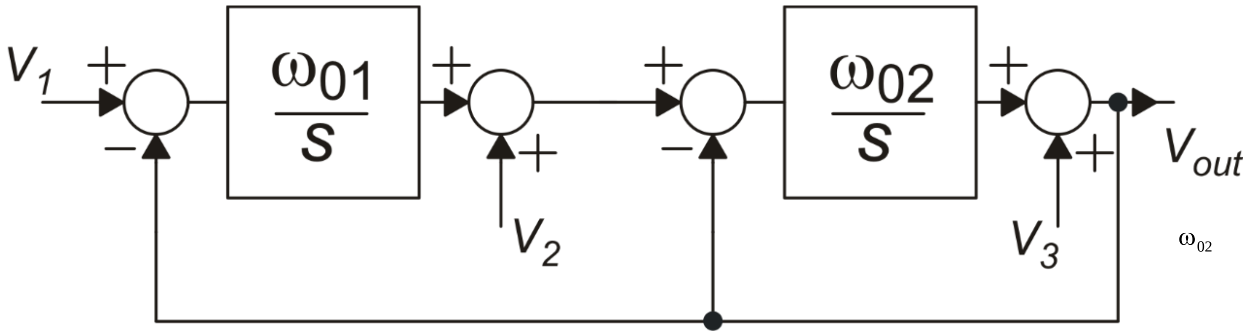
high pass

$$H(s) = \frac{s}{s + \omega_p}$$





Example: State variable Gm-C biquad



$$H(s)=\frac{B_2 s^2 + B_1\frac{\omega_p}{Q_p}s + B_0\omega_p^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

$$v_1 = B_0 v_{in}$$

$$v_2 = B_1 v_{in}$$

$$v_3 = B_2 v_{in}$$

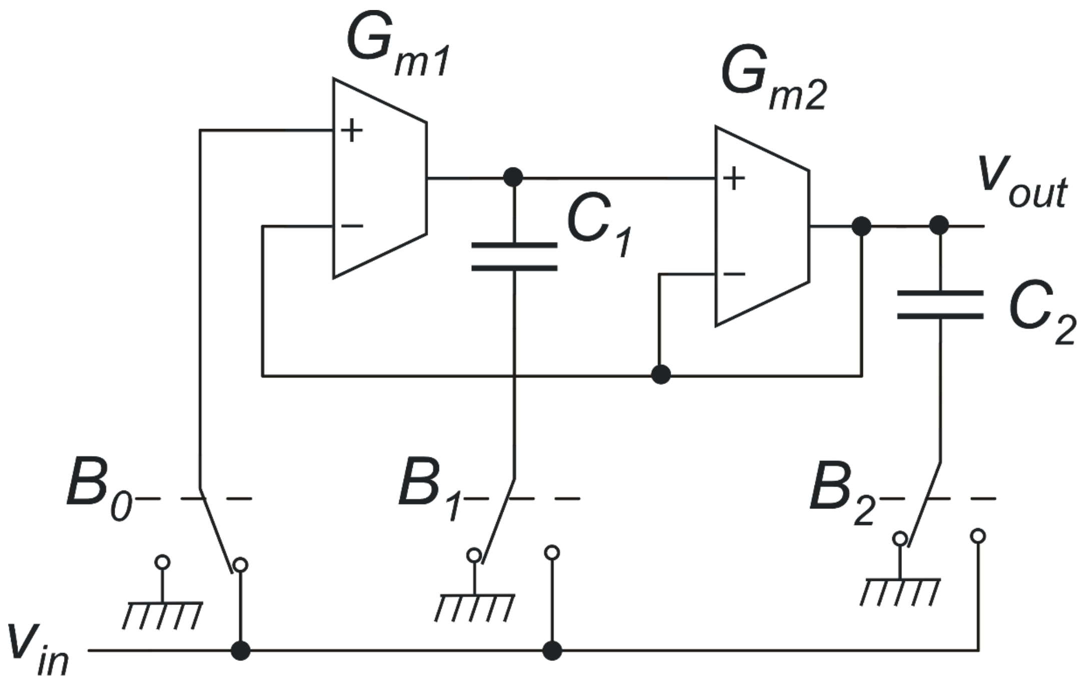
$$B_0, B_1, B_2 = \{0, 1\}$$

Flexible Biquad

$$\omega_p = \sqrt{\omega_{01}\omega_{02}}$$

$$Q_p = \sqrt{\frac{\omega_{01}}{\omega_{02}}}$$

State variable Flexible Biquad – OTA implementation



$$\omega_{01} = \frac{G_{m1}}{C_1} \qquad \omega_{02} = \frac{G_{m2}}{C_2}$$

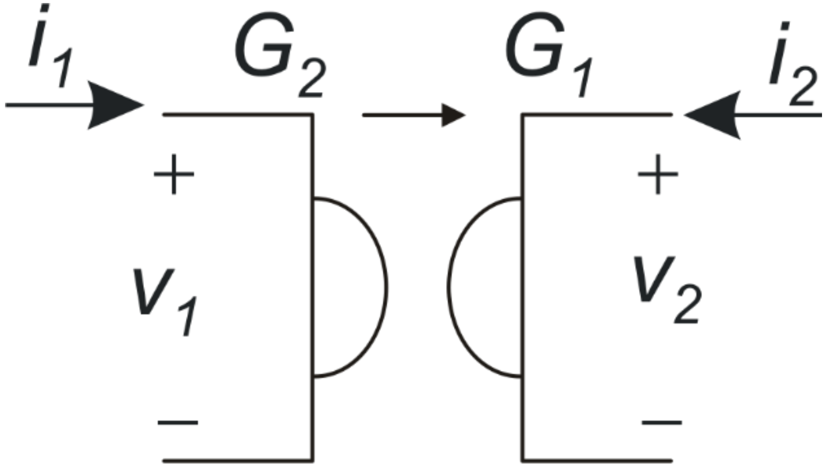
$$\omega_p = \sqrt{\frac{G_{m1}}{C_1}\frac{G_{m2}}{C_2}} \qquad Q_P = \sqrt{\frac{G_{m1}}{G_{m2}}\frac{C_2}{C_1}}$$

