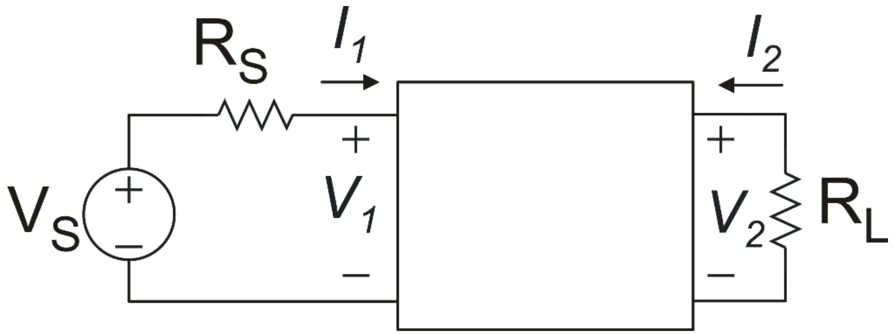
Analog Filter Design

Part. 3: Time Continuous Filter Implementation

Sect. 3-a:

- General considerations
- Passive filters

Design approaches



$$H(s) = \frac{V_2}{V_S}$$

- Passive LC (R) ladder filters
- Cascade of Biquadratic (Biquad) and Bilinear cells
- State Variable Filters
- Simulation of LC filters with active RC networks

Filter Parameters

- For a given transfer function *H(s)*, a particular implementation is characterized by several FOMs (Figures Of Merit). The most frequently used are:

➢ Dynamic Range: $DR = \frac{\max\left(V_{out}\right)}{v_{n-out}}$ $v_{n\text{-}out}$= output noise

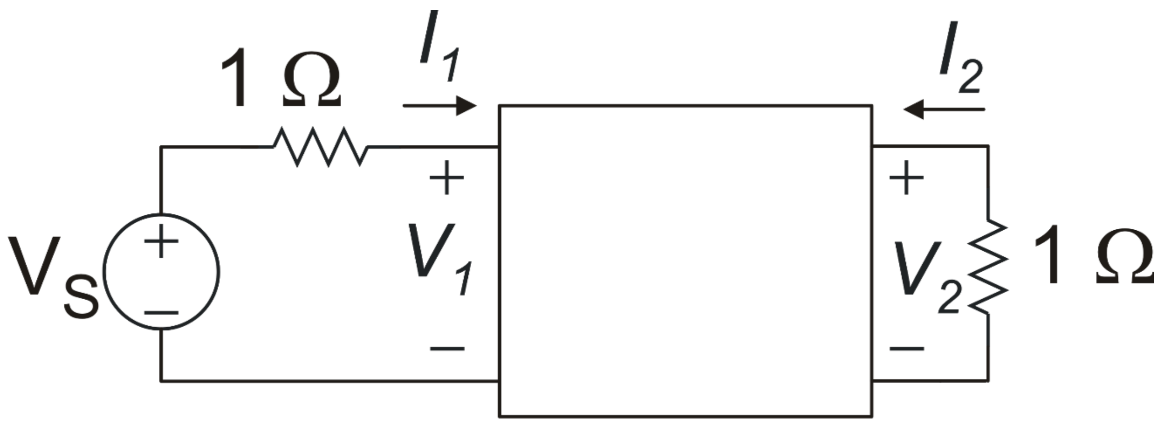
➢ Sensitivity to component variations

$$\begin{cases} S_x^{\omega_0} = \dfrac{d\omega_0/\omega_0}{dx/x} = \dfrac{x}{\omega_0}\dfrac{d\omega_0}{dx} \\ S_x^{Q} = \dfrac{x}{Q}\dfrac{dQ}{dx} \end{cases}$$

➢ Component value spread, e.g. $\frac{C_{\max}}{C_{\min}}$

LC passive filters: "The Prototype filter"

