Analog Filter Design

Part. 1: Introduction

Definition of Filter

- **Electronic filters** are linear circuits whose operation is defined in the frequency domain, i.e. they are introduced to perform different amplitude and/or phase modifications on different frequency components.

- **Digital Filters (or Numerical Filters)** operates on digital (i.e. coded) signals
- **Analog Filters** operate on analog signals, i.e. signals where the information is directly tied to the infinite set of values that a voltage or a current may assume over a finite interval (range).

Brief Filter History: The Basis

Harmonic Analysis

- 200 BC: Apollonius of Perga theory of "deferents and epicycles" (the basis of later (100 AD) Ptolemaic system of astronomy), maybe anticipated by Babylon mathematicians intuitions

- 1822 Joseph Fourier: Théorie Analytique de la Chaleur. Preceded by Euler, d'Alembert and Bernoulli works on trigonometric interpolation.

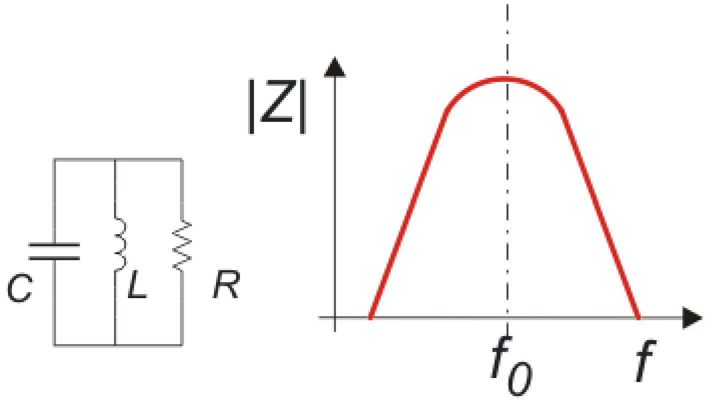
Electrical Network Analysis

- 1845 Gustav Robert Kirchhoff: Kirchhoff Circuit Laws (KCL, KVL)
- 1880-1889 Olivier Heaviside developed the Telegrapher's equations and coined the terms inductance, conductance, impedance etc.
- 1893 Charles P. Steinmetz: "Complex Quantities and Their Use in Electrical Engineering" In the same year, Arthur Kennely introduced the *complex impedance* concept

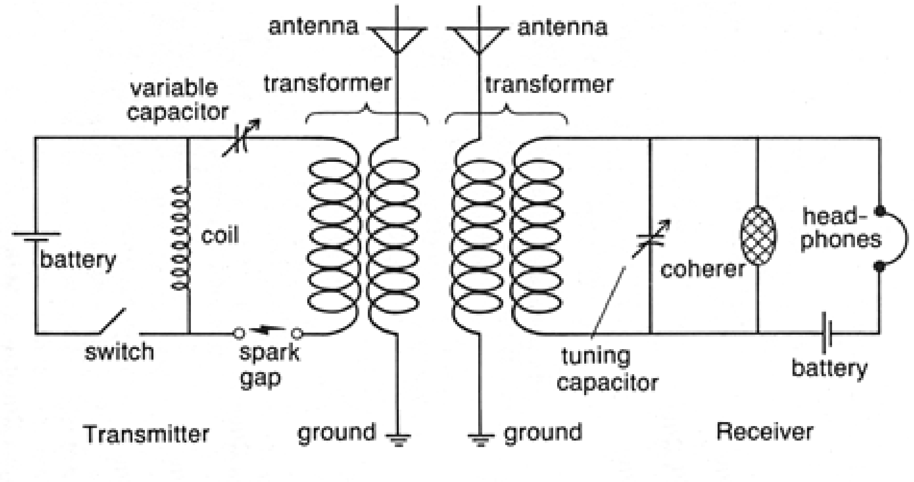
Brief Filter History: Resonance

- Acoustic resonance was know since the invention of the first musical instruments.
- Acoustical (i.e. mechanical) resonance allows selecting and enhancing individual frequencies. It suggested the first FDM (frequency division multiplexing) application to telegraph lines based on electrical resonance.

- Electrical resonance was first observed in the discharge transient of Leiden Jars



Not suitable for telephone FDM, but sufficient for Radio tuning



Marconi RTX with syntonic tuning

- **1898 – Sir Oliver Lodge: <u>Syntonic tuning</u>**

Brief filter history: The beginning of the electronics era

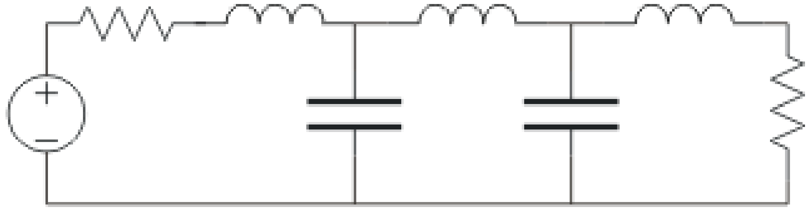
1904 – John Ambrose Fleming Valve (Vacuum Diode)

1906 – Lee De Forest “Audion” Vacuum Triode

1911 – First Triode-based Amplifiers and Oscillators

1912 – Lee De Forest: cascaded amplifier stages

1915 – G. A. Campbell – K.W. Wagner “wave filters”
first example of filter theory:
<u>Image Parameters Theory</u>



Brief Filter History: Towards Modern Filter Theory

- 1917 – Edwin Howard Armstrong: First Superhetorodyne Radio
- 1920 – Introduction of the term "feedback" (referred to positive FB)
- 1927 – Harold Stephen Black: application of negative feedback to amplifiers
- 1920-30 – Diffusion of Frequency Division Multiplexing for telephone calls.
- **1930 – Stephen Butterworth introduces maximally flat filters. (with amplifier to separate stages)**
- **1925-40 – Progressive introduction of the Network Syntesys approach to filter design. Primary contributors was Wilhelm Cauer**

Vacuum tube era

Filter operation

The role of a filter can be:

- **Modify the magnitude of different frequency components. These filters are by far the most commonly used**.

- Modify the phase of different frequency components (i.e. to compensate for an unwanted phase response of a filter of an amplifier)

Real filters generally change both the phase and magnitude of a signal

Analog filters: General properties

Common properties:

- ✓ Linearity
- ✓ Time Invariance
- ✓ Stability (BIBO: Bounded Input -> Bounded Output)

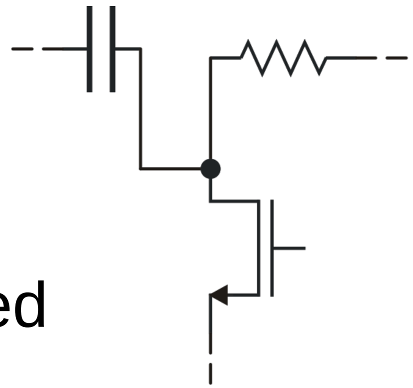
Filters may be:

- ✓ Lumped / Distributed
- ✓ Active / Passive
- ✓ Continuous-time / Discrete-Time

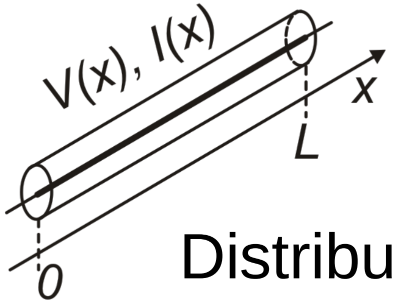
Lumped element networks

Lumped element networks are made up of components, whose state and/or behavior is completely defined by a discrete number of quantities.

Lumped element networks are simplifications or real systems, which are spatially distributed (i.e. the relevant quantities are given as function of space variables, defined over a continuous domain).