Function	B0	B1	B2
Low pass	1	0	0
High pass	0	0	1
Band-Pass	0	1	0
Notch	1	0	1

Simulation of Ladder Filters with OTAs

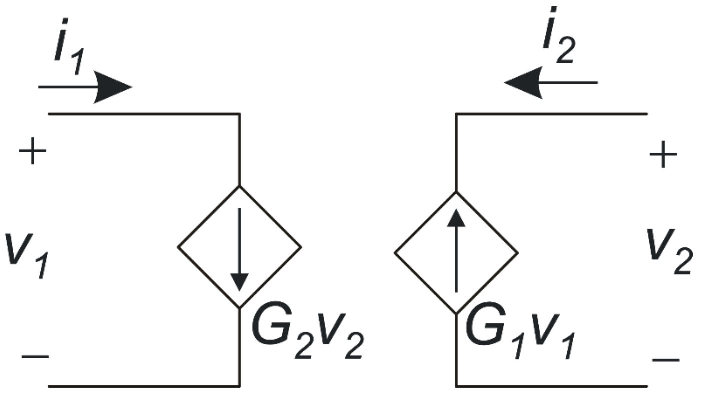
- **Simulation of the inductor: application of the OTA based Gyrator**
- **Simulation of the nodal equations by means of OTAs (signal flow path)** May require inductor simulation, depending on the transfer function to synthesize and/or architecture

Gyrator



$$i_2 = -G_1 v_1$$

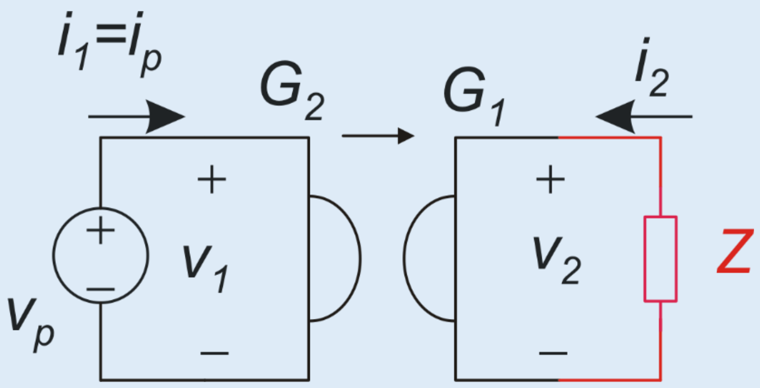
$$i_1 = G_2 v_2$$



$$Y = \begin{pmatrix} 0 & G_2 \\ -G_1 & 0 \end{pmatrix}$$

Y-parameters equivalent circuit

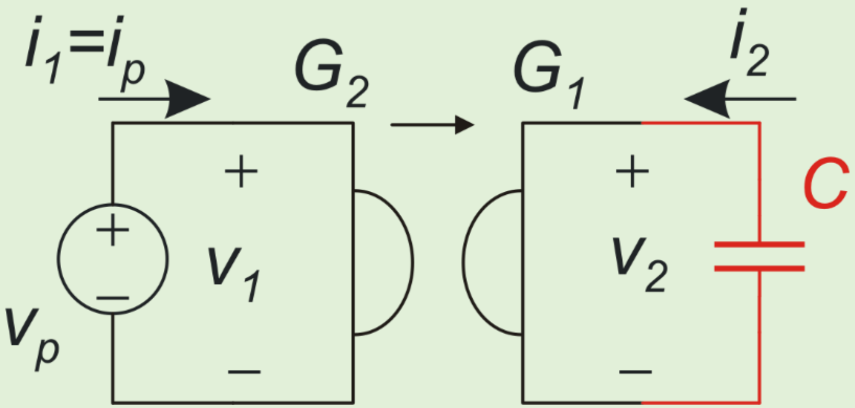
Inductance simulation by means of a gyrator and a capacitor.



