Kinetics of a 1D Slab Nuclear Reactor

1. General Two-Group Equations of Kinetics with Explicit Source

The equations of kinetics with a source are written as:

$$
\begin{aligned}
&\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla \cdot D_1 \nabla \phi_1 - \Sigma_{a,1}\phi_1 - \Sigma_{s,1\to 2}\phi_1 + (1-\beta)\left(\nu\Sigma_{f,1}\phi_1 + \nu\Sigma_{f,2}\phi_2\right) + \sum_{d=1}^{N_d}\lambda_d C_d + S_1 \\
&\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla \cdot D_2 \nabla \phi_2 - \Sigma_{a,2}\phi_1 + \Sigma_{s,1\to 2}\phi_1 + S_2 \\
&\frac{\partial C_d}{\partial t} = -\lambda_d C_d + \beta_d\left(\nu\Sigma_{f,1}\phi_1 + \nu\Sigma_{f,2}\phi_2\right) \qquad (d = 1,\ldots,N_d)
\end{aligned}
\tag{1}
$$

where it has been implicitly assumed that all fission neutrons are produced in the fast energy group.

For a 1D slab the equations are written as:

$$
\begin{aligned}
&\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \frac{\partial}{\partial x} D_1 \frac{\partial \phi_1}{\partial x} - \Sigma_{a,1}\phi_1 - \Sigma_{s,1\to 2}\phi_1 + (1-\beta)\left(\nu\Sigma_{f,1}\phi_1 + \nu\Sigma_{f,2}\phi_2\right) + \sum_{d=1}^{N_d}\lambda_d C_d + S_1 \\
&\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \frac{\partial}{\partial x} D_2 \frac{\partial \phi_2}{\partial x} - \Sigma_{a,2}\phi_2 + \Sigma_{s,1\to 2}\phi_1 + S_2 \\
&\frac{\partial C_d}{\partial t} = -\lambda_d C_d + \beta_d\left(\nu\Sigma_{f,1}\phi_1 + \nu\Sigma_{f,2}\phi_2\right) \qquad (d = 1,\ldots,N_d)
\end{aligned}
\tag{2}
$$

2. Eigenvalue problem

In steady-state conditions, assuming that no source is present, the equations of 1D kinetics reduce to the following form

$$
\begin{aligned}
&\frac{\partial}{\partial x} D_1 \frac{\partial}{\partial x}\phi_1 - \Sigma_{a,1}\phi_1 - \Sigma_{s,1\to 2}\phi_1 + (1-\beta)\left(\nu\Sigma_{f,1}\phi_1 + \nu\Sigma_{f,2}\phi_2\right) + \sum_{d=1}^{N_d}\lambda_d C_d = 0 \\
&\frac{\partial}{\partial x} D_2 \frac{\partial}{\partial x}\phi_2 - \Sigma_{a,2}\phi_2 + \Sigma_{s,1\to 2}\phi_1 = 0 \\
&\lambda_d C_d = \beta_d\left(\nu\Sigma_{f,1}\phi_1 + \nu\Sigma_{f,2}\phi_2\right) \qquad (d = 1,\ldots,N_d)
\end{aligned}
\tag{3}
$$

Reformulating the above by considering the latter expression for the delayed precursors' activity, and introducing the eigenvalue, it is:

$$
\begin{aligned}
&\frac{\partial}{\partial x} D_1 \frac{\partial \phi_1}{\partial x} - \Sigma_{a,1}\phi_1 - \Sigma_{s,1\to 2}\phi_1 + \frac{1}{k}\left(\nu\Sigma_{f,1}\phi_1 + \nu\Sigma_{f,2}\phi_2\right) = 0 \\
&\frac{\partial}{\partial x} D_2 \frac{\partial \phi_2}{\partial x} - \Sigma_{a,2}\phi_2 + \Sigma_{s,1\to 2}\phi_1 = 0
\end{aligned}
\tag{4}
$$