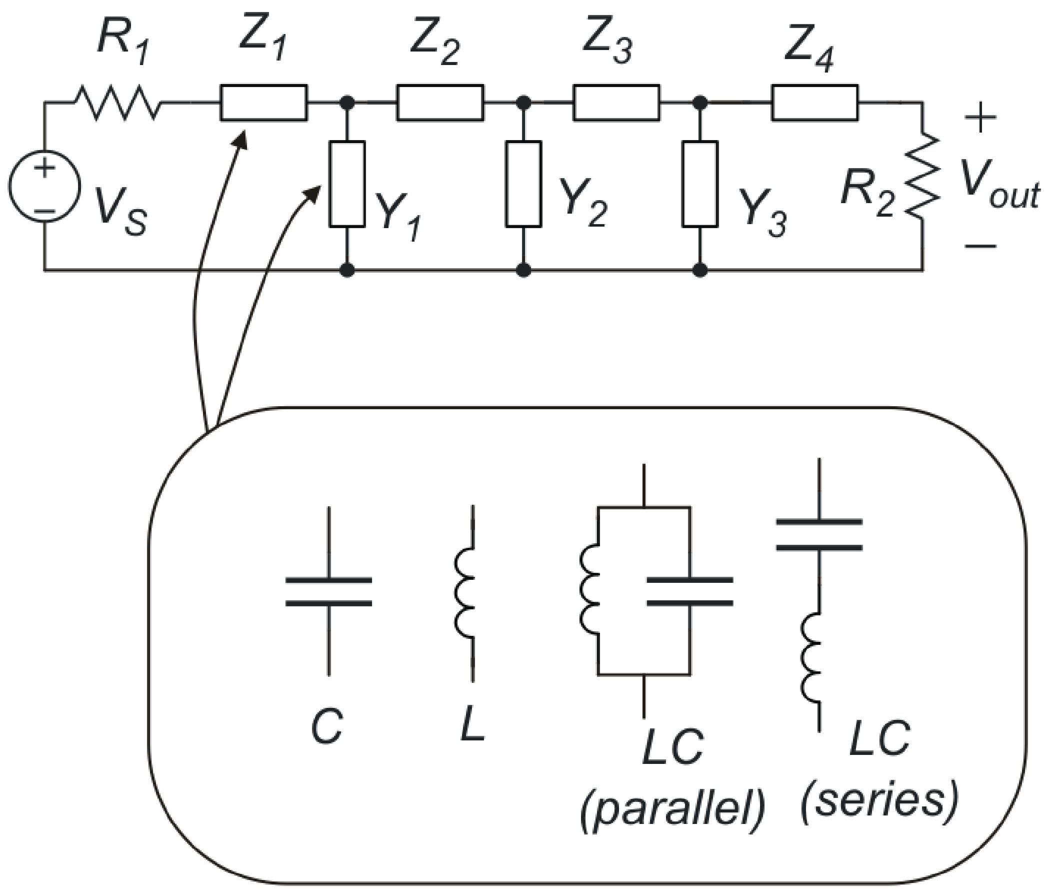
- LC ladder filters are synthetized in normalized (1 rad/s, 1 Ω) and low-pass form (prototype filter)
- Transformation rules are used to derive the required filter function (e.g. band-pass) and parameters (e.g. actual operating frequencies) from the prototype filter



