Ingegneria delle Telecomunicazioni

Satellite Communications

21. TOMTOM and beyond - GNSS Receiver Design

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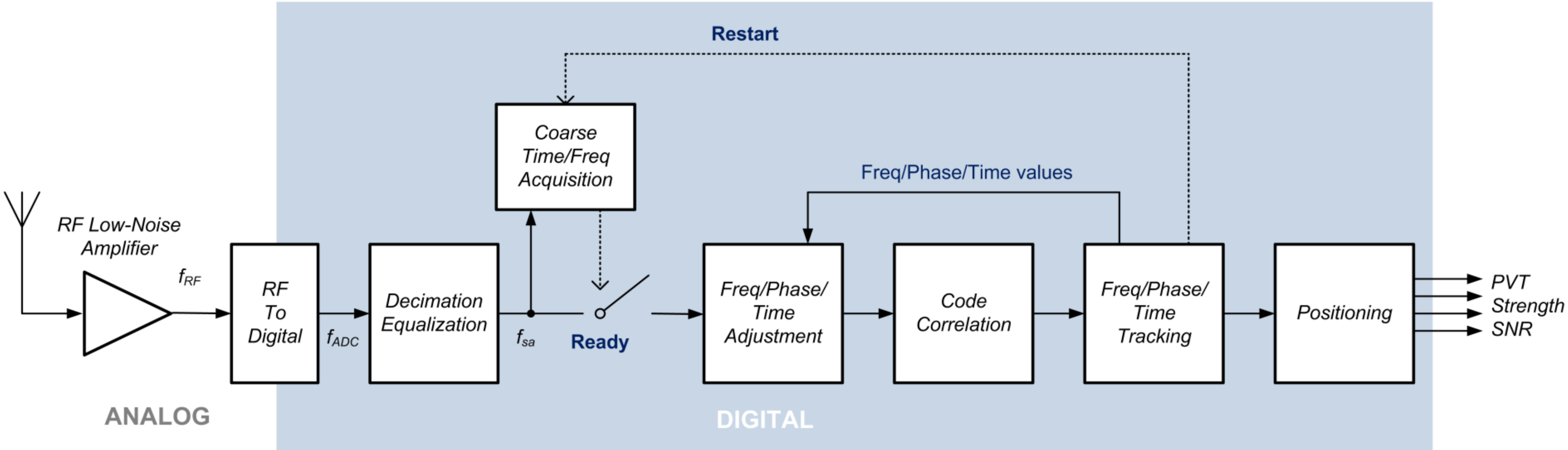
His(Her) Majesty TomTom GO, 2004

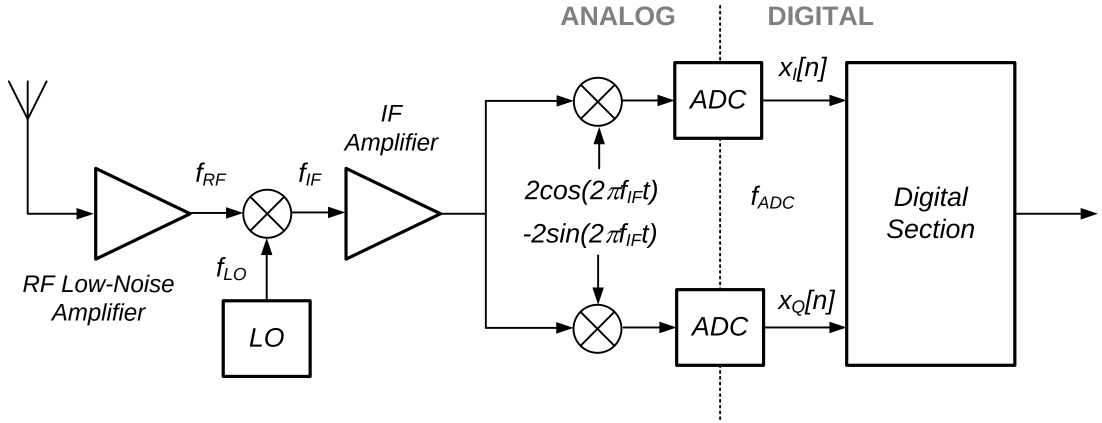
- GPS receiver
- Digital Maps
- Navigation SW
- User-Friendly GUI
- Voice Directions
- Touch-screen Display

Basic Receiver Architecture

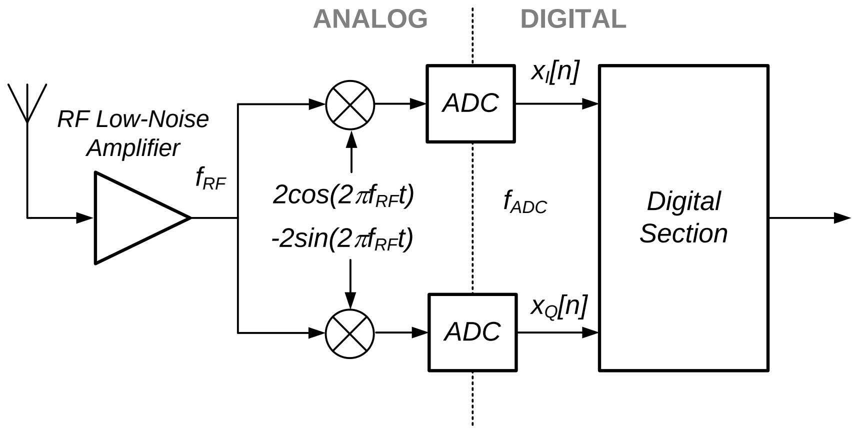
- Modern receivers perform all of the signal processing functions with DSP components and algorithms



Form RF to Digital - I

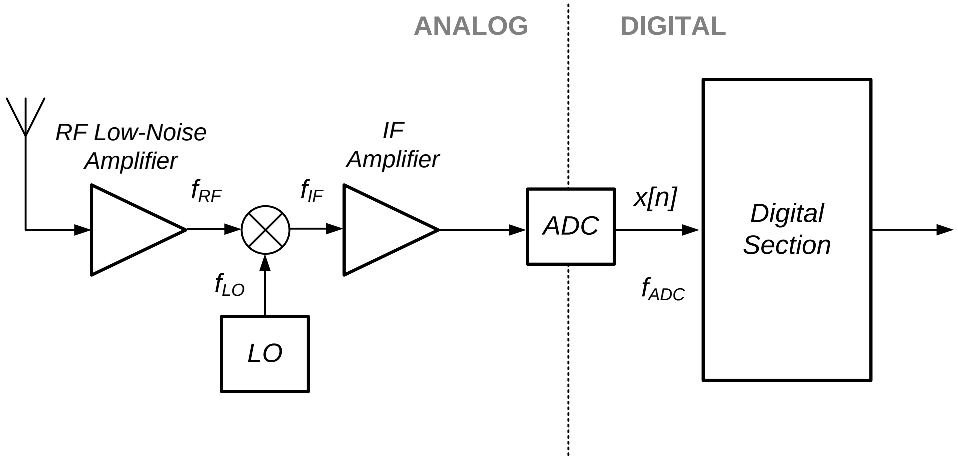


- **I/Q BaseBand Sampling with IF Conversion**

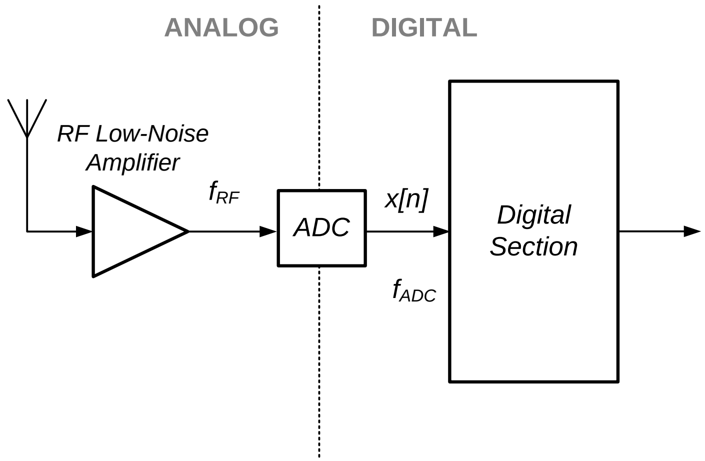


- **I/Q BaseBand Sampling with Direct Baseband Conversion (aka ZERO-IF Architecure)**