3. Numerical discretization of transient equations

By subdividing the range of the x variable (0,a) into N intervals, in each interval we have:

$$\begin{aligned}
\left(\phi_{1,i}^{n+1}-\phi_{1,i}^{n}\right)\frac{\Delta x_i}{v_1\Delta t} &= -D_{1,i}\frac{\phi_{1,i}^{n,n+1}-\phi_{1,i-1/2}^{n,n+1}}{x_i-x_{i-1/2}}+D_{1,i}\frac{\phi_{1,i+1/2}^{n,n+1}-\phi_{1,i}^{n,n+1}}{x_{i+1/2}-x_i}-\Sigma_{a,1,i}\phi_{1,i}^{n,n+1}\Delta x_i-\Sigma_{s,1\to 2}\phi_{1,i}^{n,n+1}\Delta x_i \\
&\quad +\left(1-\beta\right)\left(\nu\Sigma_{f,1}\phi_{1,i}^{n,n+1}+\nu\Sigma_{f,2}\phi_{2,i}^{n,n+1}\right)\Delta x_i+\sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n,n+1}\Delta x_i+S_1^n\Delta x_i \\
\left(\phi_{2,i}^{n+1}-\phi_{2,i}^{n}\right)\frac{\Delta x_i}{v_2\Delta t} &= -D_{2,i}\frac{\phi_{2,i}^{n,n+1}-\phi_{2,i-1/2}^{n,n+1}}{x_i-x_{i-1/2}}+D_{2,i}\frac{\phi_{2,i+1/2}^{n,n+1}-\phi_{2,i}^{n,n+1}}{x_{i+1/2}-x_i}-\Sigma_{a,2,i}\phi_{2,i}^{n,n+1}\Delta x_i+\Sigma_{s,1\to 2}\phi_{1,i}^{n,n+1}\Delta x_i+S_2^n\Delta x_i \\
C_d^{n+1} &= C_d^n\, e^{-\lambda_d\Delta t}+\frac{\beta_d}{\lambda_d}\left(\nu\Sigma_{f,1}\phi_{1,i}^{n,n+1}+\nu\Sigma_{f,2}\phi_{2,i}^{n,n+1}\right)\left(1-e^{-\lambda_d\Delta t}\right) \qquad \left(d=1,\ldots,N_d\right)
\end{aligned} \tag{5}$$

where the superscript *n,n+1* indicates an appropriate means between the values of the variable at the time t^n and t^{n+1}. As it can be noted, a semi-analytic expression was used for integrating the neutron precursors' equations. Moreover, it is assumed:

$$\phi_{1,i}^{n,n+1}=\left(1-\theta\right)\phi_{1,i}^{n}+\theta\,\phi_{1,i}^{n+1} \tag{6}$$

where $0\le\theta\le 1$; similar relationships hold for the other variables.

Concerning the fractional index fluxes, the usual relationships implying the continuity of neutron current allows eliminating them. It is:

$$-D_{1,i}\frac{\phi_{1,i}^{n,n+1}-\phi_{1,i-1/2}^{n,n+1}}{x_i-x_{i-1/2}}=-D_{1,i-1}\frac{\phi_{1,i-1/2}^{n,n+1}-\phi_{1,i-1}^{n,n+1}}{x_{i-1/2}-x_{i-1}} \tag{7}$$

and then

$$-D_{1,i}\frac{\phi_{1,i}^{n,n+1}}{x_i-x_{i-1/2}}+D_{1,i}\frac{\phi_{1,i-1/2}^{n,n+1}}{x_i-x_{i-1/2}}=-D_{1,i-1}\frac{\phi_{1,i-1/2}^{n,n+1}}{x_{i-1/2}-x_{i-1}}+D_{1,i-1}\frac{\phi_{1,i}^{n,n+1}}{x_{i-1/2}-x_{i-1}}$$