$$H_N(s) = \frac{V_2}{V_S}$$

Normalized Low-Pass Function

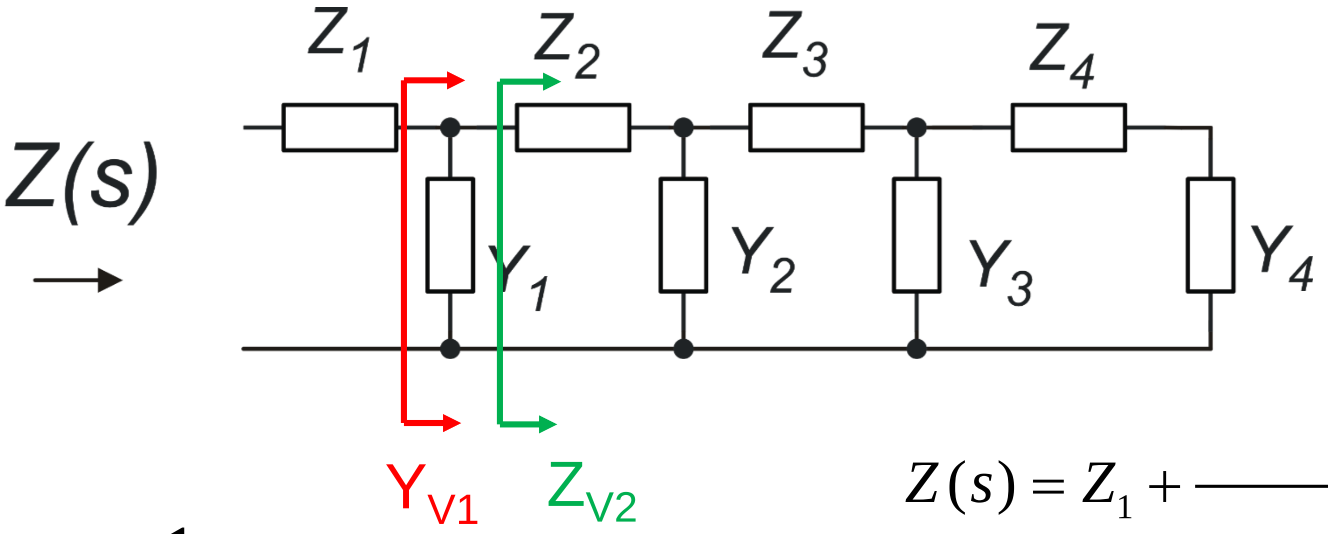
Passive Lossless Ladder Filters



- Doubly terminated LC ladder network
- Advantages: **minimum sensitivity** to component variation in the pass-band
- The lowest sensitivity is achieved with equally terminated networks (R_1=R_2).
- Can be used as starting point for the synthesis of active RC filters
- Drawback: tuning requires change of all components.

order (N) = number of capacitors + number of inductors

Ladder networks: driving point impedance (d.p.i.)



- Driving point impedances (or d.p. admittances)
- Transfer impedances (or t. admittances)

$$Z(s) = Z_1 + \frac{1}{Y_{V1}}$$

$$Z(s) = Z_1 + \frac{1}{Y_1 + \frac{1}{Z_{V2}}}$$

●●●

$$Z(s) = Z_1 + \cfrac{1}{Y_1 + \cfrac{1}{Z_2 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_3 + \cfrac{1}{Z_4 + \cfrac{1}{Y_4}}}}}}}$$

"Continued fraction"

Cauer synthesis approach for d.p.i.

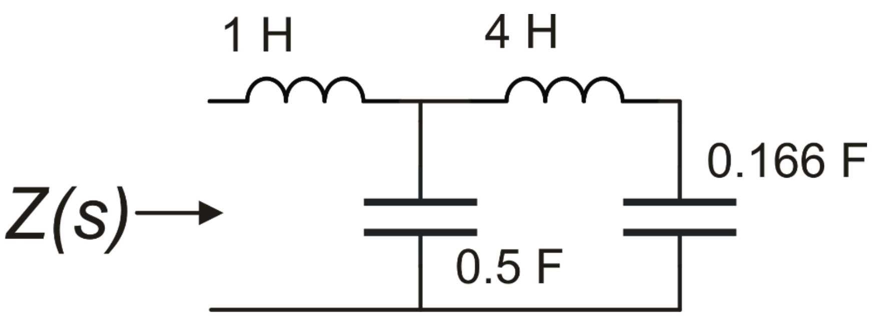
$$Z(s)=\frac{s^4+4s^2+3}{s^3+2s}\qquad Z(s)=s+\frac{2s^2+3}{s^3+2s}$$

$$\begin{cases} Z_1=s \\ \dfrac{1}{Y_{V1}}=\dfrac{2s^2+3}{s^3+2s}\end{cases}$$

$$Y_{V1}=\frac{s^3+2s}{2s^2+3}=\frac{s}{2}+\frac{s/2}{2s^2+3}$$

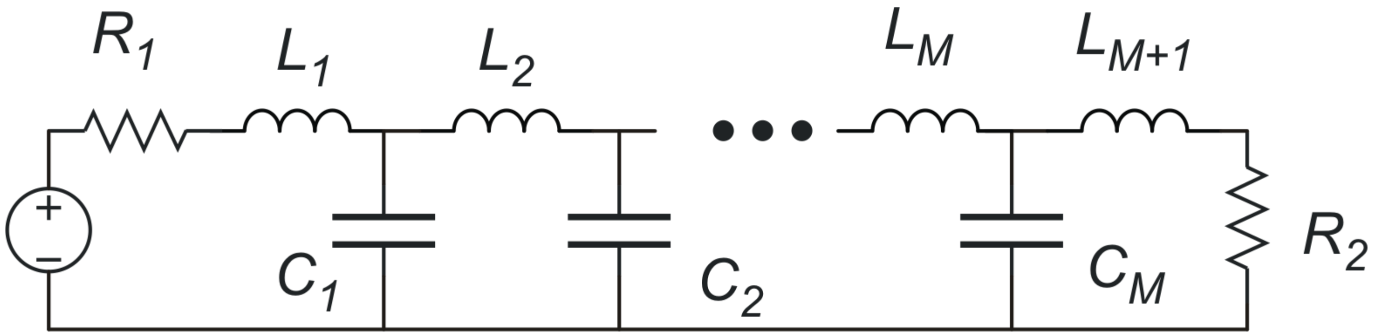
In the end …..

$$Z(s)=\underbrace{s}_{Z_1}+\cfrac{1}{\underbrace{\cfrac{s}{2}+\cfrac{1}{\underbrace{4s}_{Z_2}+\cfrac{1}{\underbrace{s/6}_{Y_2}}}}_{Y_1}}$$