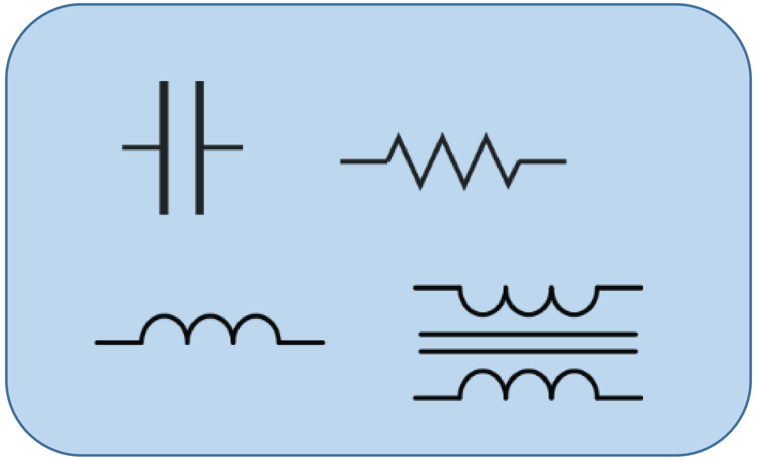


Lumped

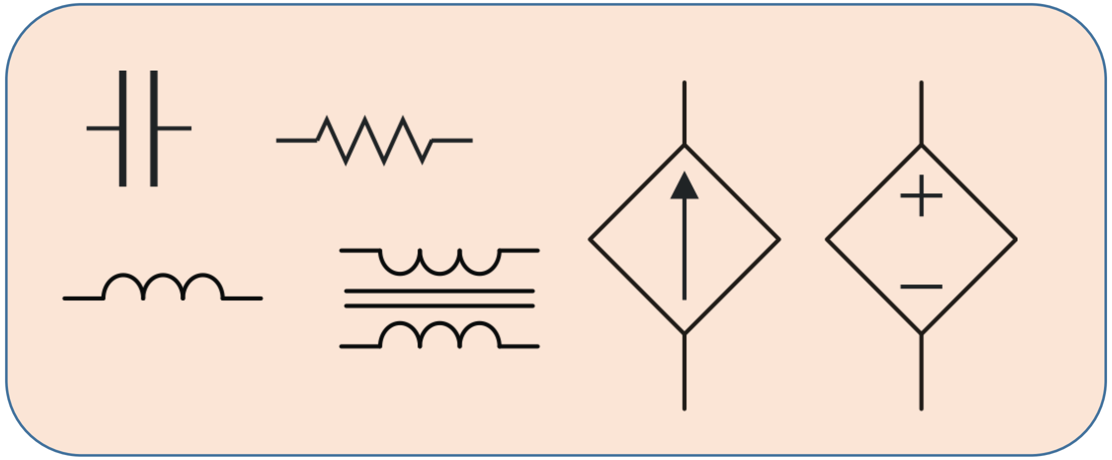


Distributed
(e.g. transmission line)

Passive and Active Networks (Linear)



Passive network

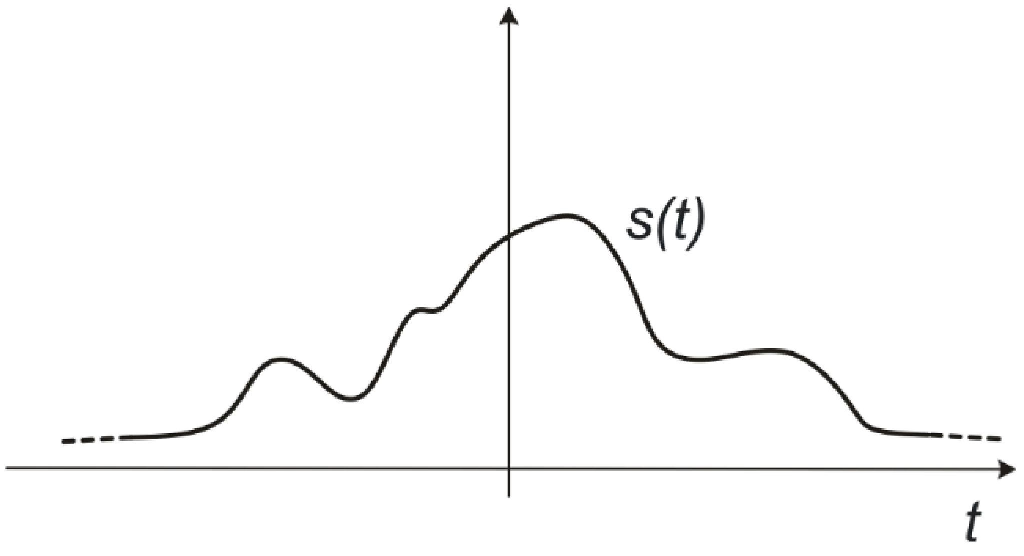


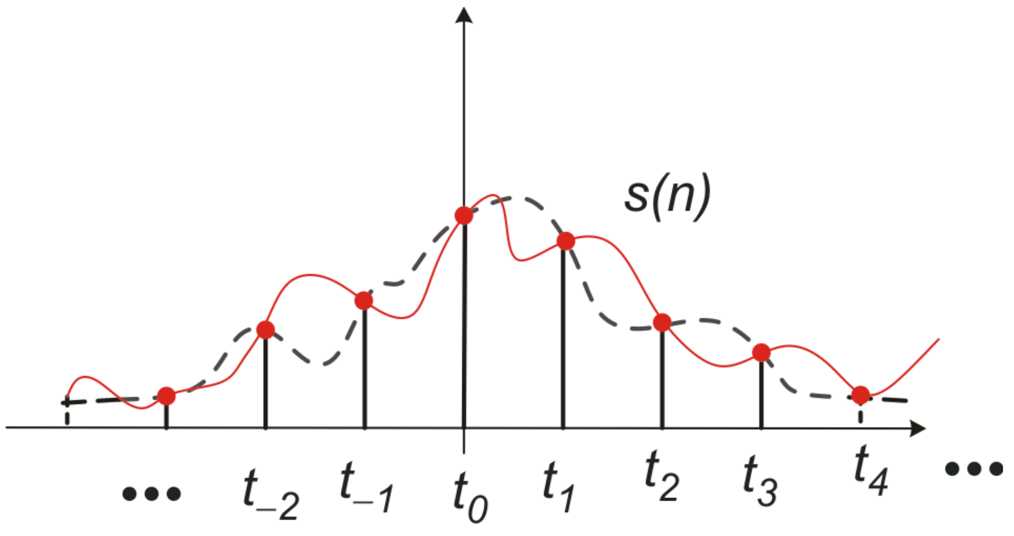
Active network

Passive network:

- Includes only passive components (resistors capacitors, inductors, transformers)
- **When connected to external independent sources, the net energy flux into the network is always positive**

Continuous and Discrete Time





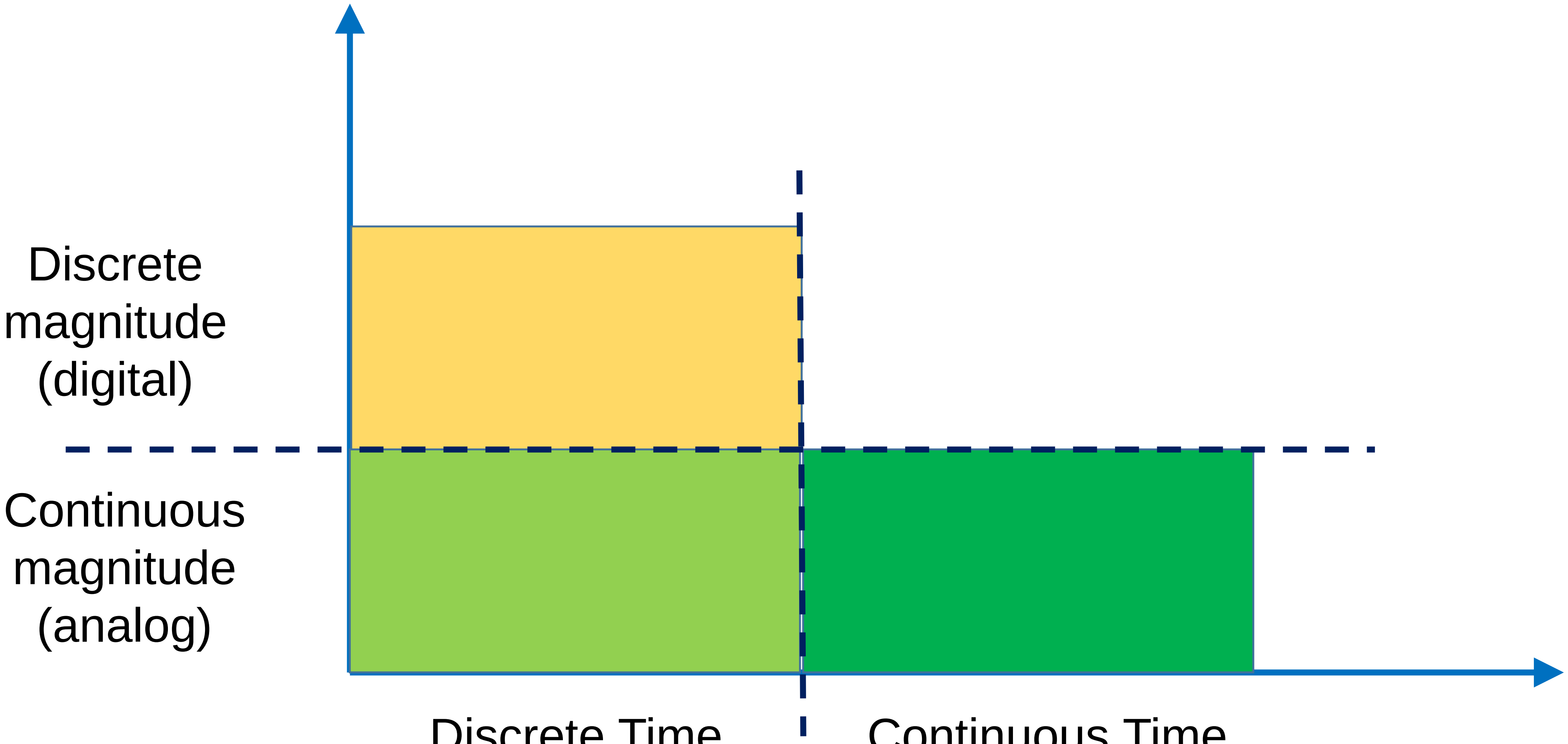
Continuous time: the signal at **all** time values belonging to a **continuous** interval are **significant**