Form RF to Digital - II



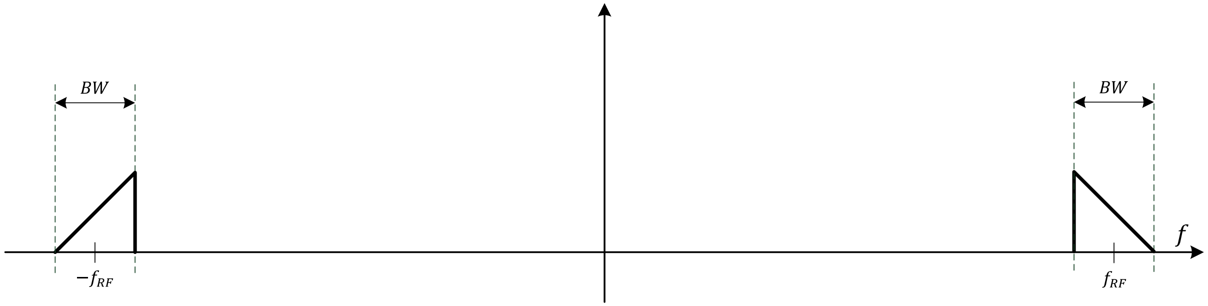
- **IF BandPass Sampling with Digital IF**



- **Direct RF Sampling with Digital IF**

Band-Pass Sampling

- *Pass-band, real signal*
  - *RF (passband) bandwidth BW*
  - *carrier frequency f_{RF}*



HERMITIAN SYMMETRIC

$$X(-f) = X(f)^*$$

- *Nyquist-sampling condition:*

$$f_s \geq 2B = 2\left(f_{RF} + \frac{BW}{2}\right)$$

- *For navigation signals in L-band, this is unrealistic!*

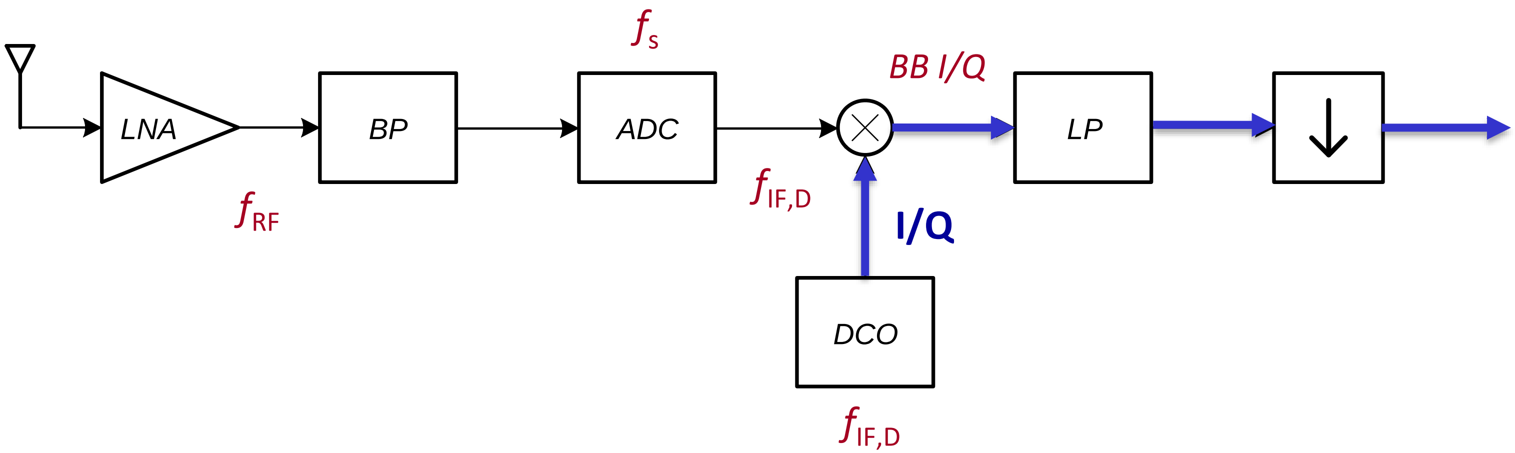
$$f_{RF} \in [1\text{ GHz}, 2\text{ GHz}] \quad \Rightarrow \quad f_s > 2 \div 4\text{GHz}$$

- *Solution: **band-pass sampling** at reduced rate :*

$$f_s \approx 2BW$$

Direct Sampling Front-End Architecture

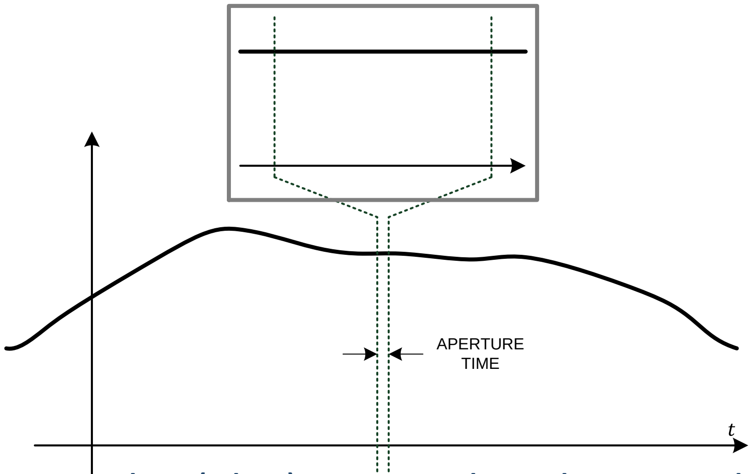
- Front-end architecture of direct-sampling receiver



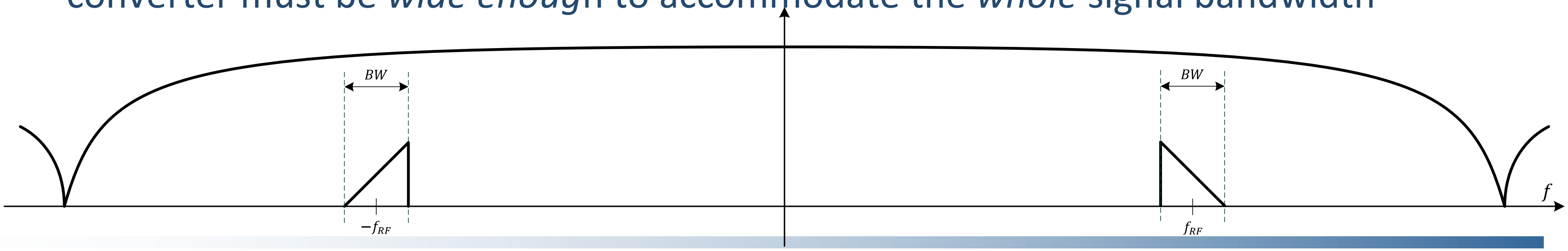
– RX signal from antenna is sampled **directly after LNA** and RF BP filter
  - no RF-to-IF downconversion stage / all-digital processing
– **Digital** IF $f_{IF,D}$ resulting from the interplay between f_{RF} and f_s (later on)
– **Digital** I/Q oscillator to convert the received signal at $f_{IF,D}$ to I/Q base-band
– low-pass filter to isolate the channel of interest
  - filter bandwidth is BW
– base-band decimation
  - base-band signal may be oversampled for many applications

Direct Sampling Requirements

- Any A/D converter is characterized by the *aperture time*, i.e., the time over which the signal must be stable (constant) to compute the digital output
  - it must be short enough to have correct signal sampling (constant level)



- In the frequency domain, this (also) means that the input bandwidth of the A/D converter must be *wide enough* to accommodate the *whole* signal bandwidth



Examples

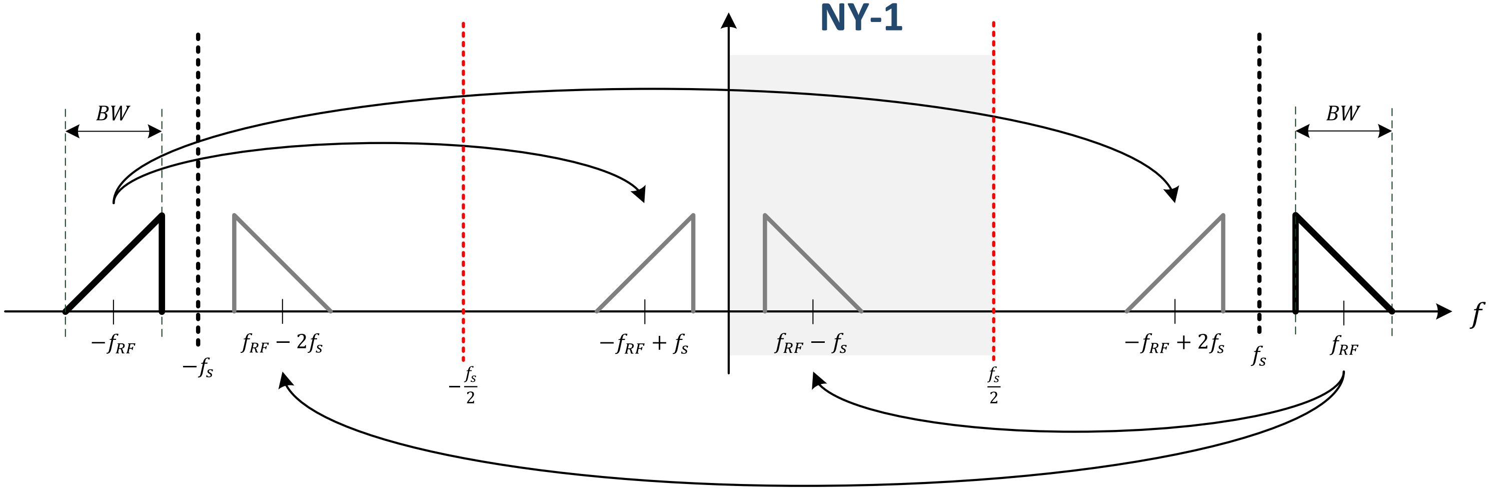
- f_{RF} = 5 MHz, BW=1 MHz
  - Conventional, Nyquist-rule f_s=2*(5+0.5)=11 MHz