$$\left[\frac{D_{1,i}}{x_i-x_{i-1/2}}+\frac{D_{1,i-1}}{x_{i-1/2}-x_{i-1}}\right]\phi_{1,i-1/2}^{n,n+1}=\left[\frac{D_{1,i-1}}{x_{i-1/2}-x_{i-1}}\phi_{1,i-1}^{n,n+1}+\frac{D_{1,i}}{x_i-x_{i-1/2}}\phi_{1,i}^{n,n+1}\right]$$

$$\left[\frac{2D_{1,i}}{\Delta x_i}+\frac{2D_{1,i-1}}{\Delta x_{i-1}}\right]\phi_{1,i-1/2}^{n,n+1}=\left[\frac{2D_{1,i-1}}{\Delta x_{i-1}}\phi_{1,i-1}^{n,n+1}+\frac{2D_{1,i}}{\Delta x_i}\phi_{1,i}^{n,n+1}\right]$$

$$\phi_{1,i-1/2}^{n,n+1}=\frac{\dfrac{D_{1,i-1}}{\Delta x_{i-1}}}{\dfrac{D_{1,i-1}}{\Delta x_{i-1}}+\dfrac{D_{1,i}}{\Delta x_i}}\phi_{1,i-1}^{n,n+1}+\frac{\dfrac{D_{1,i}}{\Delta x_i}}{\dfrac{D_{1,i-1}}{\Delta x_{i-1}}+\dfrac{D_{1,i}}{\Delta x_i}}\phi_{1,i}^{n,n+1} \tag{8}$$

Taking into account the above relationship, it is:

$$-D_{1,i}\frac{\phi_{1,i}^{n,n+1}-\phi_{1,i-1/2}^{n,n+1}}{x_i-x_{i-1/2}}=-\frac{2D_{1,i}}{\Delta x_i}\phi_{1,i}^{n,n+1}+\frac{2D_{1,i}}{\Delta x_i}\phi_{1,i-1/2}^{n,n+1}$$

$$= -\frac{2D_{1,i}}{\Delta x_i}\phi_{1,i}^{n,n+1} + \frac{2D_{1,i}}{\Delta x_i}\left[\frac{\frac{D_{1,i-1}}{\Delta x_{i-1}}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}}\phi_{1,i-1}^{n,n+1} + \frac{\frac{D_{1,i}}{\Delta x_i}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}}\phi_{1,i}^{n,n+1}\right]$$

$$= -\frac{2D_{1,i}}{\Delta x_i}\phi_{1,i}^{n,n+1} + \frac{\frac{2D_{1,i}}{\Delta x_i}\frac{D_{1,i-1}}{\Delta x_{i-1}}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}}\phi_{1,i-1}^{n,n+1} + \frac{\frac{2D_{1,i}}{\Delta x_i}\frac{D_{1,i}}{\Delta x_i}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}}\phi_{1,i}^{n,n+1}$$

$$= \frac{\frac{2D_{1,i}}{\Delta x_i}\frac{D_{1,i-1}}{\Delta x_{i-1}}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}}\phi_{1,i-1}^{n,n+1} - \frac{\frac{2D_{1,i}}{\Delta x_i}\frac{D_{1,i-1}}{\Delta x_i}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}}\phi_{1,i}^{n,n+1} = -\frac{2\frac{D_{1,i-1}}{\Delta x_{i-1}}\frac{D_{1,i}}{\Delta x_i}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}}\left(\phi_{1,i}^{n,n+1} - \phi_{1,i-1}^{n,n+1}\right)$$

Therefore, putting

$$\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1} = \frac{2\frac{D_{1,i-1}}{\Delta x_{i-1}}\frac{D_{1,i}}{\Delta x_i}}{\frac{D_{1,i-1}}{\Delta x_{i-1}}+\frac{D_{1,i}}{\Delta x_i}} \tag{9}$$

it is

$$-D_{1,i}\frac{\phi_{1,i}^{n,n+1} - \phi_{1,i-1/2}^{n,n+1}}{x_i - x_{i-1/2}} = -\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\left(\phi_{1,i}^{n,n+1} - \phi_{1,i-1}^{n,n+1}\right) \tag{10}$$