Discrete time: the signal assumes **significant values** only at time intervals that form a countable (i.e. discrete) and ordered set. The signal is then a **sequence** of values s(n)

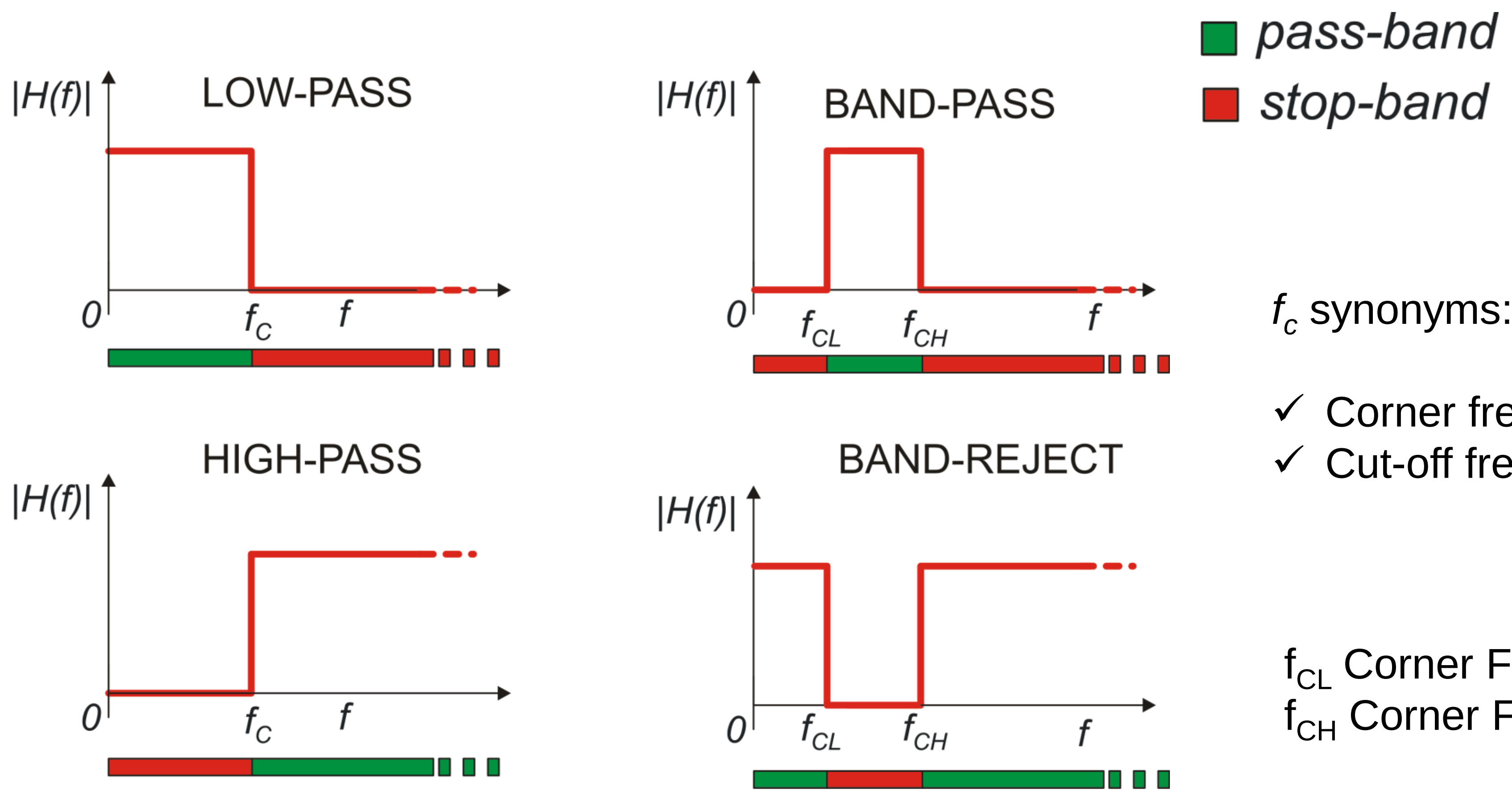
A sequence can be considered as the result of sampling a continuous-time signal

This relationship is not univocal

Time and Value Discretization



Filter types according to the ideal magnitude response

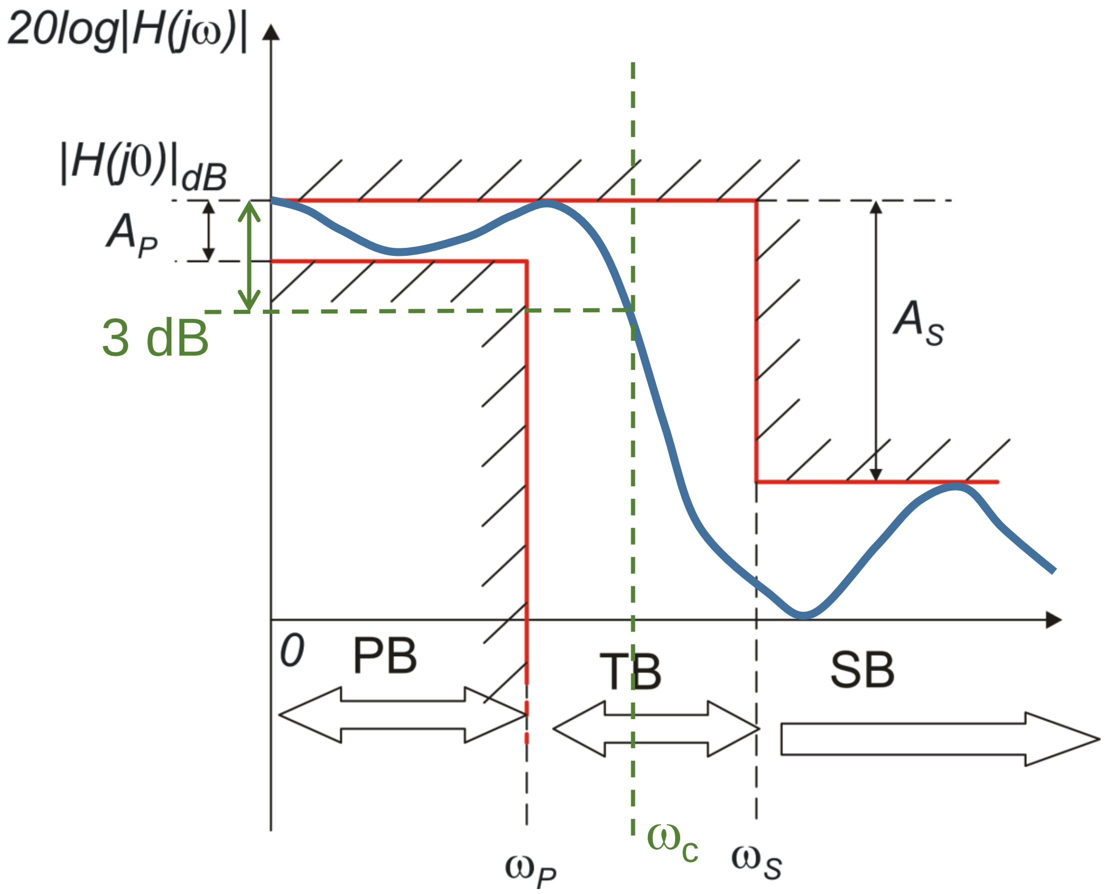


f_c synonyms:

- ✓ Corner frequency
- ✓ Cut-off frequency

f_{CL} Corner Freq – Low
f_{CH} Corner Freq – High

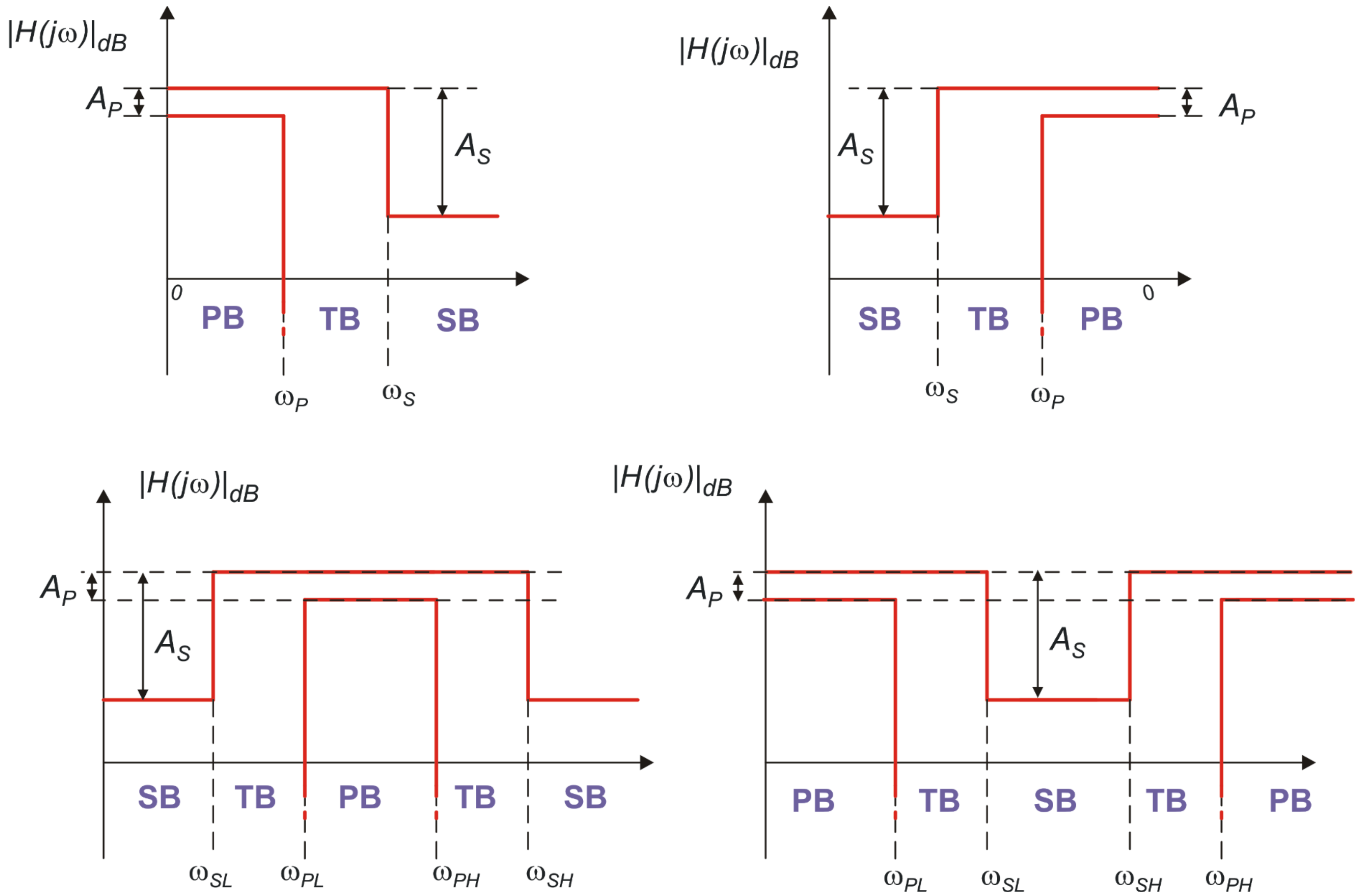
Real filters: the approximation function



- The Low Pass filter is the reference for other types
- Frequency are general given as angular frequencies (ω)
- A Transition Band (TB) is introduced

Note: ω_p is generally different from ω_c (-3 dB frequency)

Approximation parameters for high-pass, band-pass, band-stop filters



The approximation problem for

time-continuous analog filters

Approximation Problem

The first step is reducing the wide variability in filter characteristics by designing a "reference filter" from which the actual filter can be derived through "transformations".

The reference filter, $H_N(j\omega)$, is:

- Low Pass
- Normalized

Gain normalization
Characteristic frequency normalization

$$H_N(j\omega) \equiv \frac{H(j\omega \cdot \omega_N)}{H(j0)} \Rightarrow H(j\omega) = H(j0) \cdot H_N\left(j\frac{\omega}{\omega_N}\right)$$

ω_N and H(j0) depend on the actual filter

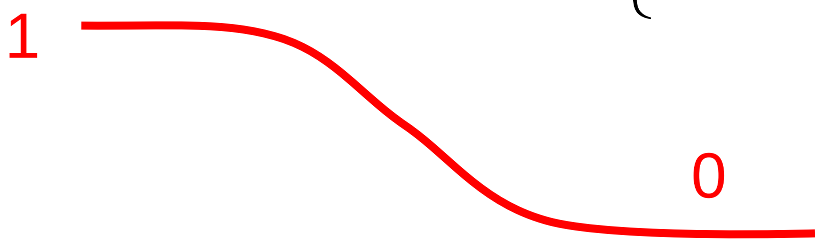
$$H_N(j\omega): \quad \omega_P \Rightarrow \omega_{PN} \equiv \frac{\omega_P}{\omega_N} \qquad \omega_S \Rightarrow \omega_{SN} \equiv \frac{\omega_S}{\omega_N}$$

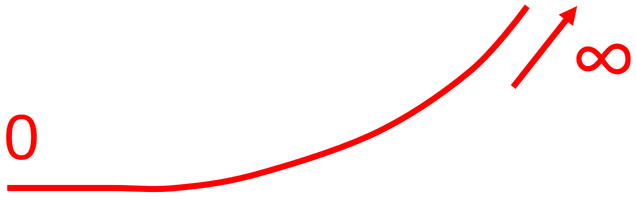
Approximation Problem

In order to classify the filters, it is convenient to define the function |k(jw)| as follows:

$$|K(j\omega)|^2 \equiv \frac{1}{|H_N(j\omega)|^2} - 1 \quad \Rightarrow \quad |H_N(j\omega)|^2 = \frac{1}{1+|K(j\omega)|^2}$$

Ideal Case: $$|H_N(j\omega)|^2 = \begin{cases} 1 & \text{in the PB} \\ 0 & \text{in the SB} \end{cases} \quad \Rightarrow \quad |K(j\omega)|^2 = \begin{cases} 0 & \text{in the PB} \\ \infty & \text{in the SB} \end{cases}$$





Approximation Problem

Real Case: ⇨ $$|K(j\omega)|^2 = \begin{cases} \le a^2 \ll 1 & \text{in the PB} \\ \ge \delta^2 \gg 1 & \text{in the SB} \end{cases}$$

$$A_P = -20\log\left(\frac{|H_N(j\omega)|}{|H_N(j0)|}\right) = 10\log\left(\frac{1}{|H_N(j\omega)|^2}\right) = 10\log\left(1+|K(j\omega)|^2\right)$$

Worst case: $$A_P = 10\log\left(1+a^2\right) \Rightarrow a = \sqrt{10^{\frac{A_P}{10}}-1}$$

Similarly: $$A_S = 10\log\left(1+\delta^2\right) \Rightarrow \delta = \sqrt{10^{\frac{A_S}{10}}-1}$$

Approximation Problem

We need to find a function K(s) such that $|K(j\omega)|^2$ satisfies the conditions:

$$|K(j\omega)|^2 = \begin{cases} \leq a^2 << 1 & \text{for } \omega \leq \omega_{PN} \\ \geq \delta^2 >> 1 & \text{for } \omega \geq \omega_{SN} \end{cases}$$

Clearly:

- There are infinite solutions to this mathematical problem
- Solutions should lead to a feasible $H_N(s)$
- Lumped element filter: $H_N(s)$ should be a rational functions:

$$H(s) = \frac{N(s)}{D(s)}$$

Where N(s) and D(s) are polynomial

Notable cases

- Maximally Flat Magnitude Filters (e.g. Butterworth Filters)
- Chebyshev Filters
- Inverse Chebyshev Filters
- Elliptical Filters
- Bessel Filters

General selection criteria:

- The lower the polynomial order, the better the solution
- Monotonic behavior in the PB can be required
- Asymptotic behavior for $\omega >> \omega_S$ could be important
- Phase response: a linear response is often required

Maximally Flat Magnitude (MFM) Approximation

$$K(s) = \varepsilon s^n$$

$$K(j\omega) = \varepsilon (j\omega)^n \quad \Rightarrow \quad |K(j\omega)|^2 = \varepsilon^2 \omega^{2n}$$

$$|H_N(j\omega)| = \frac{1}{\sqrt{1+|K(j\omega)|^2}} = \frac{1}{\sqrt{1+\varepsilon^2\omega^{2n}}}$$

$$|H_N(j\omega)| = 1 - \frac{1}{2}\varepsilon^2\omega^{2n} + \frac{3}{8}\varepsilon^4\omega^{4n} - \frac{5}{16}\varepsilon^6\omega^{6n} + \ldots.$$

All derivatives up to the (2n-1)th are zero for $\omega=0$.