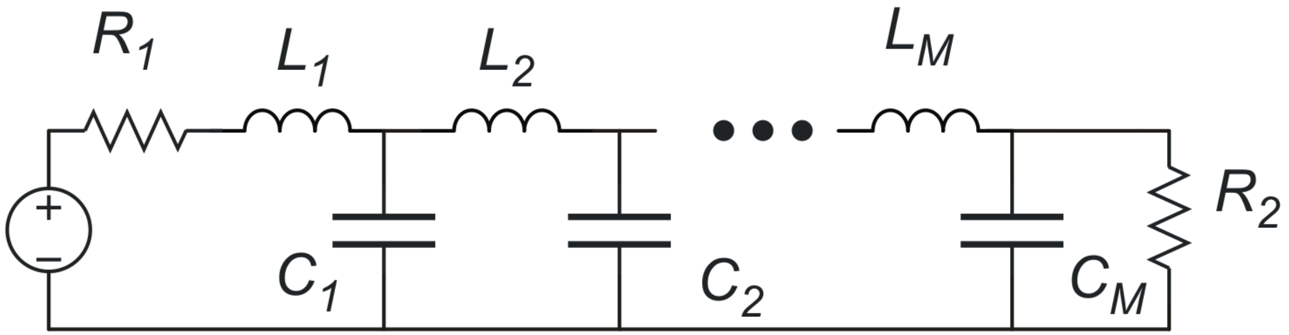


Prototype Filter Configurations (all poles)

N=2M+1 (odd order)



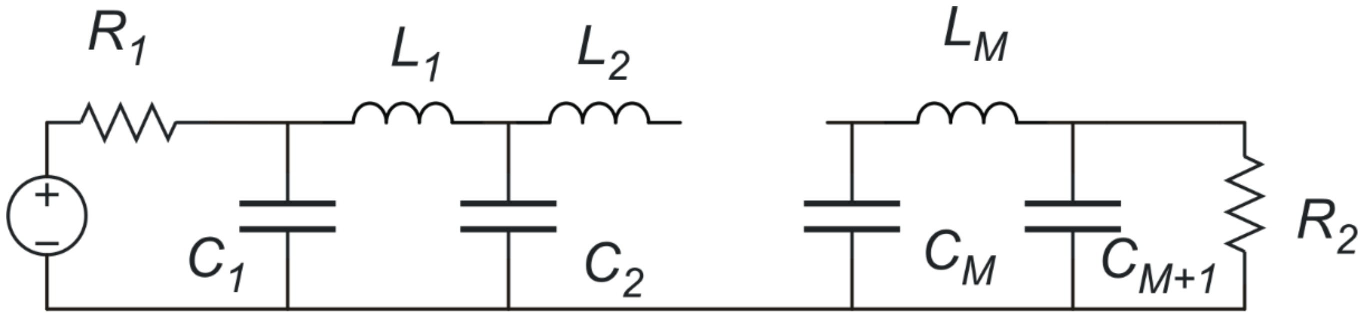
N=2M (even order)



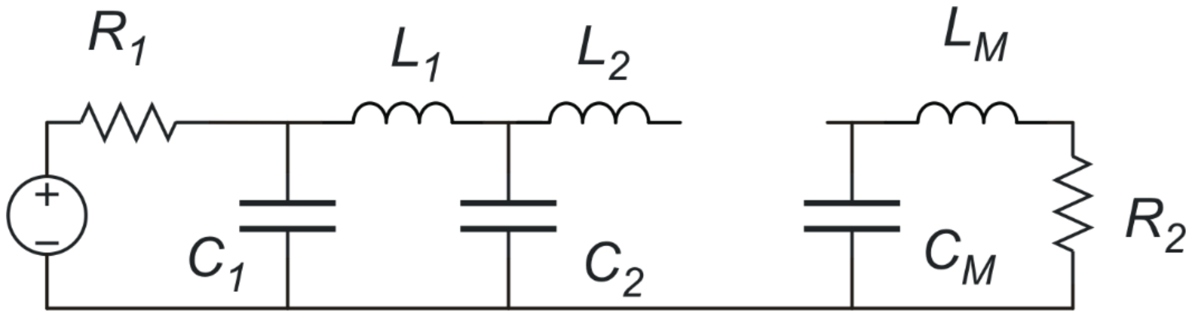
Pass-band gain: $$k = \frac{R_2}{R_1 + R_2} < 1$$ (0.5 for equally terminated networks)

Alternate solution (all poles)

N=2M+1 (odd order)