Generic Impedance Inversion

$$Z_V = \frac{v_P}{i_P} = \frac{v_P}{G_2 v_2} \qquad v_2 = -i_2 Z = G_1 v_p Z$$

$$Z_V = \frac{v_P}{i_P} = \frac{v_P}{G_2 G_1 v_P Z} = \frac{1}{G_1 G_2 Z}$$

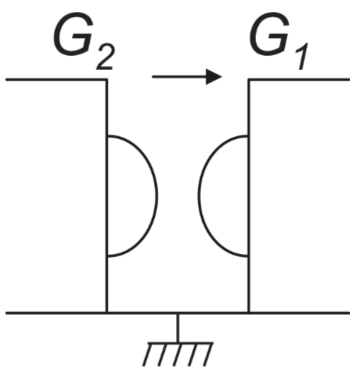


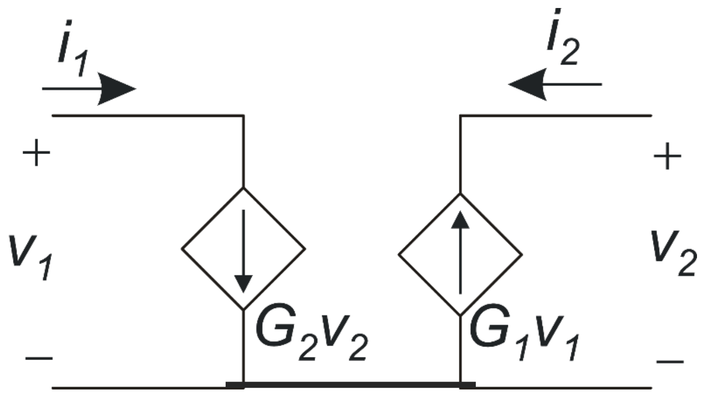
Inductor Synthesis

$$Z = \frac{1}{Cs} \Rightarrow Z_V = \frac{Cs}{G_1 G_2}$$

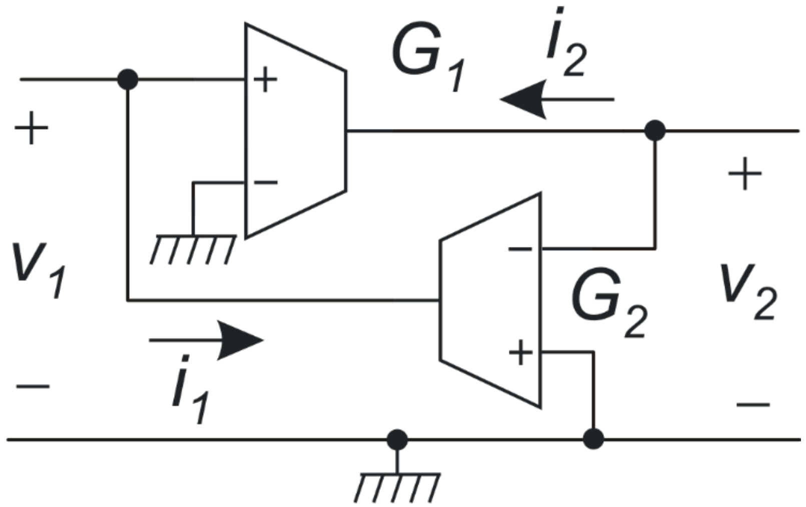
$$L_{EQ} = \frac{C}{G_1 G_2}$$

OTA Based Gyrators





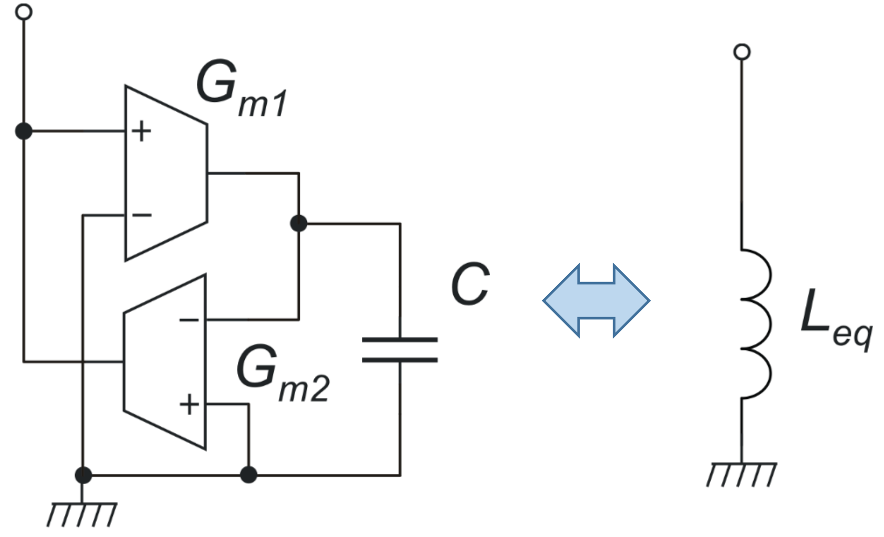
Grounded gyrator



$$i_2 = -G_1 v_1$$

$$i_1 = G_2 v_2$$

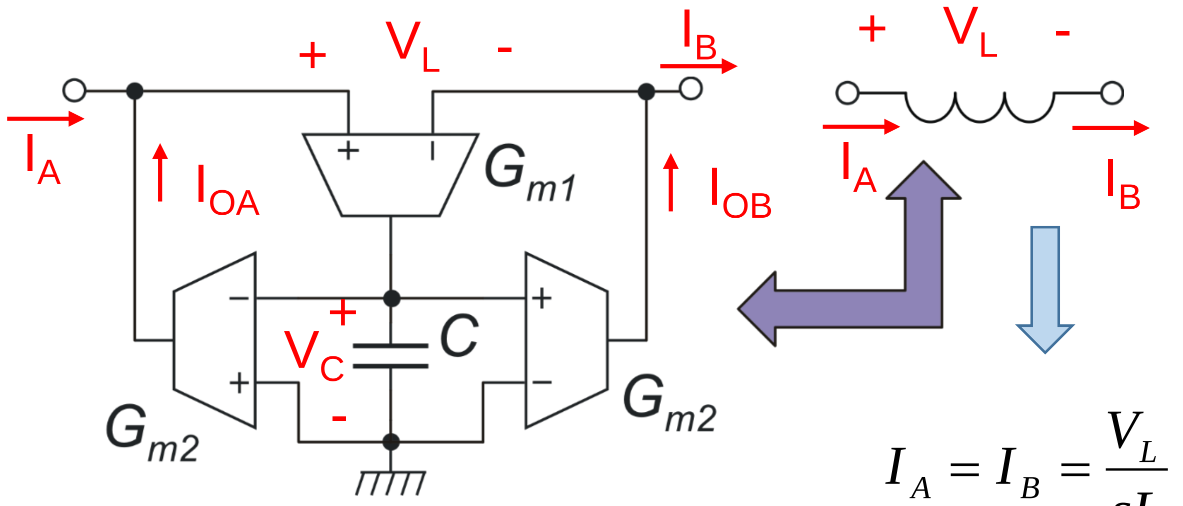
Inductor simulation with OTAs



Grounded Inductor