- *Try* f_{RF} = 4 MHz
  - … and look at the digital bandwidth –2 MHz < f < 2 MHz
  - Find $f_{IF,D}$

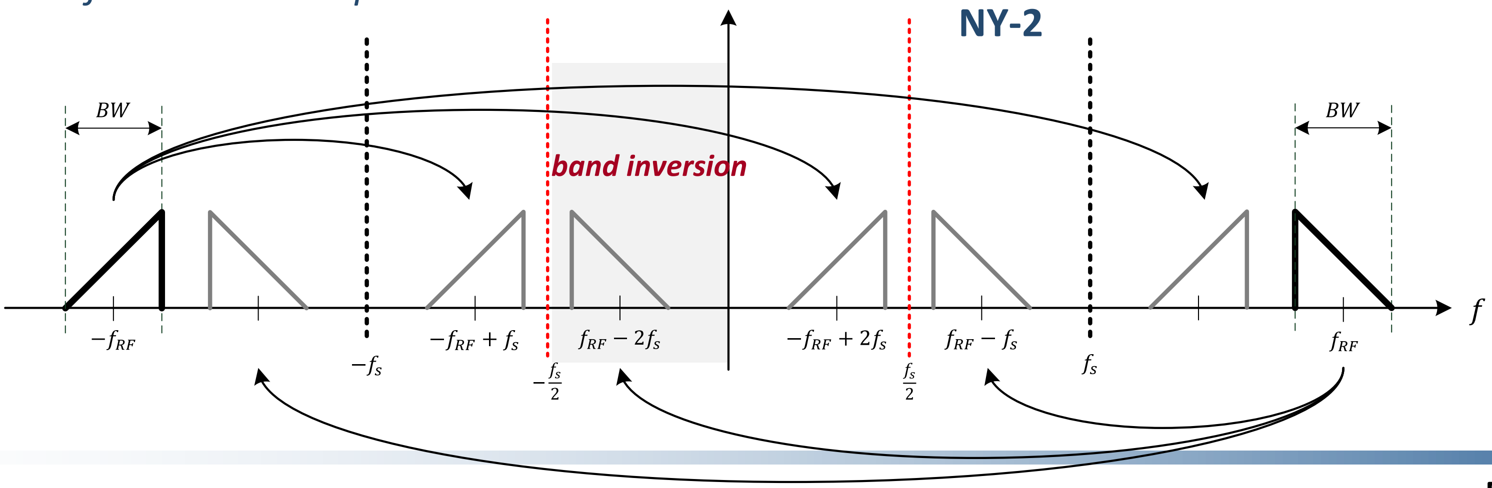
- *Then try* f_{RF} = 7 MHz
  - … and look at the digital bandwidth –3.5 MHz < f < 3.5 MHz
  - Find $f_{IF,D}$
  - Can you guess what happened ?

Pass-Band Sampling: Band Inversion

- *case I: fundamental replica in NY-1 zone:*

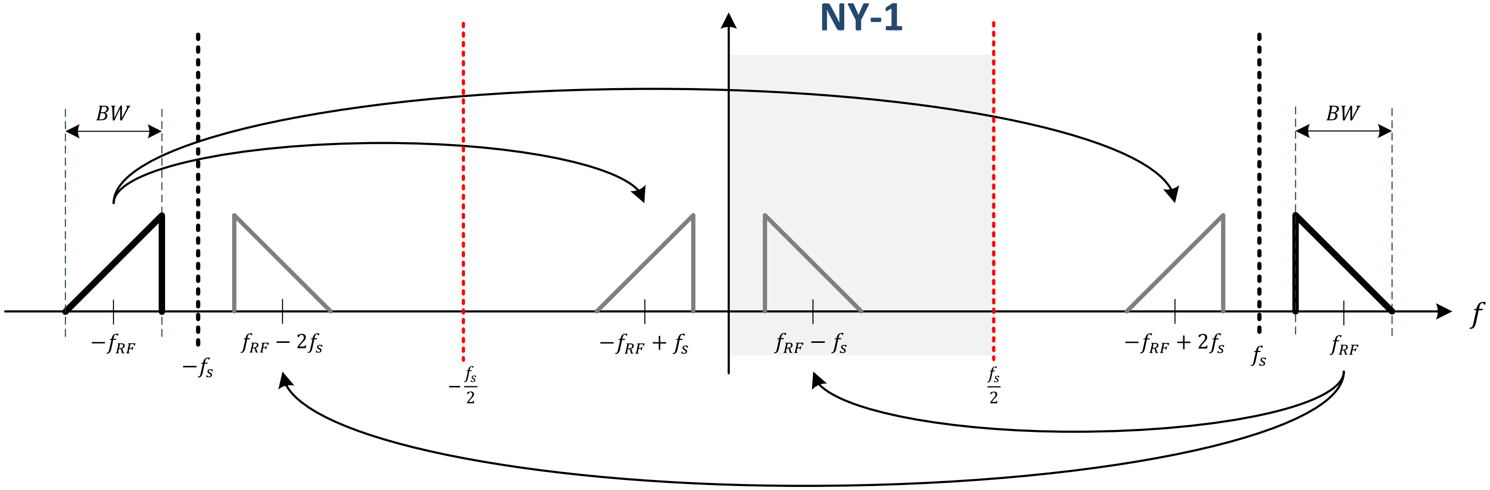


- *case II: fundamental replica in NY-2 zone:*



Pass-Band Sampling 1

- *case I: fundamental replica in NY-1 zone:*

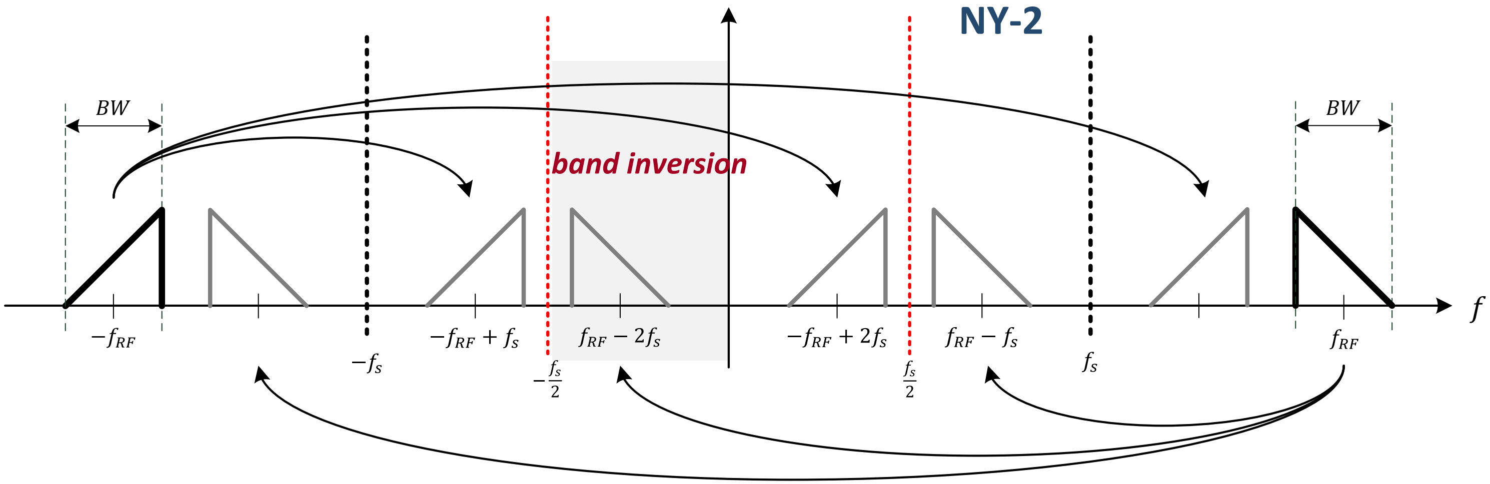


- The resulting Digital IF is derived from the relation $f_{IF,D}$=f_{RF}-mf_s where *m* is integer and is such that $f_{IF,D}$ lies in the *POSITIVE HALF* of the digital bandwidth, $0 \le f < f_s/2$

$$f_{IF,D} = f_{RF} - mf_s \quad \Rightarrow \quad f_{IF,D} = \left| f_{RF} \right|_{f_s}$$