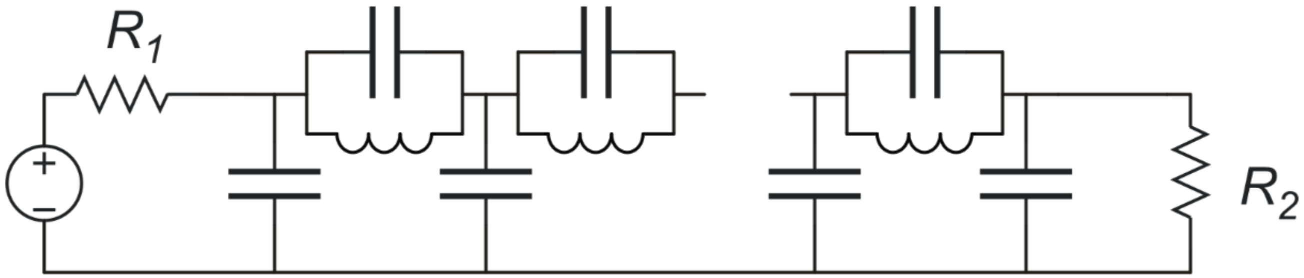


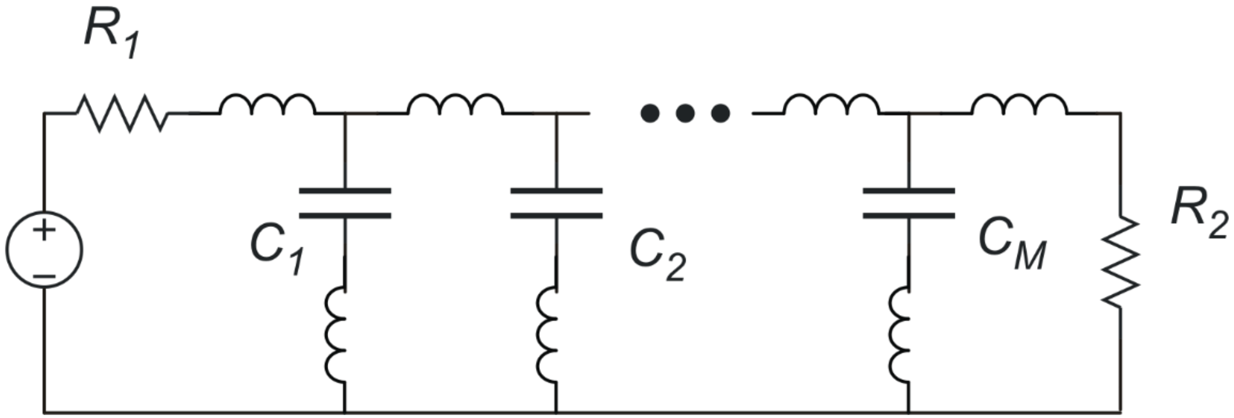
N=2M (even order)



Pass-band gain: $k = \dfrac{R_2}{R_1 + R_2} < 1$ (0.5 for equally terminated networks)

LC ladder network for TF with imaginary zeros (e.g. Inverse Chebyshev and Cauer Elliptic filters)





Frequency scaling rules

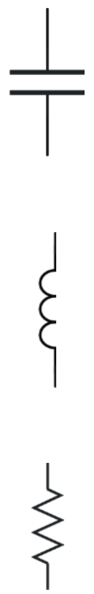
- Frequency scaling allows to change the normalization frequency, allowing transformation of the characteristic frequencies of the filter

$$s_n \rightarrow \frac{s}{\omega_N}$$

$$\frac{1}{s_n C} \rightarrow \frac{\omega_N}{sC} \Rightarrow \quad C \rightarrow \frac{C}{\omega_N}$$

$$s_n L \rightarrow s\frac{L}{\omega_N} \Rightarrow \quad L \rightarrow \frac{L}{\omega_N}$$

$$R \rightarrow R$$



Impedance Scaling Rule

- Impedance scaling is used to change component values leaving the transfer function unaltered. The target is finding feasible component values for the chosen technology

If the network includes only:

- Two terminal impedances (L,R,C components)
- Voltage Controlled Voltage Sources (VCVS) i.e Ideal voltage amplifiers.
- Current Controlled Current Sources (CCCS) i.e. ideal current amplifiers

Then: the Vout/VS transfer function is unchanged when <u>all</u> the impedances are multiplied by the same function f(s)

Impedance scaling: component transformation

An important case is when the function f(s) is a constant factor K:

$$\frac{1}{s_n C} \rightarrow K\frac{1}{sC} \Rightarrow \quad C \rightarrow \frac{C}{K}$$

$$sL \rightarrow KsL \Rightarrow \quad L \rightarrow KL$$

$$R \rightarrow KR$$

Element transformations

- **Goal:** to change the filter response from low-pass to the other three possibilities (high-pass, etc.) and perform frequency scaling at the same time.