Similarly, it is also:

$$D_{1,i}\frac{\phi_{1,i+1/2}^{n,n+1} - \phi_{1,i}^{n,n+1}}{x_i - x_{i-1/2}} = \overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\left(\phi_{1,i+1}^{n,n+1} - \phi_{1,i}^{n,n+1}\right) \tag{11}$$

and

$$-D_{2,i}\frac{\phi_{2,i}^{n,n+1} - \phi_{2,i-1/2}^{n,n+1}}{x_i - x_{i-1/2}} = -\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}\left(\phi_{2,i}^{n,n+1} - \phi_{2,i-1}^{n,n+1}\right) \tag{12}$$

$$D_{2,i}\frac{\phi_{2,i+1/2}^{n,n+1} - \phi_{2,i}^{n,n+1}}{x_i - x_{i-1/2}} = \overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}\left(\phi_{2,i+1}^{n,n+1} - \phi_{2,i}^{n,n+1}\right) \tag{13}$$

With the above definitions it is:

$$\boxed{\begin{aligned}
&\left(\phi_{1,i}^{n+1} - \phi_{1,i}^{n}\right)\frac{\Delta x_i}{v_1\Delta t} = -\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\left(\phi_{1,i}^{n,n+1} - \phi_{1,i-1}^{n,n+1}\right) + \overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\left(\phi_{1,i+1}^{n,n+1} - \phi_{1,i}^{n,n+1}\right) - \Sigma_{a,1,i}\phi_{1,i}^{n,n+1}\Delta x_i - \Sigma_{s,1\to 2}\phi_{1,i}^{n,n+1}\Delta x_i \\
&\qquad\qquad + (1-\beta)\left(\nu\Sigma_{f,1}\phi_{1,i}^{n,n+1} + \nu\Sigma_{f,2}\phi_{2,i}^{n,n+1}\right)\Delta x_i + \sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n,n+1}\Delta x_i + S_1^n\Delta x_i \\
&\left(\phi_{2,i}^{n+1} - \phi_{2,i}^{n}\right)\frac{\Delta x_i}{v_2\Delta t} = -\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}\left(\phi_{2,i}^{n,n+1} - \phi_{2,i-1}^{n,n+1}\right) + \overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}\left(\phi_{2,i+1}^{n,n+1} - \phi_{2,i}^{n,n+1}\right) - \Sigma_{a,2,i}\phi_{2,i}^{n,n+1}\Delta x_i + \Sigma_{s,1\to 2}\phi_{1,i}^{n,n+1}\Delta x_i + S_2^n\Delta x_i \\
&C_d^{n+1} = C_d^n\, e^{-\lambda_d\Delta t} + \frac{\beta_d}{\lambda_d}\left(\nu\Sigma_{f,1}\phi_{1,i}^{n,n+1} + \nu\Sigma_{f,2}\phi_{2,i}^{n,n+1}\right)\left(1 - e^{-\lambda_d\Delta t}\right) \qquad (d = 1,\ldots,N_d)
\end{aligned}} \tag{14}$$

By elaborating the equation for the first energy group, we have:

$$\left(\phi_{1,i}^{n+1}-\phi_{1,i}^{n}\right)\frac{\Delta x_i}{v_1\Delta t}=\theta\left[-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\left(\phi_{1,i}^{n+1}-\phi_{1,i-1}^{n+1}\right)+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\left(\phi_{1,i+1}^{n+1}-\phi_{1,i}^{n+1}\right)-\Sigma_{a,1,i}\phi_{1,i}^{n+1}\Delta x_i-\Sigma_{s,1\to 2}\phi_{1,i}^{n+1}\Delta x_i\right.$$
$$\left.+\left(1-\beta\right)\left(\nu\Sigma_{f,1}\phi_{1,i}^{n+1}+\nu\Sigma_{f,2}\phi_{2,i}^{n+1}\right)\Delta x_i+\sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n+1}\Delta x_i\right]$$
$$+\left(1-\theta\right)\left[-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\left(\phi_{1,i}^{n}-\phi_{1,i-1}^{n}\right)+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\left(\phi_{1,i+1}^{n}-\phi_{1,i}^{n}\right)-\Sigma_{a,1,i}\phi_{1,i}^{n}\Delta x_i-\Sigma_{s,1\to 2}\phi_{1,i}^{n}\Delta x_i\right.$$
$$\left.+\left(1-\beta\right)\left(\nu\Sigma_{f,1}\phi_{1,i}^{n}+\nu\Sigma_{f,2}\phi_{2,i}^{n}\right)\Delta x_i+\sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n}\Delta x_i\right]+S_1^n\Delta x_i$$

or

$$\begin{aligned}
&-\theta\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\phi_{1,i-1}^{n+1}\\
&+\left\{\frac{\Delta x_i}{v_1\Delta t}+\theta\left[\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,1,i}\Delta x_i+\Sigma_{s,1\to 2}\Delta x_i-\left(1-\beta\right)\nu\Sigma_{f,1}\Delta x_i\right]\right\}\phi_{1,i}^{n+1}\\
&-\theta\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\phi_{1,i+1}^{n+1}\\
&=\theta\left[\left(1-\beta\right)\left(\nu\Sigma_{f,2}\phi_{2,i}^{n+1}\right)\Delta x_i+\sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n+1}\Delta x_i\right]\\
&+\frac{\Delta x_i}{v_1\Delta t}\phi_{1,i}^{n}+\left(1-\theta\right)\left[-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\left(\phi_{1,i}^{n}-\phi_{1,i-1}^{n}\right)+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\left(\phi_{1,i+1}^{n}-\phi_{1,i}^{n}\right)-\Sigma_{a,1,i}\phi_{1,i}^{n}\Delta x_i-\Sigma_{s,1\to 2}\phi_{1,i}^{n}\Delta x_i\right.\\
&\left.+\left(1-\beta\right)\left(\nu\Sigma_{f,1}\phi_{1,i}^{n}+\nu\Sigma_{f,2}\phi_{2,i}^{n}\right)\Delta x_i+\sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n}\Delta x_i\right]+S_1^n\Delta x_i
\end{aligned}\tag{15}$$

Therefore, putting:

$$\boxed{a_{1,i}=-\theta\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}}\tag{16}$$

$$\boxed{b_{1,i}=\frac{\Delta x_i}{v_1\Delta t}+\theta\left[\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,1,i}\Delta x_i+\Sigma_{s,1\to 2}\Delta x_i-\left(1-\beta\right)\nu\Sigma_{f,1}\Delta x_i\right]}\tag{17}$$

$$\boxed{c_{1,i}=-\theta\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}}\tag{18}$$

$$\boxed{\begin{aligned}
&d_{1,i}=\theta\left[\left(1-\beta\right)\left(\nu\Sigma_{f,2}\phi_{2,i}^{n+1}\right)\Delta x_i+\sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n+1}\Delta x_i\right]\\
&+\frac{\Delta x_i}{v_1\Delta t}\phi_{1,i}^{n}+\left(1-\theta\right)\left[-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\left(\phi_{1,i}^{n}-\phi_{1,i-1}^{n}\right)+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\left(\phi_{1,i+1}^{n}-\phi_{1,i}^{n}\right)-\Sigma_{a,1,i}\phi_{1,i}^{n}\Delta x_i-\Sigma_{s,1\to 2}\phi_{1,i}^{n}\Delta x_i\right.\\
&\left.+\left(1-\beta\right)\left(\nu\Sigma_{f,1}\phi_{1,i}^{n}+\nu\Sigma_{f,2}\phi_{2,i}^{n}\right)\Delta x_i+\sum_{d=1}^{N_d}\lambda_d C_{d,i}^{n}\Delta x_i\right]+S_1^n\Delta x_i
\end{aligned}}\tag{19}$$