Pass-Band Sampling 2: Band Inversion

- *case II: fundamental replica in NY-2 zone:*



The resulting Digital IF is again derived from the relation $f_{IF,D}=f_{RF}-mf_s$ where m is integer, but now it is in the *NEGATIVE HALF* of the digital bandwidth $-f_s/2 \le f < 0$

$$f_{IF,D} = f_{RF} - mf_s \quad \Rightarrow \quad f_{IF,D} = \left| f_{RF} \right|_{f_s} - f_s$$

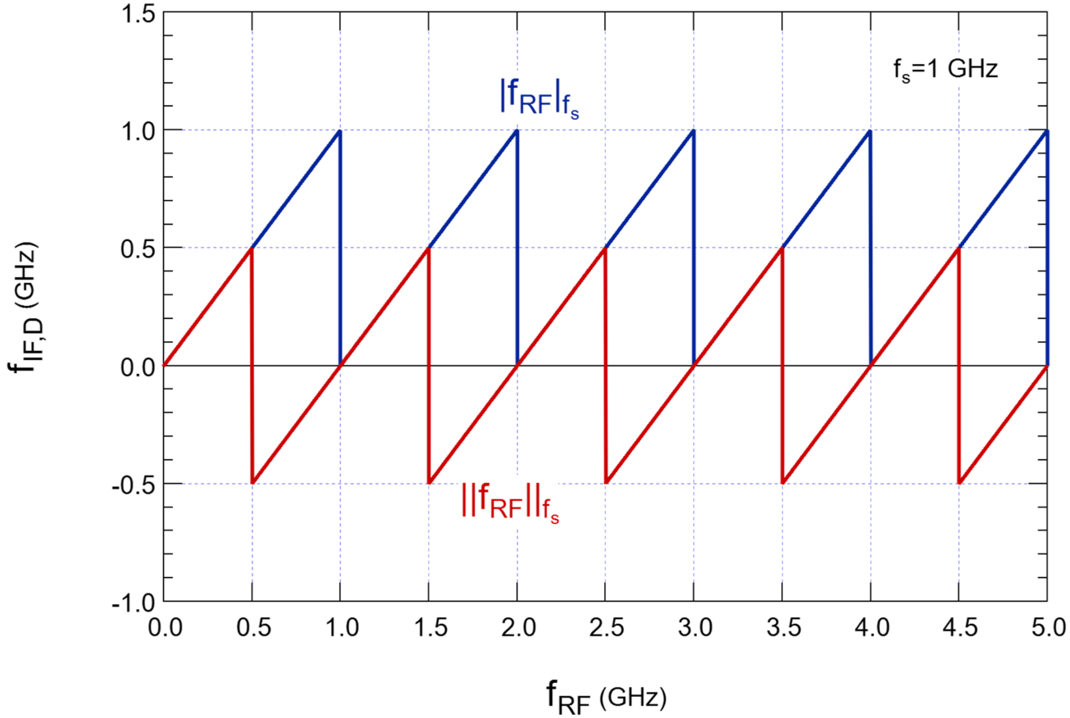
where the $-f_s$ term has the purpose of re-mapping the "modulus- f_s" function to $-f_s/2$, $f_s/2$ instead of $0,f_s$ as in the mathematical definition (leading to a digital IF out of the digital bandwidth)

Pass-Band Sampling: Computing $f_{IF,D}$

We define a "modified", symmetrical modulus function

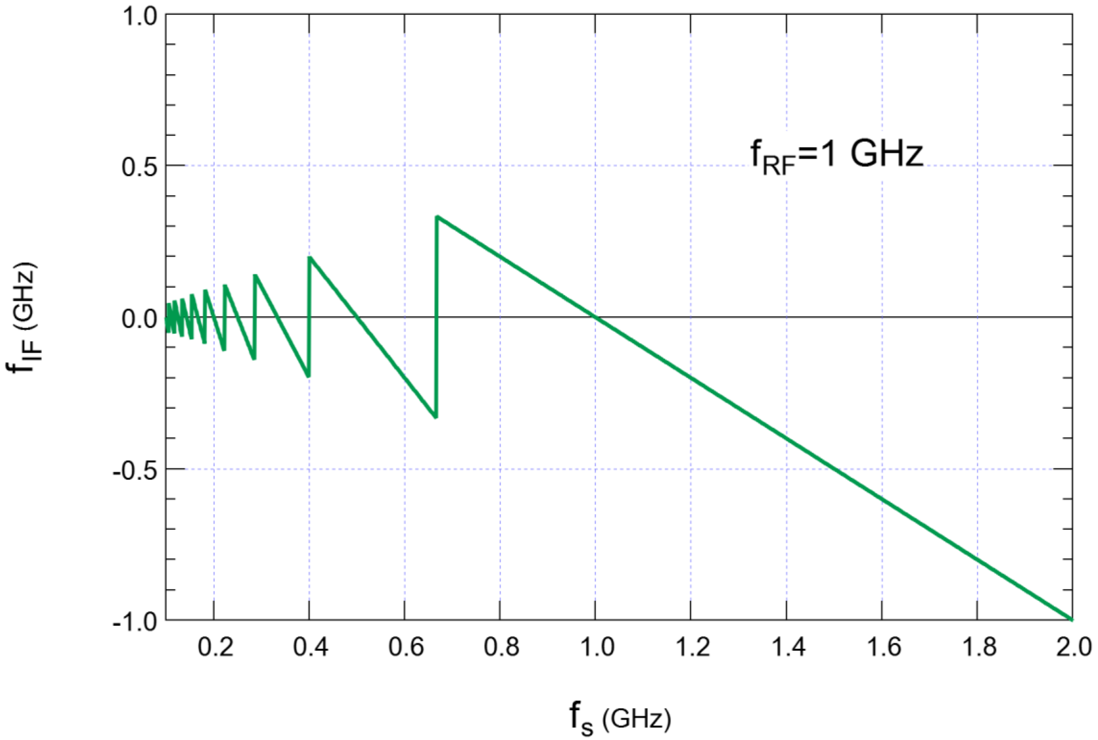
$$f_{IF,D} = \left\| f_{RF} \right\|_{f_s} \triangleq \left| f_{RF} + \frac{f_s}{2} \right|_{f_s} - \frac{f_s}{2} = \begin{cases} \left| f_{RF} \right|_{f_s} & , \quad \left| f_{RF} \right|_{f_s} \le f_s / 2 \\ \left| f_{RF} \right|_{f_s} - f_s & , \quad \left| f_{RF} \right|_{f_s} > f_s / 2 \end{cases}$$

where the $-f_s/2$ term has the purpose of re-mapping the "modulus- f_s" function to $[-f_s/2, f_s/2)$ instead of $[0, f_s)$ as in the mathematical definition. For short, we will call $f_{IF,D}$ just f_{IF}



The "Modified Modulus" as a function of f_s

$$f_{IF} = \| f_{RF} \|_{f_s}$$



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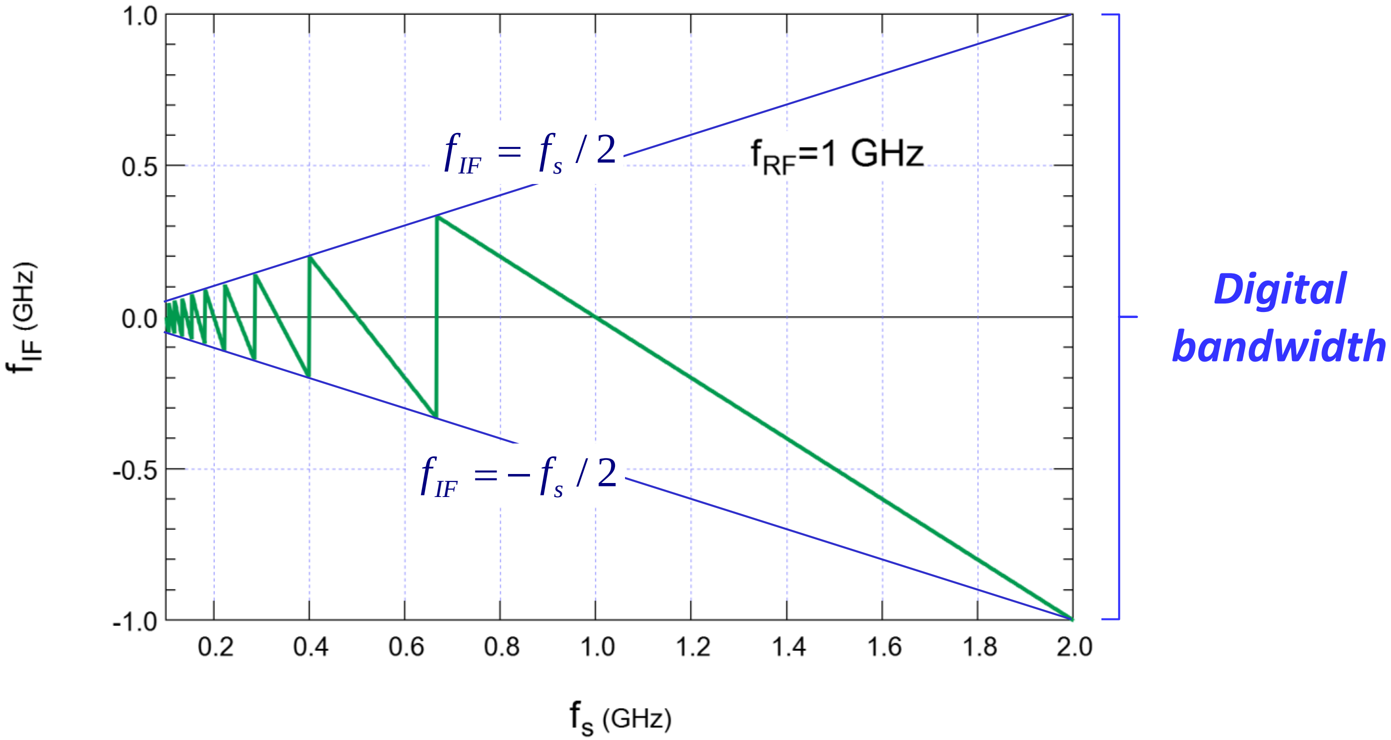