$$L_{EQ} = \frac{C}{G_{m1} G_{m2}}$$

Floating Inductor



$$I_A = I_B = \frac{V_L}{sL}$$

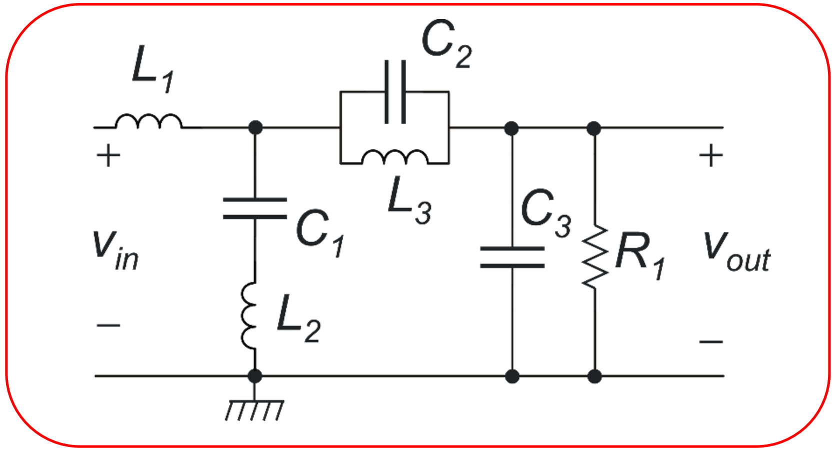
$$V_C = \frac{1}{sC} G_{m1} V_L \;;$$

$$I_A = -I_{OA} = G_{m2} V_C \;;\; I_B = I_{OB} = G_{m2} V_C$$

$$I_A = I_B = \frac{G_{m2} G_{m1}}{sC} V_L$$

$$L_{EQ} = \frac{C}{G_{m1} G_{m2}}$$

Inductor simulation with OTAs - Example



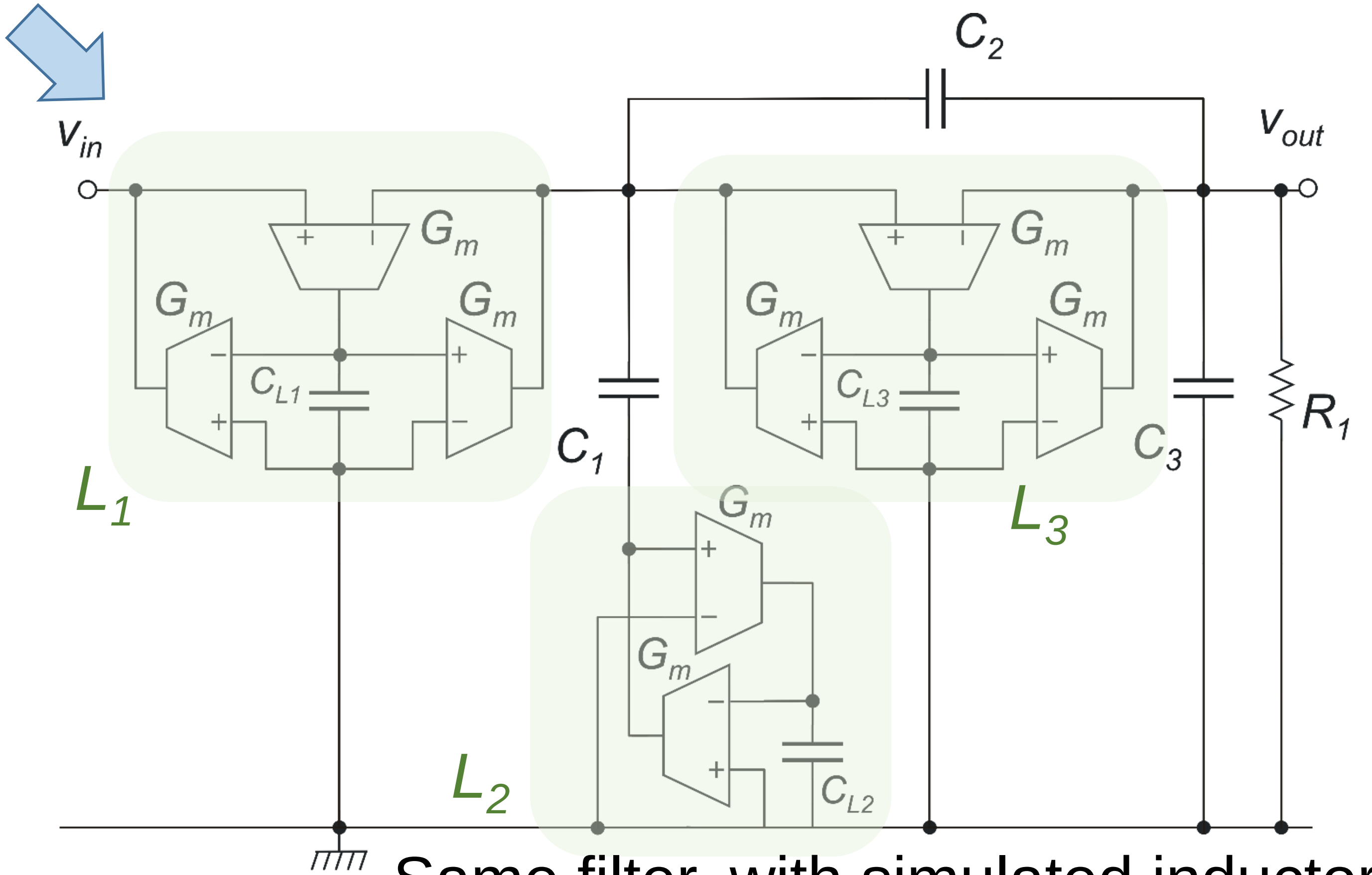
Initial passive filter

L_1,L_3: floating inductors
L_2: grounded inductor

$$C_{L1} = L_1 G_m^2$$

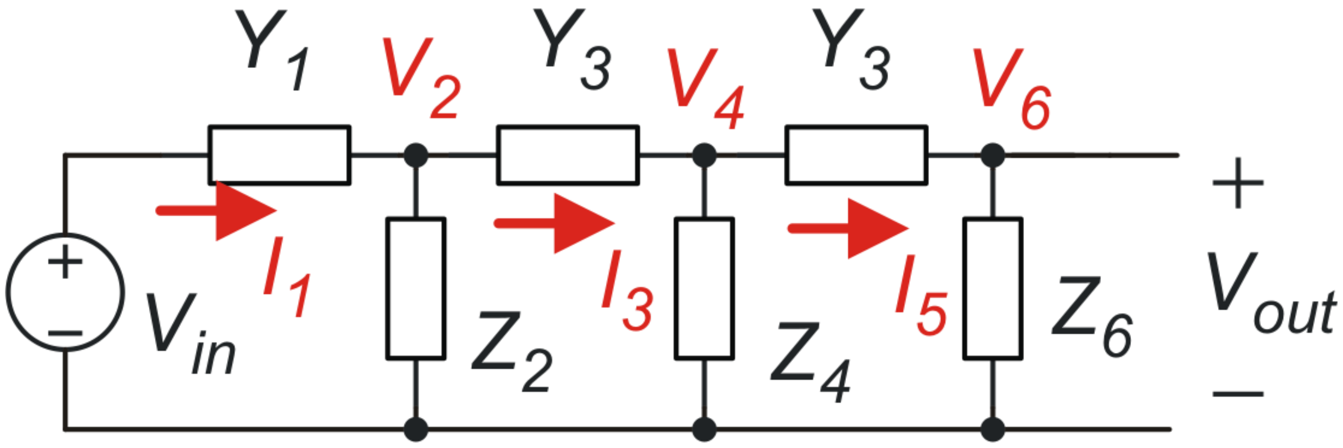
$$C_{L2} = L_2 G_m^2$$

$$C_{L3} = L_3 G_m^2$$



Same filter, with simulated inductors