Let us recall the following transformations:

From Low-Pass to:

High-Pass

$$s_n \to \frac{\omega_N}{s}$$

Band-Pass

$$s_n \to \frac{\omega_0}{B}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)$$

Band-Stop

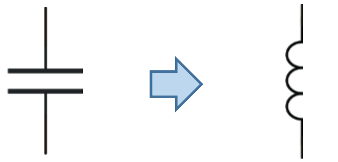
$$s_n \to \left[\frac{\omega_0}{B}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)\right]^{-1}$$

Element Transformation

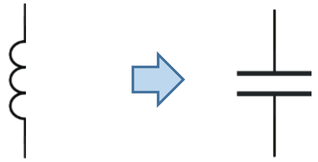
Low-pass to High-pass

$$s_n \rightarrow \frac{\omega_N}{s}$$

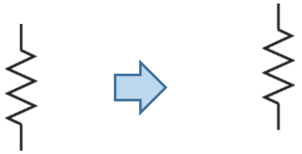
$$\frac{1}{s_n C} \rightarrow \frac{s}{\omega_N C} \quad \Rightarrow \quad C \rightarrow L = \frac{1}{\omega_N C}$$



$$s_n L \rightarrow \frac{\omega_N}{s} L \Rightarrow \quad L \rightarrow C = \frac{1}{\omega_N L}$$



$$R \rightarrow R$$



Element Transformation

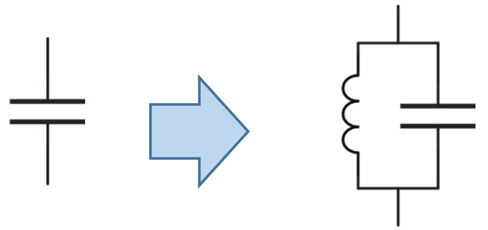
Low-pass to Band-pass

$$s_n \to \frac{\omega_0}{B}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)$$

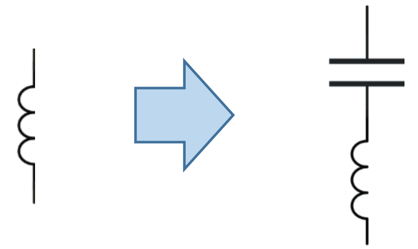
$$s_n \to \frac{s}{B}+\frac{\omega_0^2}{sB}$$

$$\frac{1}{s_n C} \to \frac{1}{\frac{sC}{B}+\frac{\omega_0^2 C}{sB}} = \frac{1}{sC_P+\frac{1}{sL_P}}$$

$$s_n L \to L\frac{s}{B}+L\frac{\omega_0^2}{sB} = sL_S+\frac{1}{sC_S}$$



$$C_P = \frac{C}{B} \qquad L_P = \frac{B}{\omega_0^2 C}$$

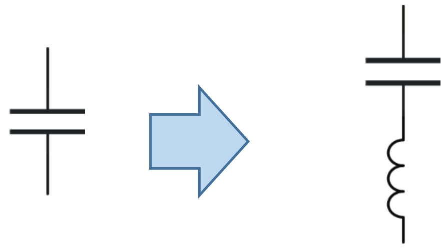


$$C_S = \frac{B}{\omega_0^2 L} \qquad L_S = \frac{L}{B}$$

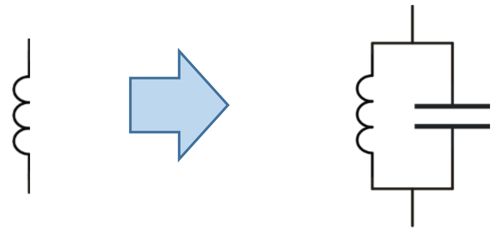
Element Transformation

Low-pass to Band-stop

$$s_n \rightarrow \left[\frac{\omega_0}{B} \left(\frac{s}{\omega_0} + \frac{\omega_0}{s} \right) \right]^{-1}$$