Maximally Flat Magnitude (MFM) Approximation

$$\left|H_N(j\omega)\right|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

Since $H_N(s)$ is the Fourier transform of a real signal (the impulsive response): $H_N^*(j\omega)=H_N(-j\omega)$. Then:

$$H_N(j\omega)H_N(-j\omega) = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

Maximally Flat Magnitude (MFM) Approximation

$$s = j\omega \quad \Rightarrow \quad \omega = -js$$

$$H_N(s)H_N(-s) = \frac{1}{1+\varepsilon^2\left(-s^2\right)^n}$$

$$H_N(s) = \frac{1}{D(s)}$$

$$H_N(s)H_N(-s) = \frac{1}{D(s)D(-s)} = \frac{1}{1+\varepsilon^2\left(-s^2\right)^n}$$

Thus, $H_N(s)$ is an all-poles function:

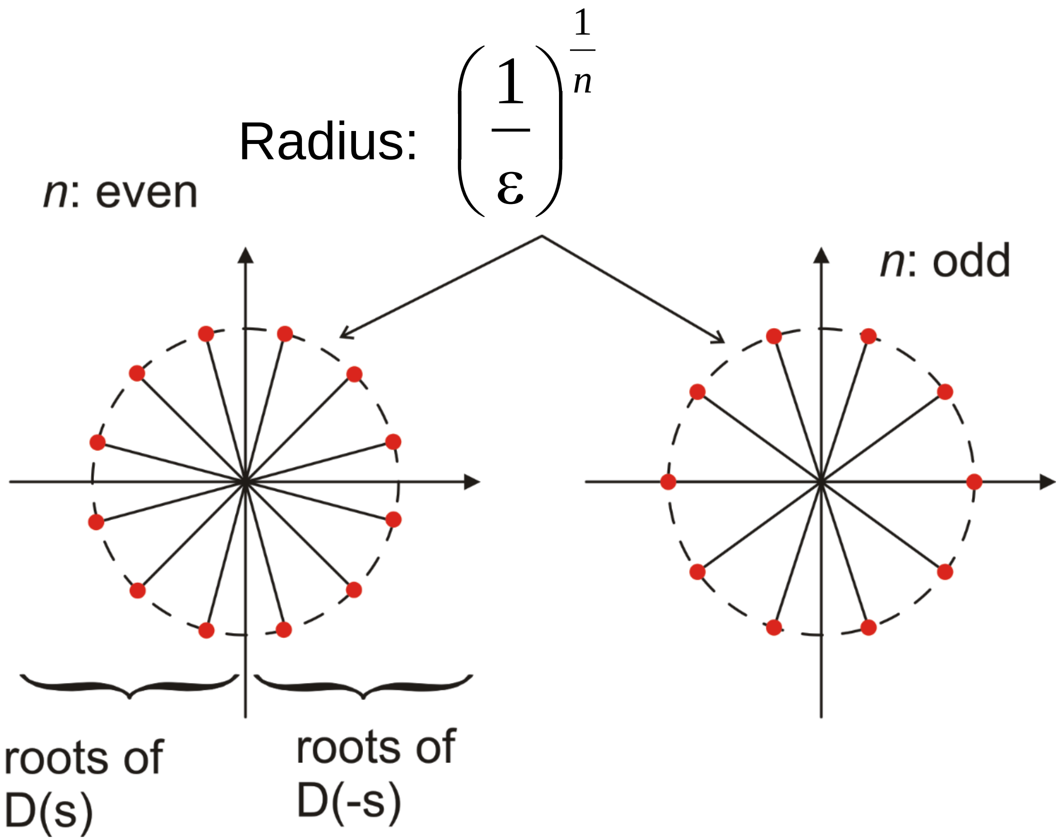
The roots of *D(s)D(-s)* are the solutions of $1+\varepsilon^2(-s^2)^n=0$

Maximally Flat Magnitude (MFM) Approximation

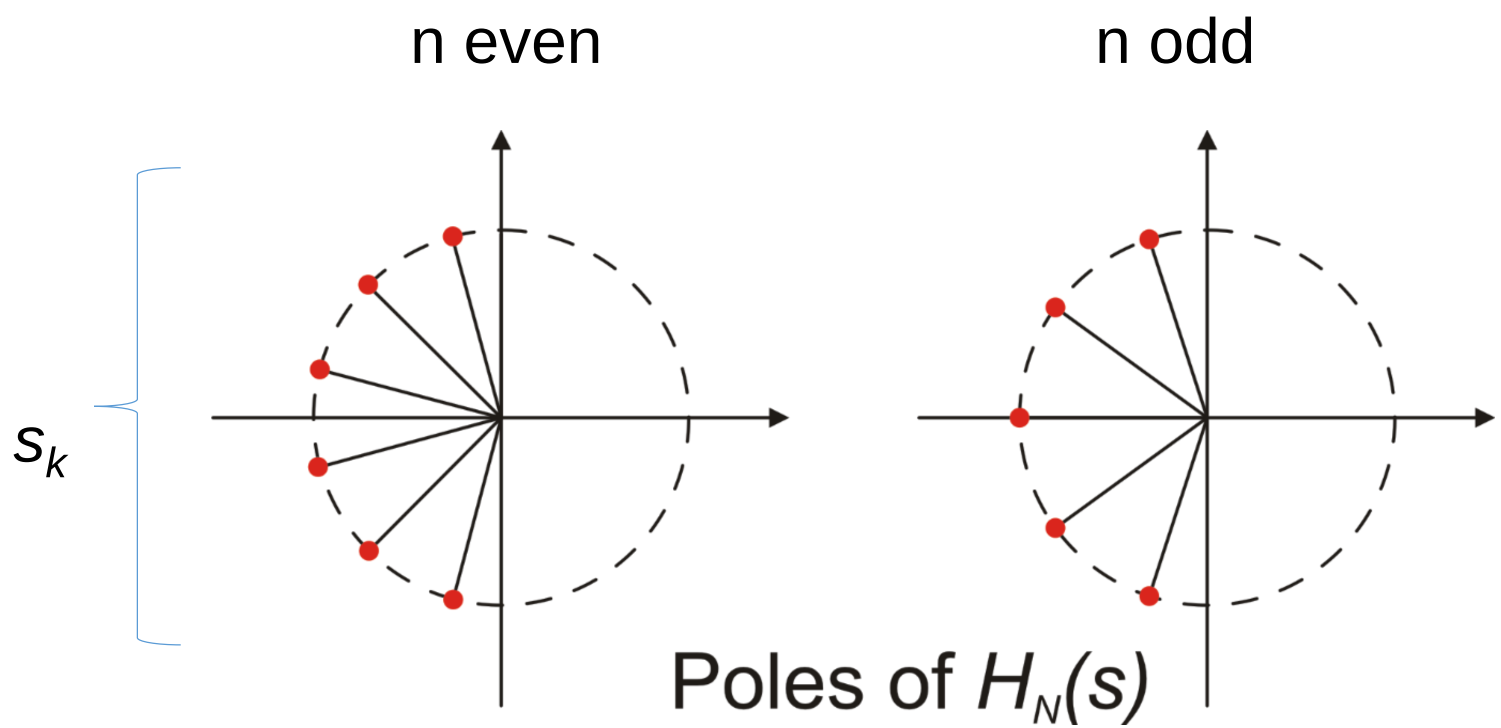
$$\left(-s^2\right)^n = -\frac{1}{\varepsilon^2} \quad \Rightarrow \quad s^{2n} = (-1)^{n-1}\frac{1}{\varepsilon^2} = e^{j\pi(n-1)}\frac{1}{\varepsilon^2}$$

$$s_k = \left(\frac{1}{\varepsilon}\right)^{\frac{1}{n}} \exp\left(j\pi\frac{2k+n-1}{2n}\right) \quad k: \text{integer}$$

$$\text{angle}: \begin{cases} \text{odd multiples of } \dfrac{\pi}{2n} \text{ for n even} \\ \text{even multiples of } \dfrac{\pi}{2n} \text{ for n odd} \end{cases}$$