the first energy group equation takes the form

$$\boxed{a_{1,i}\,\phi_{1,i-1}^{n+1}+b_{1,i}\,\phi_{1,i}^{n+1}+c_{1,i}\,\phi_{1,i+1}^{n+1}=d_{1,i}} \tag{20}$$

though this three-point equation hides in the known term the presence of $\phi_{2,i}^{n+1}$ and $C_{d,i}^{n+1}$, requiring a proper treatment by iterations.

Similarly, elaborating the second energy group equation, it is:

$$\begin{aligned}
&-\theta\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}\phi_{2,i-1}^{n,n+1}\\
&+\left\{\frac{\Delta x_i}{v_2\Delta t}+\theta\left[\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,2,i}\Delta x_i\right]\right\}\phi_{2,i}^{n,n+1}\\
&-\theta\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}\phi_{2,i+1}^{n,n+1}\\
&=\frac{\Delta x_i}{v_2\Delta t}\phi_{2,i}^{n}+\theta\Sigma_{s,1\to 2}\phi_{1,i}^{n+1}\Delta x_i\\
&(1-\theta)\left[-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}\left(\phi_{2,i}^{n}-\phi_{2,i-1}^{n}\right)+\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}\left(\phi_{2,i+1}^{n}-\phi_{2,i}^{n}\right)-\Sigma_{a,2,i}\phi_{2,i}^{n}\Delta x_i+\Sigma_{s,1\to 2}\phi_{1,i}^{n}\Delta x_i\right]+S_2^n\Delta x_i
\end{aligned}$$

and then, putting

$$\boxed{a_{2,i}=-\theta\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}} \tag{21}$$

$$\boxed{b_{2,i}=\frac{\Delta x_i}{v_2\Delta t}+\theta\left[\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,2,i}\Delta x_i\right]} \tag{22}$$

$$\boxed{c_{2,i}=-\theta\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}} \tag{23}$$

$$\boxed{\begin{aligned}
&d_{2,i}=\frac{\Delta x_i}{v_2\Delta t}\phi_{2,i}^{n}+\theta\Sigma_{s,1\to 2}\phi_{1,i}^{n+1}\Delta x_i\\
&+(1-\theta)\left[-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}\left(\phi_{2,i}^{n}-\phi_{2,i-1}^{n}\right)+\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}\left(\phi_{2,i+1}^{n}-\phi_{2,i}^{n}\right)-\Sigma_{a,2,i}\phi_{2,i}^{n}\Delta x_i+\Sigma_{s,1\to 2}\phi_{1,i}^{n}\Delta x_i\right]+S_2^n\Delta x_i
\end{aligned}} \tag{24}$$

the formulation is reached

$$\boxed{a_{2,i}\,\phi_{2,i-1}^{n+1}+b_{2,i}\,\phi_{2,i}^{n+1}+c_{2,i}\,\phi_{2,i+1}^{n+1}=d_{2,i}} \tag{25}$$

where the three point equation again hides the presence of $\phi_{1,i}^{n+1}$ in the known term, requiring iterations.

For the equation precursors' concentrations, the closed form solution

$$\boxed{C_d^{n+1}=C_d^n\,e^{-\lambda_d\Delta t}+\frac{\beta_d}{\lambda_d}\left(\nu\Sigma_{f,1}\phi_{1,i}^{n,n+1}+\nu\Sigma_{f,2}\phi_{2,i}^{n,n+1}\right)\left(1-e^{-\lambda_d\Delta t}\right)\qquad\left(d=1,\ldots,N_d\right)} \tag{26}$$

Is already available for integration adopted.