Signal flow simulation of ladder (LC) networks with OTAs



Network Equations

$$I_1 = Y_1(V_{in} - V_2)$$
$$V_2 = Z_2(I_1 - I_3)$$
$$I_3 = Y_3(V_2 - V_4)$$
$$V_4 = Z_4(I_3 - I_5)$$
$$I_5 = Y_5(V_4 - V_6)$$
$$V_6 = Z_6 I_5$$

Variable transformations

Target: Transform current variables (I_1,I_3,I_5) into voltage variables

$$V_1 = \frac{1}{g} I_1 \quad V_3 = \frac{1}{g} I_3 \quad V_5 = \frac{1}{g} I_5$$

→ Homogeneous equivalent equations →

$$V_1 = \frac{Y_1}{g}\left(V_{in} - V_2\right)$$

$$V_2 = gZ_2\left(V_1 - V_3\right)$$

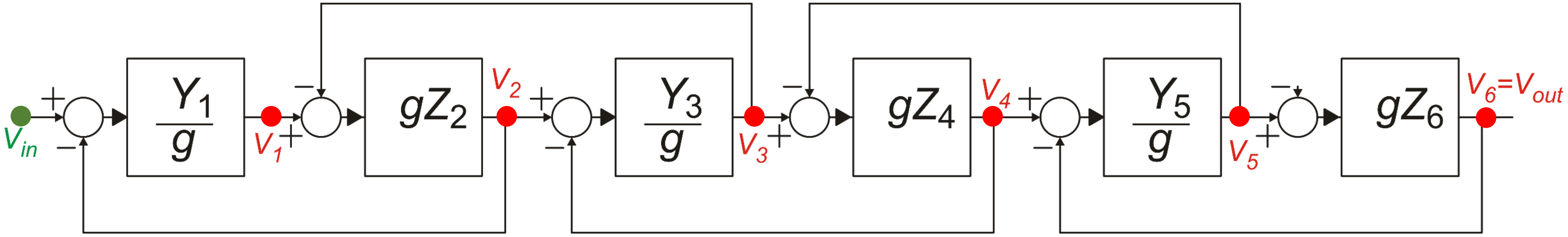
$$V_3 = \frac{Y_3}{g}\left(V_2 - V_4\right)$$