Maximally Flat Magnitude (MFM) Approximation



$$H_N(s) = \frac{1}{\prod_{k=1}^{n}(s - s_k)}$$

Butterworth Filters

- The Butterworth filter is a MFM filter with ε=1
- It can be shown that the case ε≠1 is identical to ε=1 but with simple frequency transformation

$$|H_N(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

$D_n(s)$ are the Butterworth polynomials

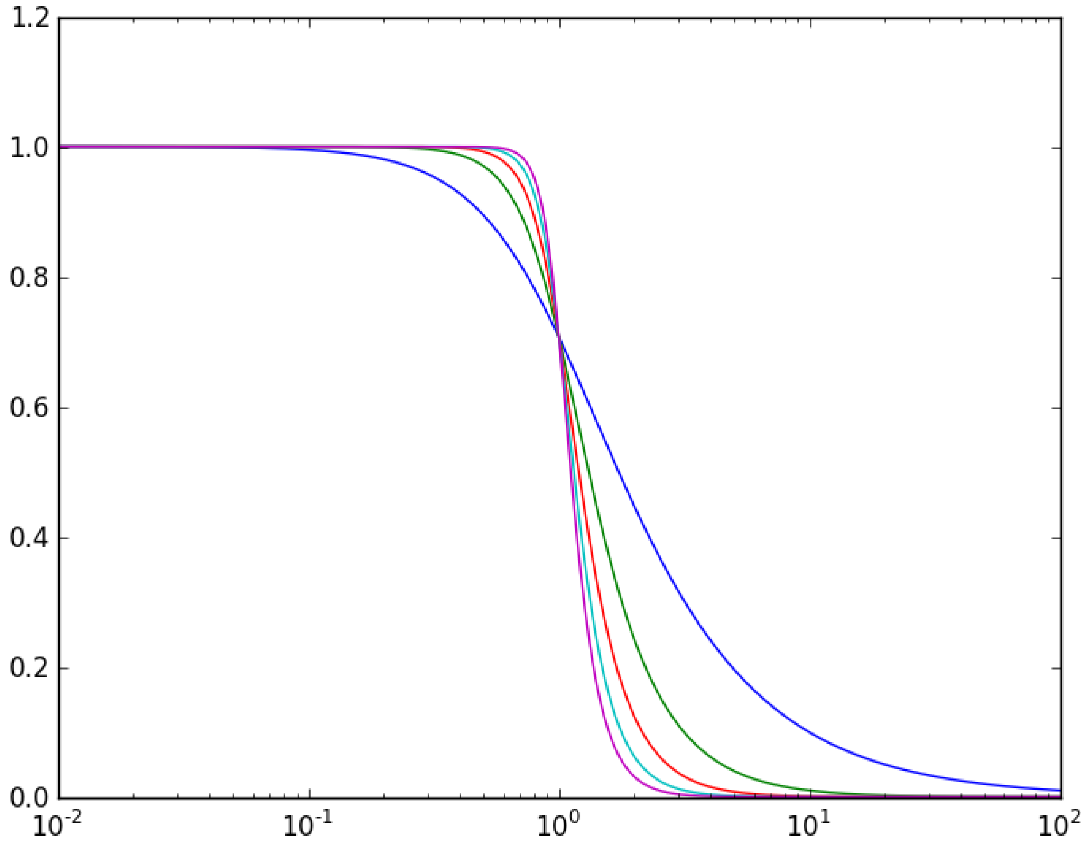
Non normalized (actual) filter ⇨ $$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_N}\right)^{2n}}}$$

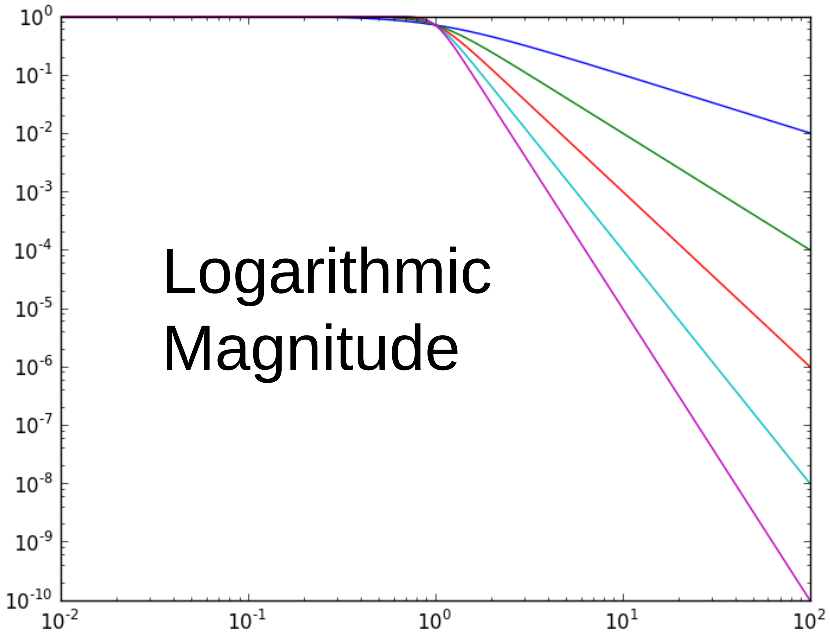
Butterworth Filters

$$|H_N(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}} \Rightarrow |H_N(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}} \Rightarrow -3dB \Rightarrow \omega_{CN} = 1$$

$$\Downarrow$$

$$\omega_N \equiv \omega_C$$





Logarithmic Magnitude

Filter Parameter Determination

$$|K(j\omega_{SN})| = \delta \quad |K(j\omega_{PN})| = a$$

$$a = \sqrt{10^{\frac{A_P}{10}} - 1} \quad \delta = \sqrt{10^{\frac{A_S}{10}} - 1}$$

$$|K(j\omega)| = \omega^n$$

$$\left(\frac{\omega_{SN}}{\omega_{PN}}\right)^n = \left(\frac{\omega_S}{\omega_P}\right)^n = \frac{\delta}{a} = \sqrt{\frac{10^{\frac{A_S}{10}} - 1}{10^{\frac{A_p}{10}} - 1}} = \sqrt{\eta}$$

$$n = \left\lceil \frac{\log(\eta)}{2\log\left(\frac{\omega_S}{\omega_P}\right)} \right\rceil$$

$$\omega_{PN}^n = a \Rightarrow \omega_{PN} = a^{\frac{1}{n}}$$

$$\frac{\omega_P}{\omega_N} = \omega_{PN} \Rightarrow \omega_N = \omega_C = \omega_P a^{-\frac{1}{n}}$$

Butterworth filter : example

f_P=1 kHz
f_S=2 kHz
A_P=1 dB
A_S=60 dB

$$\frac{\omega_S}{\omega_P} = \frac{f_S}{f_P}$$

$$a = \sqrt{10^{\frac{A_P}{10}} - 1} = 0.509$$

$$\delta = \sqrt{10^{\frac{A_S}{10}} - 1} = 1000$$

$$\sqrt{\eta} = \frac{\delta}{a} = 1965$$

$$\eta = 3.86 \times 10^6$$

$$n = \left\lceil \frac{\log(\eta)}{2\log\left(\frac{\omega_S}{\omega_P}\right)} \right\rceil = \lceil 10.94 \rceil = 11$$

$$\omega_C = \omega_P a^{-\frac{1}{n}} = 2\pi f_P \cdot 1.063$$

Chebyshev Filters

$$\left|K(j\omega)\right| = \varepsilon C_n(\omega)$$

$C_n(\omega)$: n-th order Chebyshev polynomial

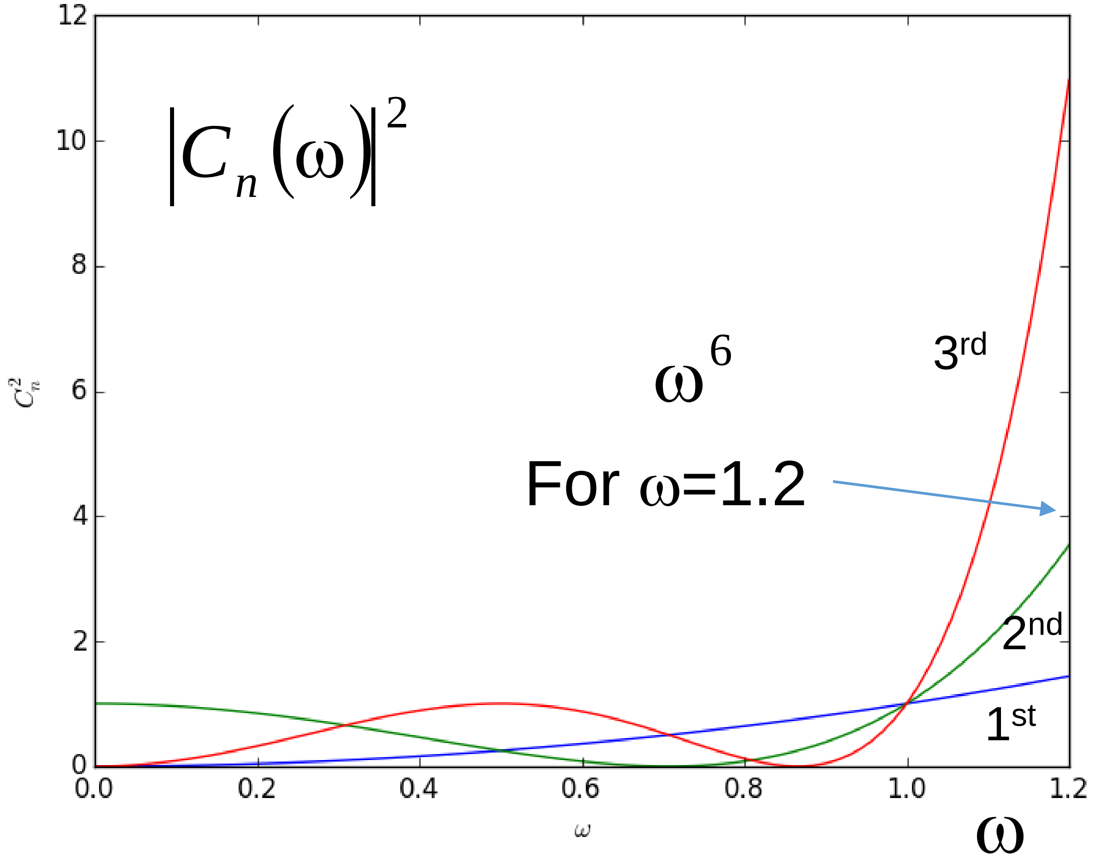
$$C_n(\omega) = \begin{cases} \cos\left[n \cdot \arccos(\omega)\right] & |\omega| < 1 \\ \cosh\left[n \cdot \operatorname{arccos h}(\omega)\right] & |\omega| > 1 \end{cases}$$