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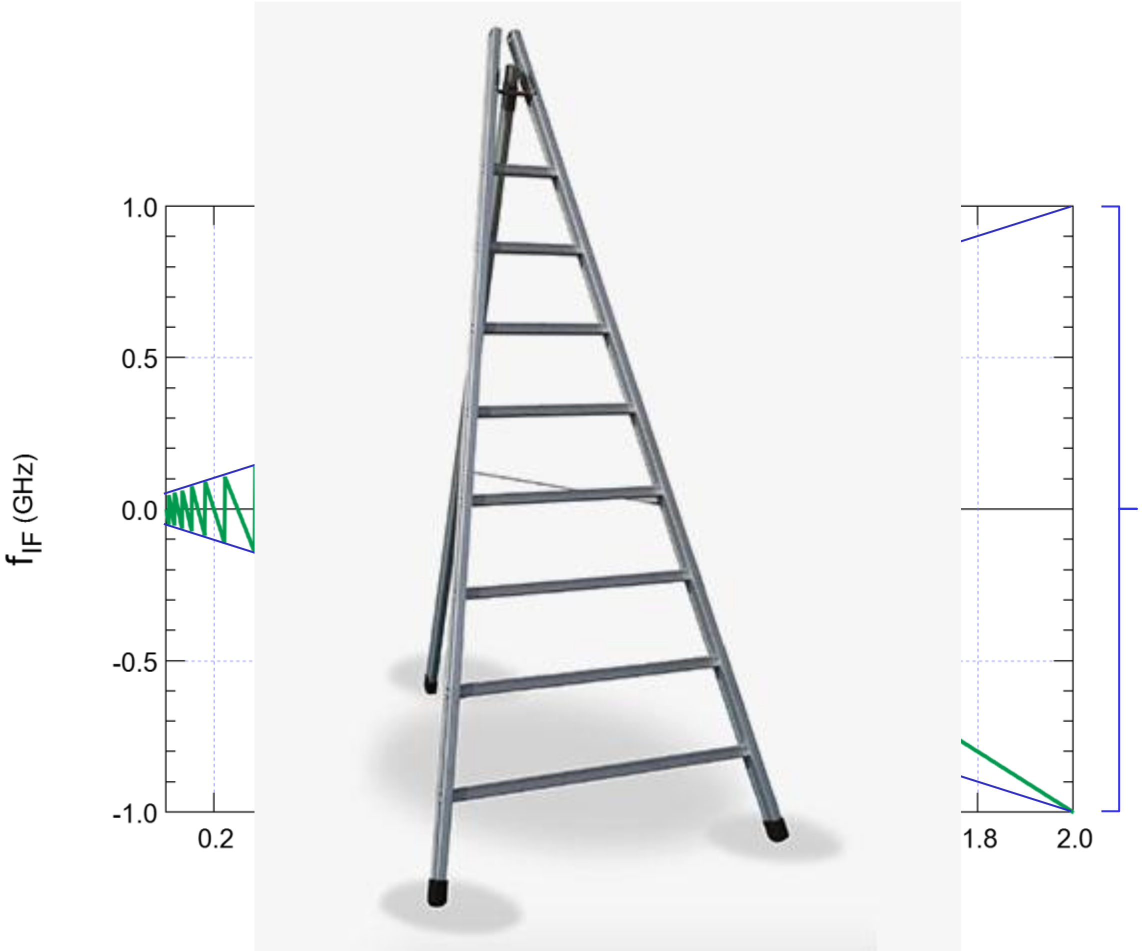


Digital bandwidth

The Ladder Diagram I

- *Digital IF frequency:*

$$f_{IF} = \| f_{RF} \|_{f_s} = \left| f_{RF} + \frac{f_s}{2} \right|_{f_s} - \frac{f_s}{2}$$ *(Symmetric Modulus Function)*

- *Case-I (NY-1):*

$$0 \leq f_{IF} < \frac{f_s}{2}$$

  - *Just frequency conversion*

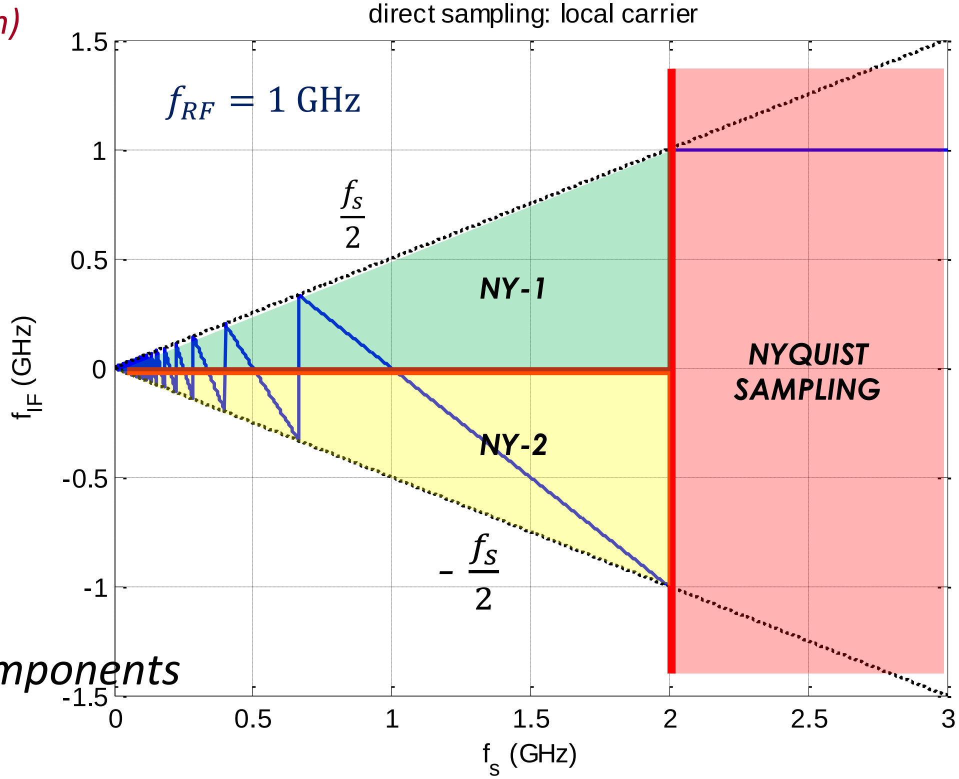
- *Case II (NY-2):*

$$-\frac{f_s}{2} \leq f_{IF} < 0$$

  - *Frequency conversion AND* ***band inversion****, i.e., f_{IF} can be considered positive but the Q components is sign-changed*

case study:

$$f_{RF} = 1 \text{ GHz}$$



- ***OK with f_{IF}, but what is the relation of f_s with the (bandpass) signal BW ?***

Updated Nyquist Rule

- Selection of sampling rate
  - ***no aliasing within the signal bandwidth***

$$\begin{cases} f_{IF} + \dfrac{BW}{2} \le \dfrac{f_s}{2} \\ f_{IF} - \dfrac{BW}{2} \ge 0 \end{cases} \qquad \begin{cases} f_{IF} + \dfrac{BW}{2} \le 0 \\ f_{IF} - \dfrac{BW}{2} \ge -\dfrac{f_s}{2} \end{cases} \Rightarrow \begin{cases} |f_{IF}| \ge \dfrac{BW}{2} \\ |f_{IF}| \le \dfrac{f_s}{2} - \dfrac{BW}{2} \end{cases}$$