$$V_4 = gZ_4\left(V_3 - V_5\right)$$

$$V_5 = \frac{Y_5}{g}\left(V_4 - V_6\right)$$

$$V_6 = gZ_6 V_5$$

Leap-Frog architecture



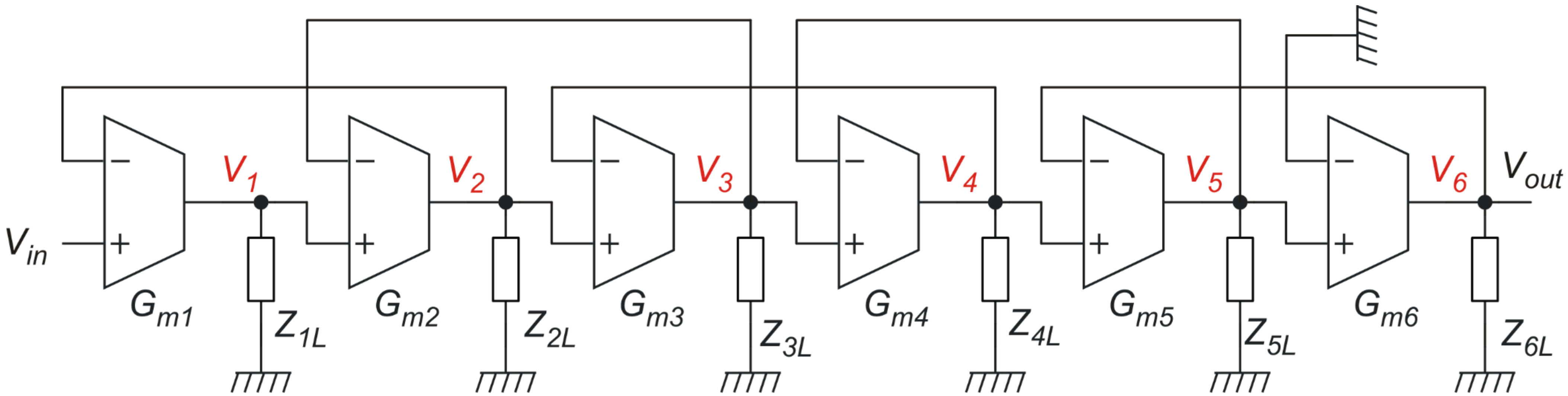
Homogeneous equivalent equations

$$V_1 = \frac{Y_1}{g}\left(V_{in} - V_2\right) \qquad V_4 = gZ_4\left(V_3 - V_5\right)$$

$$V_2 = gZ_2\left(V_1 - V_3\right) \qquad V_5 = \frac{Y_5}{g}\left(V_4 - V_6\right)$$

$$V_3 = \frac{Y_3}{g}\left(V_2 - V_4\right) \qquad V_6 = gZ_6 V_5$$

OTA implementation of the Leap-Frog structure



$$V_1 = \frac{Y_1}{g}\left(V_{in} - V_2\right)$$

$$V_2 = gZ_2\left(V_1 - V_3\right)$$

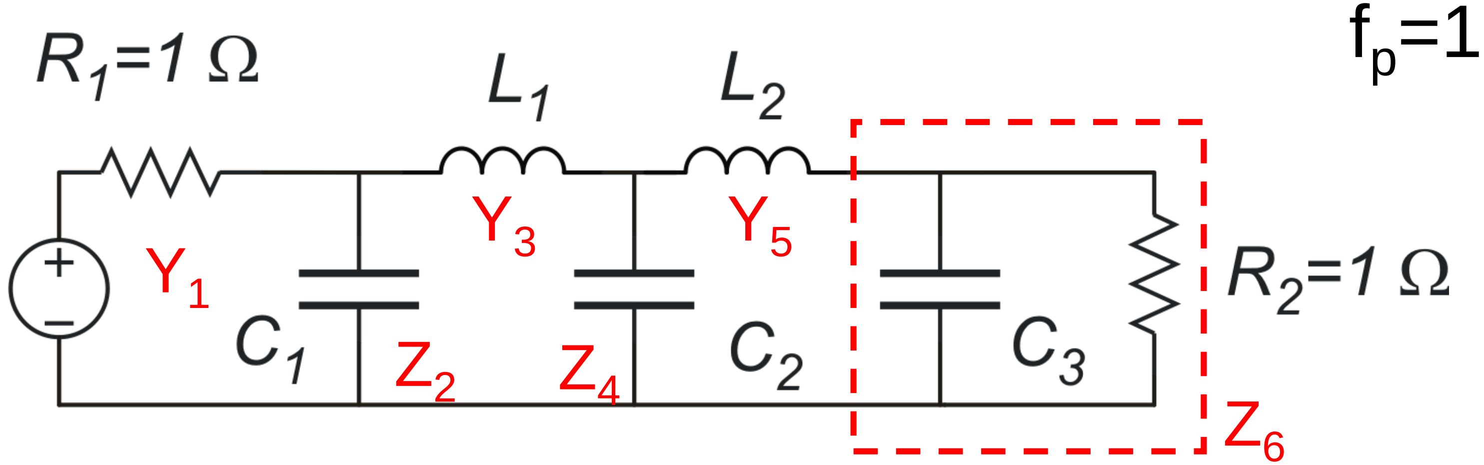
$$V_1 = Z_{1L}G_{m1}\left(V_{in} - V_2\right)$$