4. Numerical discretization of the eigenvalue problem

Adopting the same spatial discretization used in the case of the transient equation, it is

$$
\begin{aligned}
&-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\phi_{1,i-1}+\left[\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,1,i}\Delta x_i+\Sigma_{s,1\to 2,i}\Delta x_i\right]\phi_{1,i}-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\phi_{1,i+1}=\frac{1}{k}\left(\nu\Sigma_{f,1,i}\phi_{1,i}+\nu\Sigma_{f,2,i}\phi_{2,i}\right)\Delta x_i\\
&-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}\phi_{2,i-1}+\left[\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,2,i}\Delta x_i\right]\phi_{2,i}-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}\phi_{2,i+1}=\Sigma_{s,1\to 2,i}\,\phi_{1,i}\Delta x_i
\end{aligned}
\tag{27}
$$

The following external (power) iteration process can be therefore considered

$$
\boxed{
\begin{aligned}
&-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}\phi_{1,i-1}^{(n+1)}+\left[\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,1,i}\Delta x_i+\Sigma_{s,1\to 2,i}\Delta x_i\right]\phi_{1,i}^{(n+1)}-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}\phi_{1,i+1}^{(n+1)}=\frac{1}{k^{(n)}}\left(\nu\Sigma_{f,1,i}\phi_{1,i}^{(n)}+\nu\Sigma_{f,2,i}\phi_{2,i}^{(n)}\right)\Delta x_i\\
&-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}\phi_{2,i-1}^{(n+1)}+\left[\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,2,i}\Delta x_i\right]\phi_{2,i}^{(n+1)}-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}\phi_{2,i+1}^{(n+1)}=\Sigma_{s,1\to 2,i}\,\phi_{1,i}^{(n+1)}\Delta x_i
\end{aligned}}
\tag{28}
$$

The generational formulation for advancing the eigenvalue is taken as:

$$
\boxed{k^{(n+1)}=k^{(n)}\,\frac{\sum_{i=1}^{N}\left(\nu\Sigma_{f,1,i}\phi_{1,i}^{(n+1)}+\nu\Sigma_{f,2,i}\phi_{2,i}^{(n+1)}\right)\Delta x_i}{\sum_{i=1}^{N}\left(\nu\Sigma_{f,1,i}\phi_{1,i}^{(n)}+\nu\Sigma_{f,2,i}\phi_{2,i}^{(n)}\right)\Delta x_i}}
\tag{29}
$$

Similar positions as in the transient case can be adopted for solving the two-group equations by the classical internal-external iteration scheme.

$$
\boxed{
\begin{aligned}
&\underbrace{-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}}_{a_{1,i}}\phi_{1,i-1}^{(n+1)}+\underbrace{\left[\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,1,i}\Delta x_i+\Sigma_{s,1\to 2,i}\Delta x_i\right]}_{b_{1,i}}\phi_{1,i}^{(n+1)}\underbrace{-\overline{\left(\frac{D_1}{\Delta x}\right)}_{i,i+1}}_{c_{1,i}}\phi_{1,i+1}^{(n+1)}=\underbrace{\frac{1}{k^{(n)}}\left(\nu\Sigma_{f,1,i}\phi_{1,i}^{(n)}+\nu\Sigma_{f,2,i}\phi_{2,i}^{(n)}\right)\Delta x_i}_{d_{1,i}}\\
&\underbrace{-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}}_{a_{2,i}}\phi_{2,i-1}^{(n+1)}+\underbrace{\left[\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i-1}+\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}+\Sigma_{a,2,i}\Delta x_i\right]}_{b_{2,i}}\phi_{2,i}^{(n+1)}\underbrace{-\overline{\left(\frac{D_2}{\Delta x}\right)}_{i,i+1}}_{c_{2,i}}\phi_{2,i+1}^{(n+1)}=\underbrace{\Sigma_{s,1\to 2,i}\,\phi_{1,i}^{(n+1)}\Delta x_i}_{d_{2,i}}
\end{aligned}}
\tag{30}
$$