$$C_n(x) = 2xC_{n-1}(x) - C_{n-2}$$

$$C_0(x) = 1; \quad C_1(x) = x \qquad C_2(x) = 2x^2 - 1$$

Chebyshev polynomials: properties

$$C_n(\omega) = \begin{cases} \cos[n \cdot \arccos(\omega)] & |\omega| < 1 \\ \cosh[n \cdot \operatorname{arccos\,h}(\omega)] & |\omega| > 1 \end{cases}$$



For $0<\omega<1$ oscillates between 0 and 1
For $\omega=0$: 0 if n odd, 1 if n even
For $\omega=1$: 1 for every n
For $\omega>0$ increase monotonically

Chebyshev polynomial have the highest leading term coefficient than any other Polynomial constrained to be less than 1 (in modulus) for ω between 0 and 1

Chebyshev filters: Pass Band Attenuation

$$\left|K(j\omega)\right| = \varepsilon C_n(\omega)$$

$$\left|H_N(j\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2(\omega)}}$$

$$0 < \omega < 1 \Rightarrow \frac{1}{\sqrt{1+\varepsilon^2}} \le \left|H_N(j\omega)\right| \le 1$$

$$\omega_N \equiv \omega_P \qquad \frac{1}{\sqrt{1+\varepsilon^2}} \le \left|\frac{H(j\omega)}{H(j0)}\right| = H_N\left(j\frac{\omega}{\omega_N}\right) \le 1$$

$$a = \varepsilon = \sqrt{10^{\frac{A_P}{10}} - 1}$$

$$\text{e.g.} \begin{cases} A_P = 1\ \text{dB} \Rightarrow \varepsilon \cong 0.5 \\ A_P = 3\ \text{dB} \Rightarrow \varepsilon \cong 1 \end{cases}$$

Chebyshev filters: Stop Band Attenuation

$$\left|K(j\omega_{SN})\right| = \varepsilon C_n(\omega_{SN}) = \varepsilon \cdot \cosh\left(n \cdot \operatorname{arccosh}(\omega_{SN})\right) = \delta$$

$$\cosh\left(n \cdot \operatorname{arccosh}(\omega_{SN})\right) = \frac{\delta}{\varepsilon} = \sqrt{\eta}$$

$$\eta \equiv \frac{\delta^2}{\varepsilon^2} = \frac{10^{\frac{A_S}{10}} - 1}{10^{\frac{A_P}{10}} - 1}$$

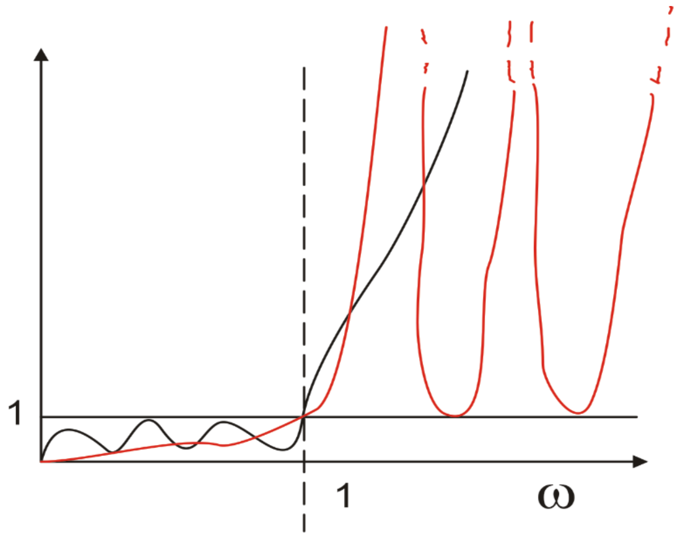
$$n_{\min} = \left\lceil \frac{\operatorname{arccosh}(\sqrt{\eta})}{\operatorname{arccosh}(\omega_{SN})} \right\rceil$$

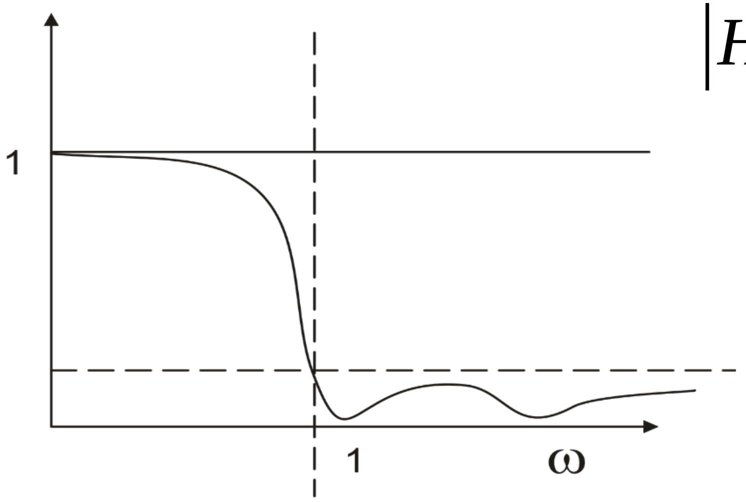
Inverse Chebyshev filters (or Chebyshev type II)

Target: Obtain monotonic behavior in the pass-band (no ripple) and ripple
In the stop-band

$$|K(j\omega)| = \frac{1}{\varepsilon C_n\left(\frac{1}{\omega}\right)}$$

$$|H_N(j\omega)| = \frac{1}{\sqrt{1+\frac{1}{\varepsilon^2 C_n^2\left(\frac{1}{\omega}\right)}}}$$





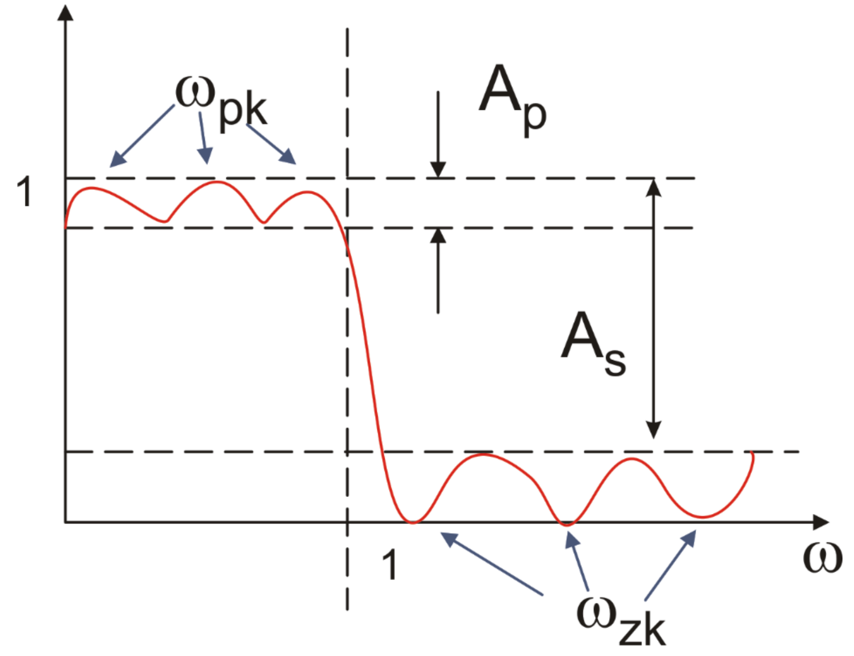
$$|H_N(j1)| = \frac{1}{\sqrt{1+\frac{1}{\varepsilon^2}}}$$

Elliptic (or Cauer) Filters

$$|K(j\omega)| = \varepsilon R_n(\omega) \qquad |H_N(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 R_n^2(\omega)}}$$

R_n : "elliptical" rational function : $N_R(\omega) / D_R(\omega)$

$$R_n(\omega) = \begin{cases} M \prod_{k=1}^{n/2} \frac{\omega^2 - \omega_{pk}^2}{\omega^2 - \omega_{zk}^2} & \text{for n even} \\ M\omega \prod_{k=1}^{(n-1)/2} \frac{\omega^2 - \omega_{pk}^2}{\omega^2 - \omega_{zk}^2} & \text{for n odd} \end{cases}$$



$$\omega_{pk}\omega_{zk} = \omega_s$$

M such that $|R_n(\omega)| \leq 1$ for $|\omega| \leq 1$

Phase, delay, group delay

In order to maintain the shape of a generic signal, the following conditions must be respected:

- All the significant frequency components of the signal fall into the filter pass-band, which should be as flat as possible:
- The filter phase response in the pass-band should be of the type:

$$\phi = -\omega t_R$$

If this condition is fulfilled, the input signal is simply delayed by time t_R.