$$C_S = \frac{CB}{\omega_0^2} \qquad L_S = \frac{1}{BC}$$



$$C_P = \frac{1}{BL} \qquad L_P = \frac{BL}{\omega_0^2}$$

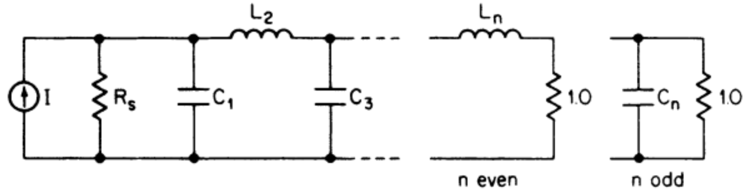
Design of LC ladder passive filters

- A procedure that allows designing an arbitrary transfer function with a ladder structure does not exist.
- All-pole functions (e.g. Butterworth, Chebyshev I, Bessel) can be designed with a standard approach, where the branches of the ladder (Z and Y elements) are pure capacitors or inductors. Given a class of networks, not all functions are feasible.
- The rigorous design of Cauer (elliptic) filters is less straightforward.
- **Tables** are available for the most frequently used ladder topologies and transfer functions. Several **CAD design tools** are also available.

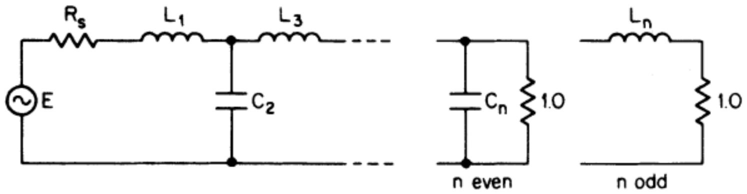
Table example

Williams & Taylor
«Electronic Filter Design Handbook»
2006, McGraw-Hill

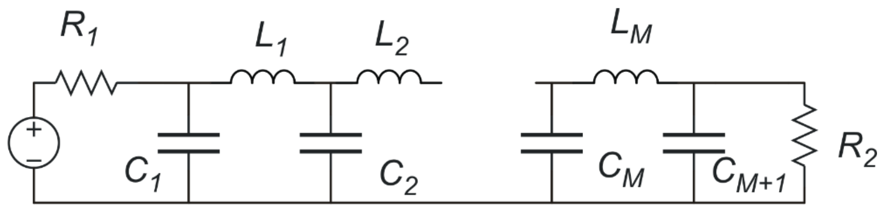
TABLE 11-29 0.25-dB Chebyshev *LC* Element Values

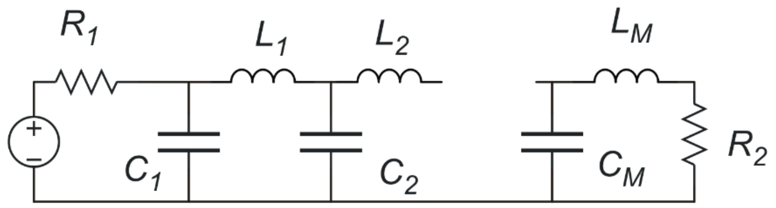


n	R_s	C_1	L_2	C_3	L_4
2	2	0.6552	2.7632		
	3	0.3740	4.3118		
	4	0.2637	5.7389		
	8	0.1215	11.2589		
	Inf.	1.3584	0.8902		
3	1	1.6325	1.4360	1.6325	
	0.5	3.2663	1.0775	1.6325	
	0.333	4.8988	0.9572	1.6325	
	0.25	6.5326	0.8971	1.6325	
	0.125	13.0639	0.8081	1.6325	
	Inf.	1.5348	1.5285	0.8169	
4	2	0.6747	3.6860	1.0247	1.8806
	3	0.4149	6.2744	0.7682	2.1302
	4	0.3020	8.8161	0.6667	2.2533
	8	0.1448	19.0204	0.5334	2.4516
	Inf.	1.4817	1.8213	1.5068	0.7853
n	$1/R_s$	L_1	C_2	L_3	C_4