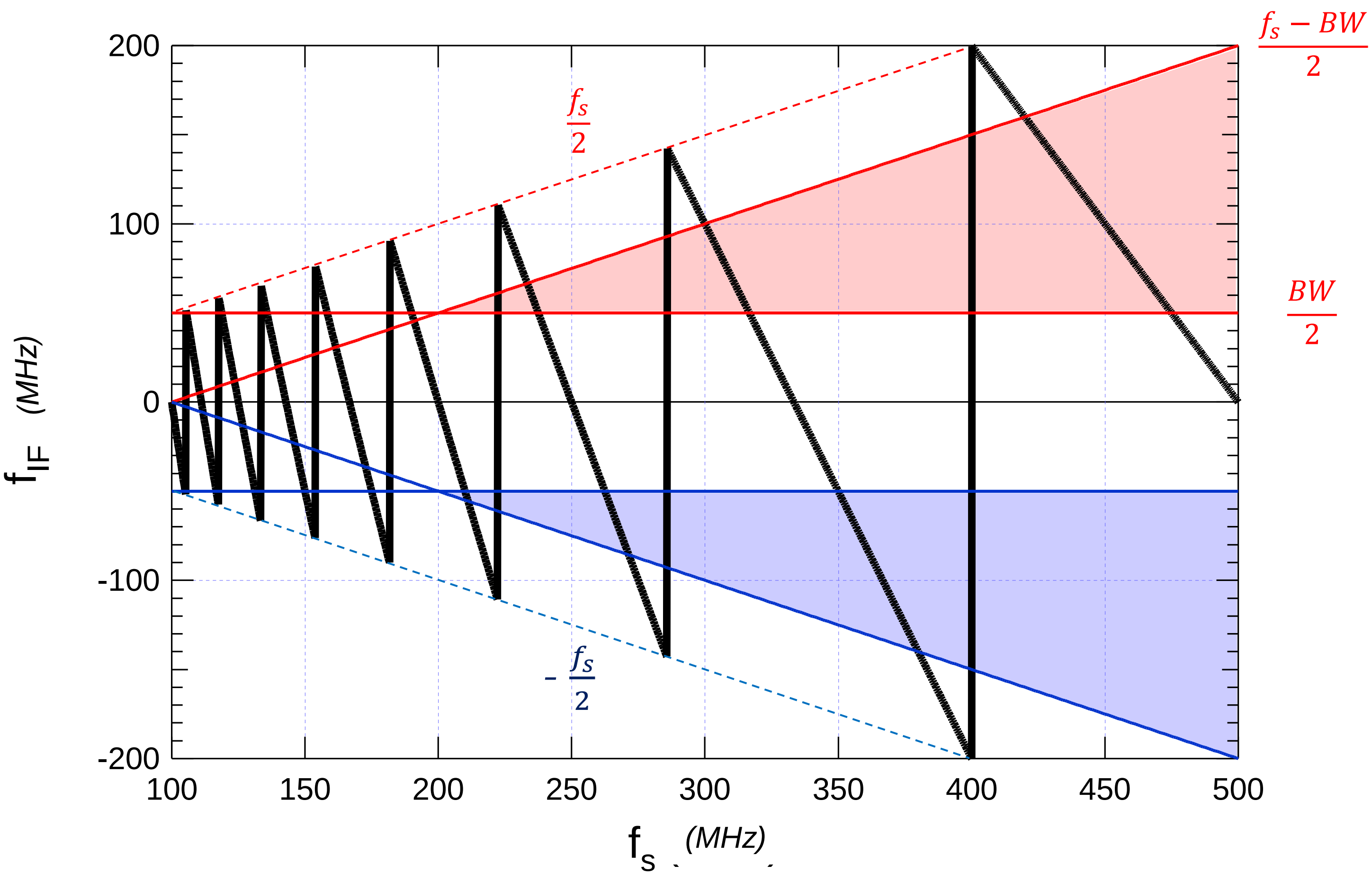
f_{IF} >0 ***f_{IF} <0***

$$\frac{BW}{2} \le |f_{IF}| \le \frac{f_s}{2} - \frac{BW}{2}$$

- We can visualize the inequality on the Ladder diagram to actually find the allowed values of the digital carrier f_{IF} AND of the sampling frequency f_s

The Ladder Diagram II

- *Case study: f_{RF}=1 GHz and BW=100 MHz*

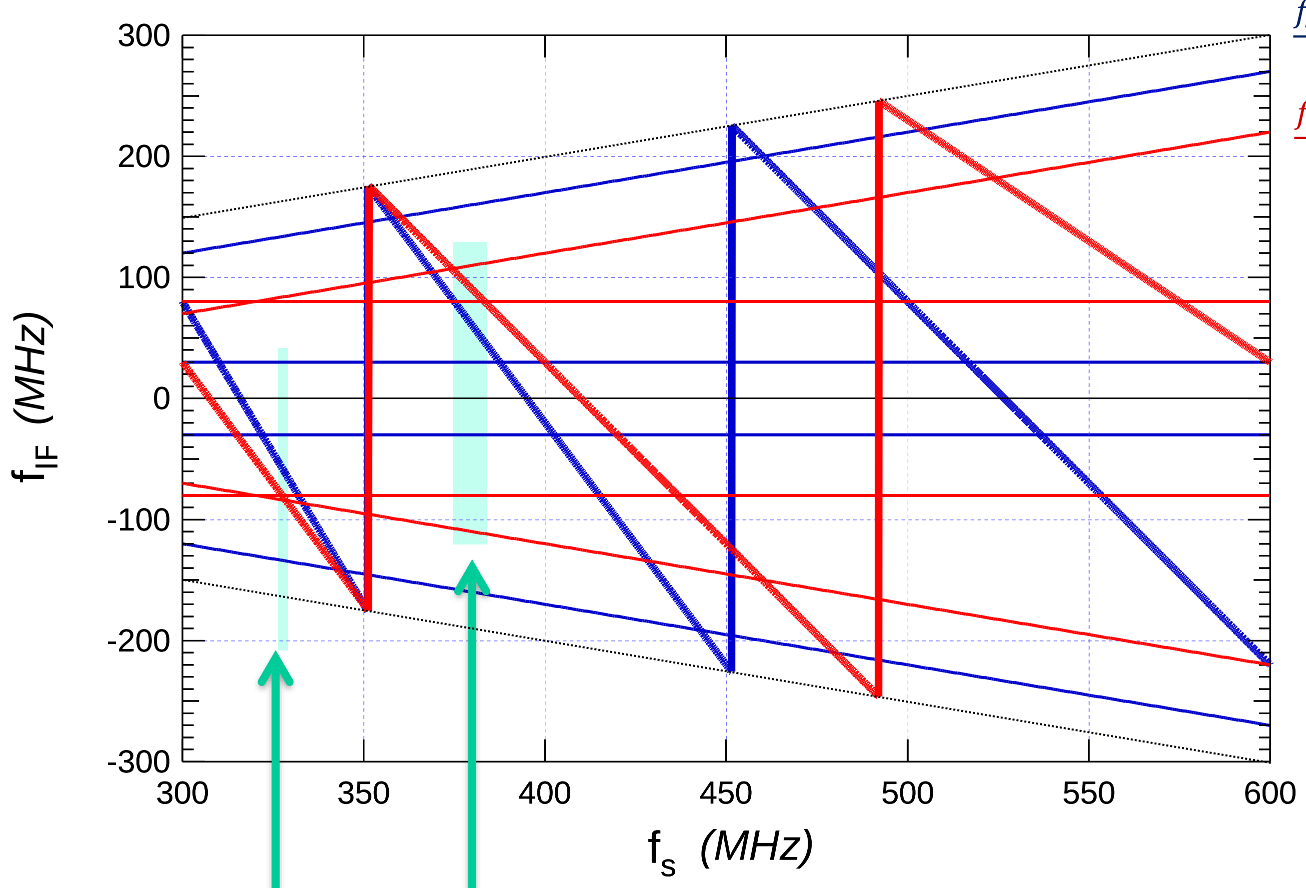


Sample Design: GNSS Direct-Sampling Front-End

N.	RIMS V3 Req	Satellite System	Band	Lower/Upper	Signal	Carrier (MHz)	Carrier_BB (MHz)	ID_Comb_Carrier-BW	ID_BW_RF	BW @ RF (MHz)
1	Baseline	GALILEO	E5a	Lower	E5aI-E5aQ	1176,45	-135,55	1	1	20,46
2	Baseline	GPS	L5	Lower	I5-Q5	1176,45	-135,55	1	1	20,46
3	Baseline	GEO	L5	Lower	L5I-L5Q	1176,45	-135,55	1	1	20,46
4	Expandability	HEO	L5	Lower	L5I-L5Q	1176,45	-135,55	1	1	20,46
5	Expandability	GALILEO	E5b	Lower	E5bI-E5bQ	1207,14	-104,86	2	1	20,46
6	Optional	BEIDOU	E5b	Lower	B2I	1207,14	-104,86	2	1	20,46
7	Optional	GALILEO	E5	Lower	E5	1191,795	-120,205	3	2	40,92
8	Baseline	GPS	L2	Lower	L2CM/L2CL	1227,6	-84,4	4	1	20,46
9	Baseline	GPS	L2	Lower	L2P	1227,6	-84,4	4	1	20,46
10	Optional	GLONASS	L2	Lower	L2OF	1246	-66	5	3	7
11	Expandability	GALILEO	E6	Lower	E6B-E6C	1278,75	-33,25	6	4	10,23
12	Baseline	GALILEO	E1	Upper	E1B-E1C	1575,42	-64,58	7	5	24,552
13	Baseline	GPS	L1	Upper	L1CA	1575,42	-64,58	8	6	30,69
14	Expandability	GPS	L1	Upper	L1C-D/P	1575,42	-64,58	8	6	30,69
15	Baseline	GEO	L1	Upper	L1I-L1Q	1575,42	-64,58	8	6	30,69
16	Expandability	HEO	L1	Upper	L1I-L1Q	1575,42	-64,58	8	6	30,69
17	Optional	GLONASS	L1	Upper	L1OF	1602	-38	9	7	9
18	Optional	BEIDOU	L1	Upper	B1I	1561,098	-78,902	10	1	20,46