In other words, the group delay should be constant. The group delay is defined as:

$$\tau_G = -\frac{d\phi}{d\omega}$$

Bessel Filters

Constant group delay is very important in systems that has to handle digital transmissions, where signal distortion may result in high BER (Bit Error Rate), or even in unrecoverable signal.

The Bessel filter is obtained by considering an all-pole function:

$$H_N(s) = \frac{K}{D(s)}$$

We start from a generic polynomial D(s), substitute s=jω and than calculate the phase of H_N by:

$$\phi = -\arctan\left(\frac{\mathrm{Im}(D(j\omega))}{\mathrm{Re}(D(j\omega))}\right)$$

Bessel Filters

The Bessel Filters are derived by:

- Taylor’s expansion of the arctan() functionis calculated obtaining a polynomial approximation of the phase;
- The first derivative of the phase approximation is calculated
- The constant term of the derivative is set to 1 (group delay=1)
- Higher order terms are set to zero; this corresponds to setting the derivatives of the group delay to zero. The number of derivatives that can be nulled depends on the filter order.

➢ The result is a maximally flat group delay

➢ Can be used to purposely introduce a delay

Bessel filters, examples

Denominators of various order for unity group delay

$$1: \; D(s) = s + 1; \qquad K = 1$$

$$2: \; D(s) = s^2 + 3s + 3; \qquad K = 3$$

$$3: \; D(s) = s^3 + 6s^2 + 15s + 15; \qquad K = 15$$

....

$$\omega_{-3dB} \cong \sqrt{(2n-1)\ln(2)}$$

Approximate expression for n ≥ 3

$$D_n = (2n-1)D_{n-1} + s^2 D_{n-2}$$ (Bessel polynomials, recursive expression)

$$\omega \rightarrow \frac{\omega}{\omega_N} \Rightarrow \tau_D = \frac{1}{\omega_N}$$ (pass-band group delay after frequency scaling)

The Bessel filter is less selective than a Butterworth filter of same order, but its phase response is much more linear

Summary of filter characteristics

- ***Butterworth***: maximally flat in the pass-band and monotonic everywhere
- ***Chebyshev***: More selective than Butterworth (sharper transition), but ripple in the pass-band (monotonic in the stop-band)
- ***Inverse Chebyshev***: Same selectivity than Chebyshev, but ripple in the stop-band (flat in the pass-band). Magnitude do not decrease asymptotically in the stop-band
- ***Elliptic***: Best selectivity, but ripple in both the pass-band and stop-band. Magnitude do not decrease asymptotically in the stop-band
- ***Bessel:*** The least selective of all other filters, but the best in terms of phase linearity (constant group-delay in the pass-band)

Other continuous time filters

- Optimum L-filter (Papoulis)
  Obtains the best selectivity with a monotonic response. Compared with a Butterworth of the same order filter it is sharper in the transition band, but less flat (but still monotonic) in the pass-band.
- **All pass filters (phase equalizers)**. Their common characteristic is that for each pole they have a zero with opposite real part. As a result, they have RHP (right half plane) zeros and their step response is generally preceded by a glitch in the opposite direction with respect to the final value.
  For step-like signals, low-pass phase equalizers (e.g. Bessel filters) are to be preferred.
- **Filters based on Padè approximations**: The Padè approximation is the best n-order rational function that approximate an arbitrary function. It is used for the approximation of the ideal delay: $\exp(-j\omega t_D)$. The all-pass functions are a particular case of Padè approximation.

Frequency transformations

The aim of frequency transformations are:

- Change the characteristic frequencies with respect to the normalized case

$$s_n \to \frac{s}{\omega_N}$$

All characteristic frequencies are multiplied by ω_N

- Change the low pass response into an high pass, band-pass etc.

$$s_n \to \frac{\omega_N}{s}$$

High pass

$$s_n \to \frac{\omega_0}{B}\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)$$

Band-Pass

$$s_n \to \left[\frac{\omega_0}{B}\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)\right]^{-1}$$

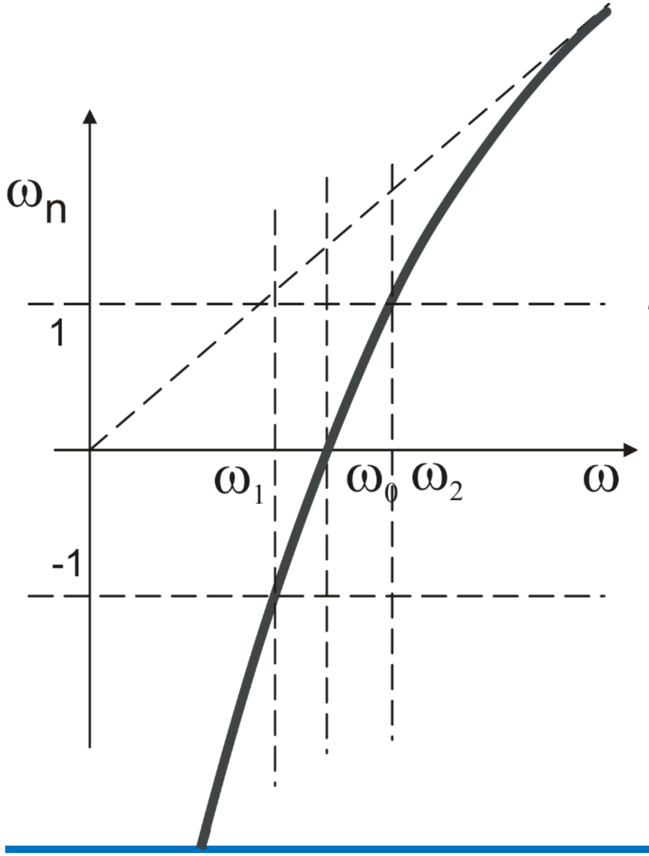
Band Stop

Pass Band transformation: meaning of B, ω_0

$$s_n \to \frac{\omega_0}{B}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)$$

$$j\omega_n \to \frac{\omega_0}{B}\left(\frac{j\omega}{\omega_0}+\frac{\omega_0}{j\omega}\right) = j\frac{\omega_0}{B}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

$$\omega_n \to \frac{\omega_0}{B}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$



for small variations around ω_0, such that: $\frac{\delta\omega}{\omega_0} << 1$

$$\omega_0 + \delta\omega \to \frac{\omega_0}{B}\left(1+\frac{\delta\omega}{\omega_0}-\frac{1}{1+\frac{\delta\omega}{\omega_0}}\right) \cong 2\frac{\delta\omega}{B}$$

Pass Band transformation: meaning of B, ω_0

$$\omega_n = 1 \Rightarrow \omega = \omega_2 = \omega_0\sqrt{1+\left(\frac{B}{2\omega_0}\right)^2} + \frac{B}{2} \qquad \omega_n = -1 \Rightarrow \omega = \omega_1 = \omega_0\sqrt{1+\left(\frac{B}{2\omega_0}\right)^2} - \frac{B}{2}$$

$$\omega_2 - \omega_1 = B$$

$$\frac{\omega_2 + \omega_1}{2} = \omega_0\sqrt{1+\left(\frac{B}{2\omega_0}\right)^2}$$

$$\cong \quad \omega_0$$

$$\text{for}: B \ll \omega_0$$

Pass Band transformation: meaning of B, ω_0

- When $\omega=\omega_0$, $\omega_n=0$. Then, the response of the pass-band filter at ω_0 is the D.C. value ($\omega_n=0$) of the prototype low pass filter.
- For ω variations from ω_0, ω_n moves away from the origin. When $\omega<\omega_0$, ω_n is negative, so that $H(\omega)$ is the complex conjugate of the values at $\omega>\omega_0$ (see the phase diagram in the figure)
- The bandwidth B is the difference between the frequencies ω_1 and ω_2, for which the absolute value of the normalized frequency is unity.
- If the bandwidth B is much smaller than frequency ω_0 (selective filter), than ω_1 and ω_2 are symmetrical with respect to ω_0.