Example: Butterworth Prototype Filter



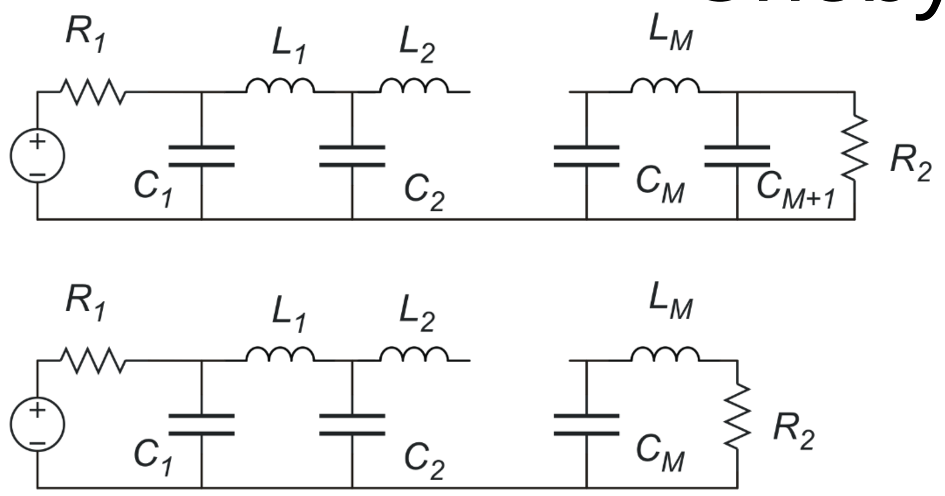


radians – per - seconds

$\omega=1$

N	C1	L1	C2	L2	C3	L3	C4	L4	C5	L5
2	1.4142	1.4142								
3	1.0000	2.0000	1.0000			Butterworth (1 rps passband)				
4	0.7654	1.8478	1.8478	0.7654						
5	0.6180	1.6180	2.0000	1.6180	0.6180					
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2740	0.4450			
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129

Chebyshev 1 dB ripple



Note: In these tables, generally $\omega=1$ is the **-3 dB angular frequency,** regardless of ripple (and A_p)
e.g. Williams & Taylor «Electronic Filter Design Handbook»

n	C1	L1	C2	L2	C3	L3	C7
2	0.572	3.132					
3	2.216	1.088	2.216				
4	0.653	4.411	0.814	2.535			
5	2.207	1.128	3.103	1.128	2.207		
6	0.679	3.873	0.771	4.711	0.969	2.406	
7	2.204	1.131	3.147	1.194	3.147	1.131	2.204

Example

- Design a LC ladder Chebyshev filter with the following characteristics:

f_pass =10 kHz, Maximum Pass-band attenuation 1 dB

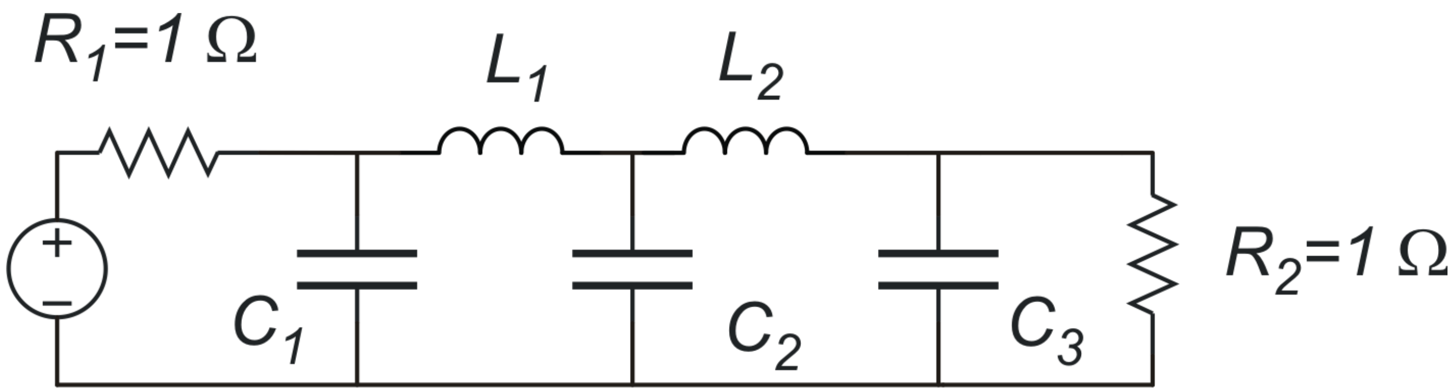
f_stop = 20 kHz   Minimum Stop-Band Attenuation: 40 dB

Python: cheb1ord: Order=5, ω_P=62.8 krad/s

But: tables are normalized to ω_{-3dB} . From the magnitude plot:

$$f_{-3\mathrm{dB}} \cong 10.34 \ \ \mathrm{kHz} \qquad \omega'_N = \omega_{-3\mathrm{dB}} \cong 65 \ \ \mathrm{krad/s}$$

LC Filter design using Tables



From table: prototype filter

$\omega'_N=\omega_{-3dB}$=1 rad/s

C1=2.207 F
L1=1.128 H
C2=3.103 F
L2=1.128 H
C3=2.207 F

$$C \rightarrow \frac{C}{\omega'_N}$$

$$L \rightarrow \frac{L}{\omega'_N}$$

ω_{-3dB}=65 krad/s

C1=33.9 μF
L1=17.35 μH
C2=47.7 μF
L2=17.35 μH
C3=33.9 μF

Frequency scaled filter