$$V_2 = Z_{2L}G_{m2}\left(V_1 - V_3\right)$$

$$Z_{1L} = \frac{Y_1}{gG_{m1}}$$ same for all odd indexes

$$Z_{2L} = \frac{gZ_2}{G_{m2}}$$ same for all even indexes

Example: 5th Order Chebyshev Filter



f_p=10 kHz

C1=33.9 μF
L1=17.35 μH
C2=47.7 μF
L2=17.35 μH
C3=33.9 μF

$$Z_{1L} = \frac{1}{R_1}\frac{1}{gG_{m1}} \qquad g = 1\ \mathrm{S} \qquad \mathrm{R}_1 = 1\,\Omega \qquad Z_{1L} = \frac{1}{G_{m1}} \qquad Z_{1L} = 1\ \mathrm{k\Omega} \Rightarrow G_{m1} = 1\ \mathrm{mS}$$

$$Z_{2L} = \frac{gZ_2}{G_{m2}} = \frac{g}{sC_1G_{m2}} \Rightarrow Z_{2L} = \frac{1}{sC_{2L}} \qquad C_{2L} = \frac{G_{m2}}{g}C_1 \qquad g = 1\ \mathrm{S} \qquad G_{m2} = 10\ \mu\mathrm{S} \qquad C_{2L} = 339\ \mathrm{pF}$$

Example: 5th Order Chebyshev Filter

$$Z_{3L}=\frac{Y_3}{gG_{m1}}\qquad Y_3=\frac{1}{sL_1}\;\Rightarrow\; Z_{3L}=\frac{1}{sL_1 gG_{m1}}=\frac{1}{sC_{3L}}\;\Rightarrow\; C_{3L}=L_1 gG_{m1}$$

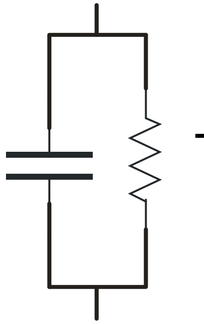
$$g=1\ \text{S}\qquad G_{m3}=10\ \mu\text{S}\;\Rightarrow\; C_{3L}=173.5\ \text{pF}$$

Z_{4L}, Z_{5L}
Same procedure

Z_{6L}

$$Z_{6L}=\frac{gZ_6}{G_{m6}}=\frac{g}{G_{m6}}\frac{1}{\frac{1}{R_2}+sC_3}=g\frac{1}{\frac{G_{m6}}{R_2}+sG_{m6}C_3}$$

$\frac{G_{m6}C_3}{g}$



$\frac{R_2}{gG_{m6}}$

$R_{1L}=1\ \text{k}\Omega$
$C_{2L}=339\ \text{pF}$
$C_{3L}=173.5\ \text{pF}$
$C_{4L}=47.7\ \text{pF}$
$C_{5L}=173.5\ \text{pF}$
$Z_{6L}=100\ \text{k}\Omega \| 339\ \text{pF}$
$G_{m1}=1\text{mS}$
$G_{m2-6}=10\ \mu\text{S}$

Frequent choices in active ladder filters

- Inductor synthesis :

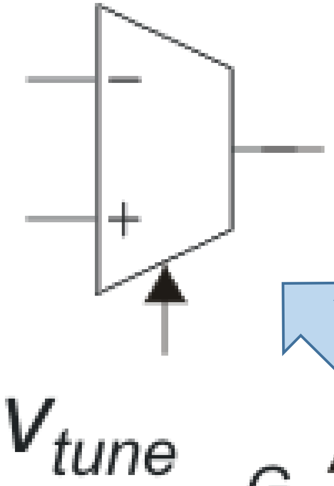
  HP filters (ideal for all-grounded inductors)
  BP filters
  LP – filters with zeroes

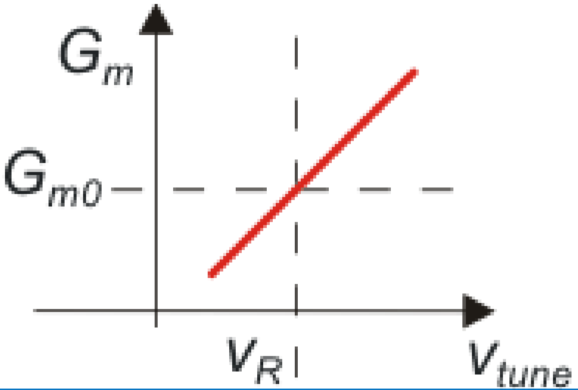
- Leapfrog architectures:

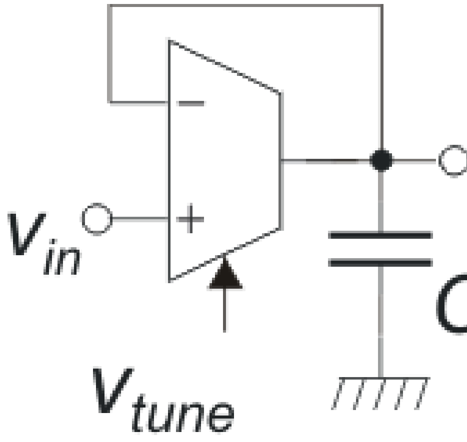
  LP all – pole filters
  BP filters (resonant groups simulated by biquads)