- *Lower-band:*
  - *SIS from 1 to 11* $(1150 \text{ to } 1310 \text{ MHz})$
  - $BW_L = 160 \text{ MHz}\,;\, f_{RF,L} = 1230 \text{ MHz}$
- *Upper-band*
  - *SIS from 12 to 18* $(1550 \text{ to } 1610 \text{ MHz})$
  - $BW_U = 60 \text{ MHz}\,;\, f_{RF,U} = 1580 \text{ MHz}$

GNSS Direct-Sampling Front-End: Ladder(s)



$\frac{f_s - 60}{2}$

$\frac{f_s - 160}{2}$

BOTH ladders (lower and upper bands) has to work at the same time !

$375\ \mathrm{MHz} \le f_s < 383\ \mathrm{MHz}$

$327.5\ \mathrm{MHz} \le f_s < 328.5\ \mathrm{MHz}$

GNSS Direct-Sampling Front-End: Final Choice

- *Selection of sampling rate: Ladder diagram*
  - *requirement: no aliasing of the signal bandwidth*

NY-1 $\begin{cases} f_{IF} + \dfrac{BW}{2} \le \dfrac{f_s}{2} \\ f_{IF} - \dfrac{BW}{2} \ge 0 \end{cases}$ **NY-2** $\begin{cases} f_{IF} + \dfrac{BW}{2} \le 0 \\ f_{IF} - \dfrac{BW}{2} \ge -\dfrac{f_s}{2} \end{cases}$ $\Rightarrow$ $\begin{cases} |f_{IF}| \ge \dfrac{BW}{2} \\ |f_{IF}| \le \dfrac{f_s}{2} - \dfrac{BW}{2} \end{cases}$

$\Downarrow$

$$\frac{BW}{2} \le |f_{IF}| \le \frac{f_s}{2} - \frac{BW}{2}$$

Lowest-frequency solution (NY-2):

$$327.5 \text{ MHz} \le f_s < 328.5 \text{ MHz}$$

$\Downarrow$

$$\boxed{f_s = 328\text{MHz}}$$

next solution (NY-1):

$$375 \text{ MHz} \le f_s < 383 \text{ MHz}$$

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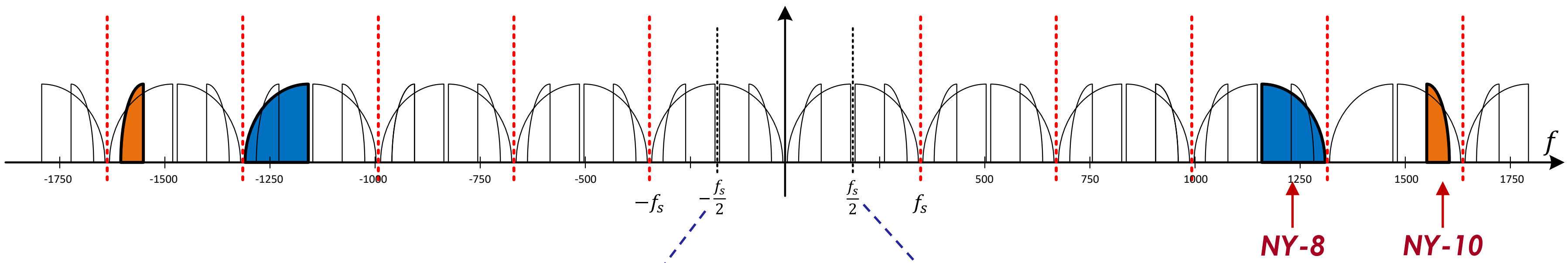
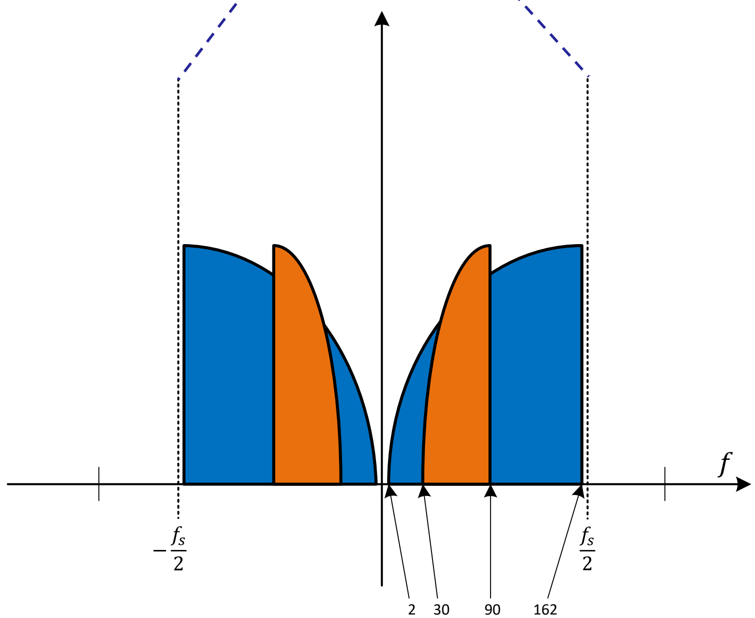
GNSS Direct-Sampling Front-end: Spectra

- *Sampling frequency* $f_s = 328$ MHz



- *Upper-band:*
  - *[-90; -30] MHz*
  - $f_{IF,U} = -60\text{MHz}$
- *Lower-band:*
  - *[-162; -2] MHz*
  - $f_{IF,L} = -82\text{MHz}$



Basic Modeling of the I/Q received signal

- N_{sat} : number of satellites in visibility *(elevation larger than ≈10 degrees)*
- i : satellite identifier
- C_i : received signal power
- τ_i : time of flight in the user time scale
- Δf_i : Doppler shift
- θ_i : carrier phase

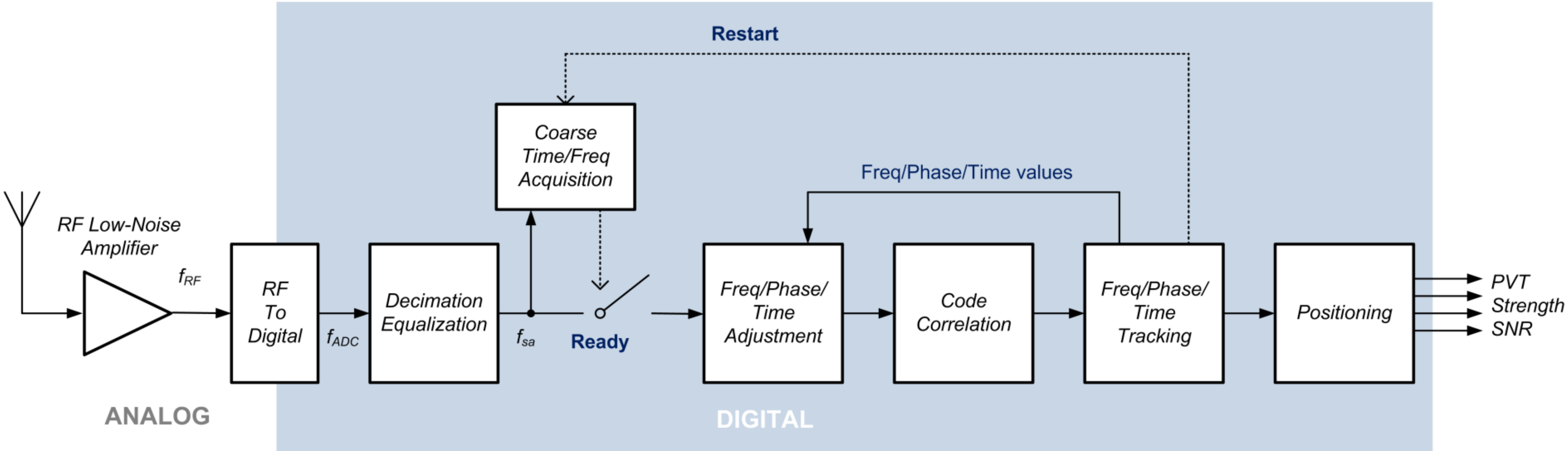
$$r(t) = \sum_{i=1}^{N_{sat}} \sqrt{2C_i}\, s_i(t-\tau_i) \exp\left[j\left(2\pi \Delta f_i t + \theta_i\right)\right] + w(t)$$