Self-Tuning of OTA Filters

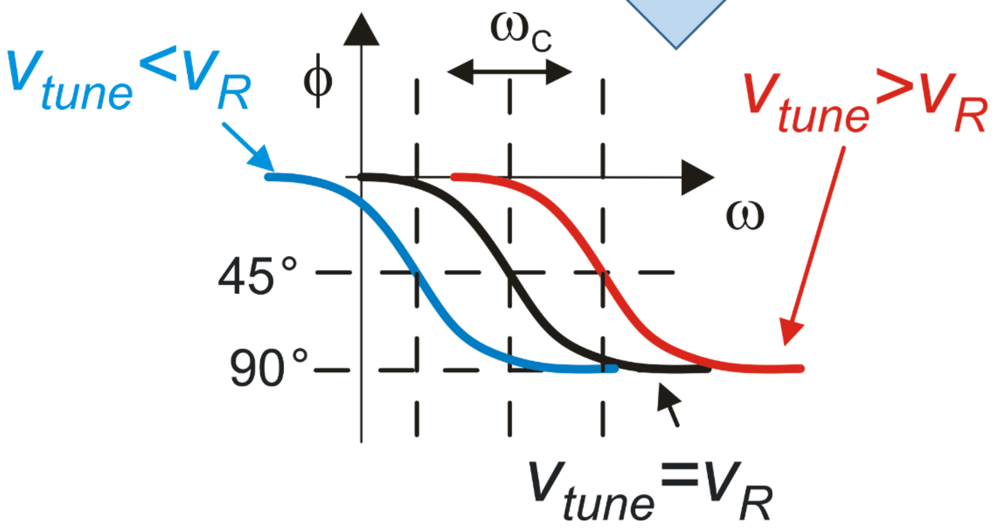
In integrated circuits, Gm's and capacitances are strongly affected by PVT variations (up to ± 30 % variations). For these reasons, in most OTAs the Gm can be controlled by means of a voltage applied to a proper terminal (Vtune). In this way self-tuning of the filter can be accomplished.





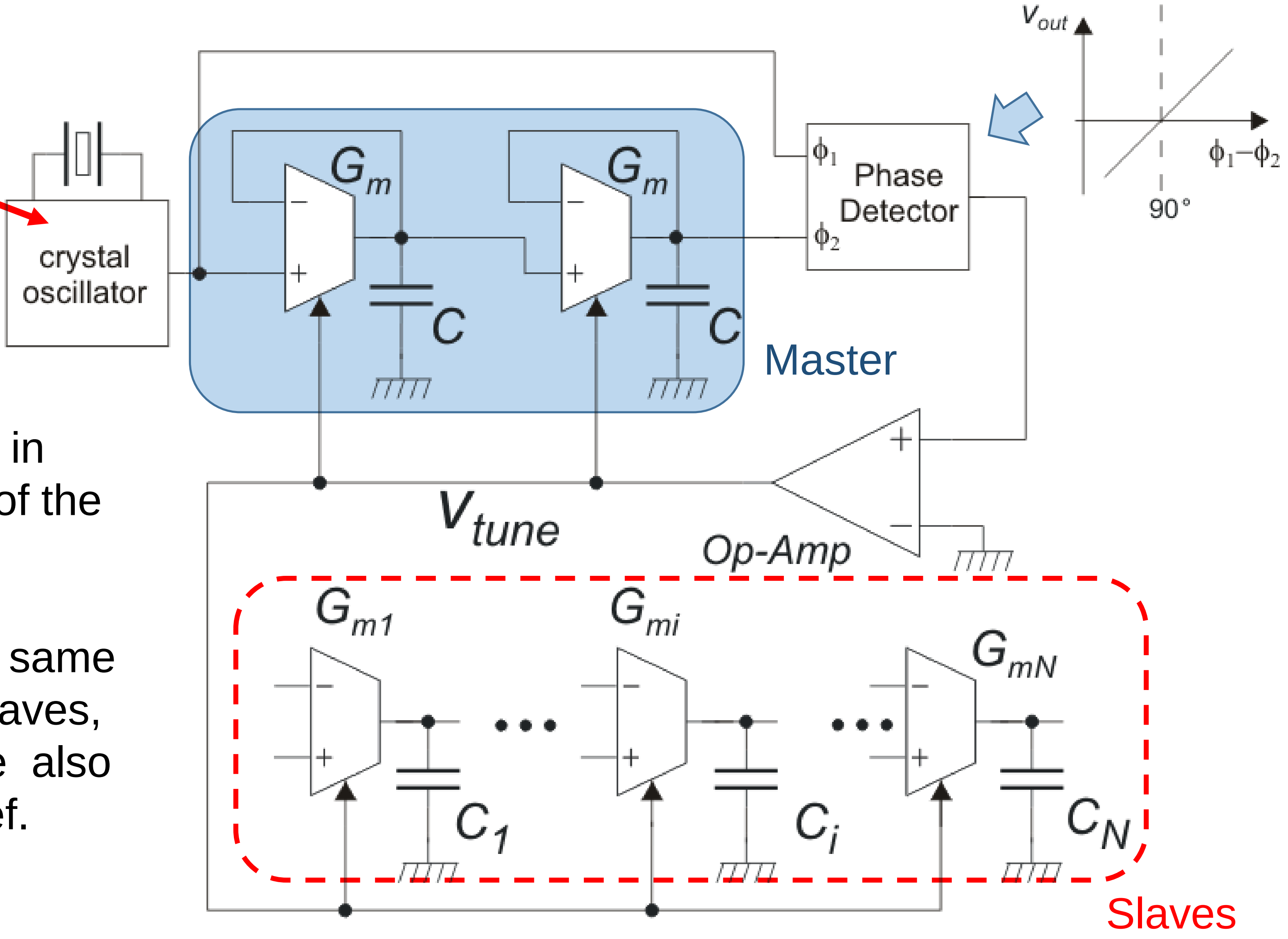


To understand the self-tuning principle, consider the effect of tuning on the phase response of a 1st order LP filter



Self-Tuning of OTA Filters: master slave approach

Reference frequency source



The loop varies V_{tune} in such a way that wC of the two master LP filters equals the reference frequency. Since the same Vtune is fed to the slaves, their Gm/C ratios are also proportional to the ref. frequency.