Whatever the architecture, at the output of the **RF to Digital** Section:

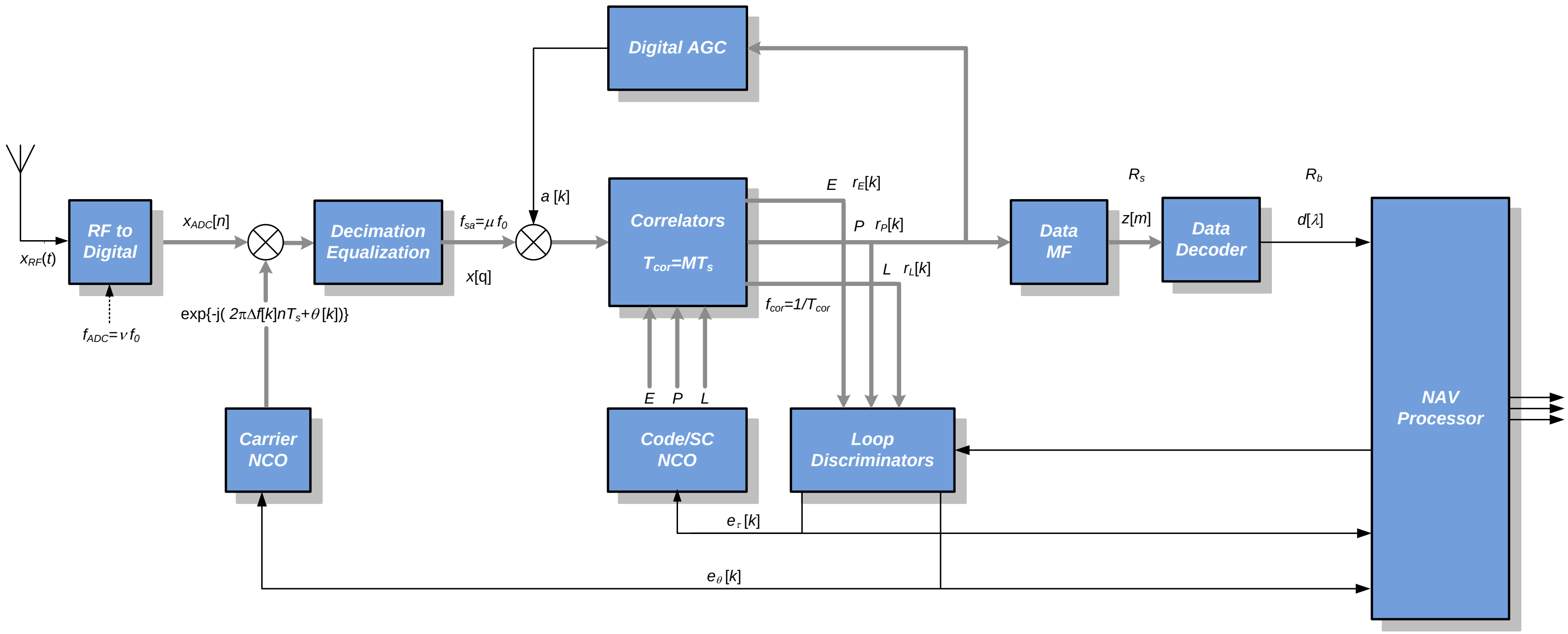
$$x_{ADC}[n] = \sum_{i=1}^{N_{sat}} \sqrt{2C_i}\, s_i(nT_s-\tau_i) \exp\left[j\left(2\pi \Delta f_i nT_s + \theta_i\right)\right] + w[nT_s]$$

Basic Receiver Architecture

- Modern receivers perform all of the signal processing functions with DSP components and algorithms



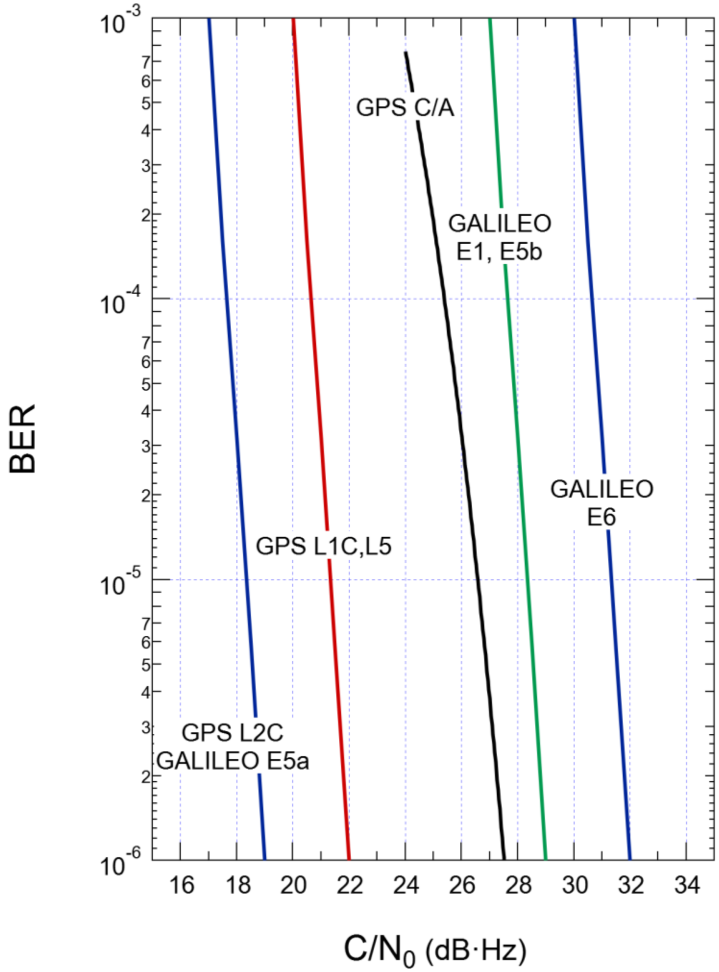
Detailed Receiver Architecture



E=Early, L=Late, correlations for the DLL
P=Prompt for Data Detection (may also be used for the DLL)

NAV Message Data Reception

- **Data detection performance depend on the bit-rate and on the kind of channel coding that is used on the different carriers/channels/systems**



A Case Study: Code Delay Tracking

- N_{sat}=1
- *i=1* satellite identifier
- C_1=1/2 received signal power
- $\tau_1=\tau$ time of flight in the user time scale
- Δf_i=0 Doppler shift
- θ_i=0 carrier phase
- Pilot signal only (no Q component needed)

$$r(t) = \sqrt{2C}\, c(t-\tau) + w(t)$$

Let us concentrate on delay tracking…

To maximize $\int_0^{T_0} r(t)c(t-\tilde{\tau})dt$ ***we do:***

$$\frac{d}{d\tilde{\tau}}\int_0^{T_0} r(t)c(t-\tilde{\tau})dt = 0 \Rightarrow \int_0^{T_0} r(t)\frac{d}{d\tilde{\tau}}c(t-\tilde{\tau})dt = 0$$

$$\frac{d}{d\tilde{\tau}}c(t-\tilde{\tau}) \cong -\frac{c(t-\tilde{\tau}+\Delta)-c(t-\tilde{\tau}-\Delta)}{2\Delta} \Rightarrow \int_0^{T_0} r(t)\frac{c(t-\tilde{\tau}+\Delta)-c(t-\tilde{\tau}-\Delta)}{2\Delta}dt = 0$$

$$\int_0^{T_0} r(t)c(t-\tilde{\tau}+\Delta)dt - \int_0^{T_0} r(t)c(t-\tilde{\tau}-\Delta)dt = 0$$

T_0 is the correlation (observation) time

Error Function

$$e(\tilde{\tau}) = \int_0^{T_0} r(t)c(t-\tilde{\tau}+\Delta)dt - \int_0^{T_0} r(t)c(t-\tilde{\tau}-\Delta)dt$$

Early correlation $\tilde{\tau}$ ***Late correlation***

Can be computed Real-Time on the k-th ***correlation interval*** $[(k-1)T_0, kT_0)$

$$e(\tilde{\tau}[k-1]) = \int_{(k-1)T_0}^{kT_0} r(t)c(t-\tilde{\tau}[k-1]+\Delta)dt - \int_{(k-1)T_0}^{kT_0} r(t)c(t-\tilde{\tau}[k-1]-\Delta)dt$$

the variable k *runs at the* ***correlation time*** T_0

Recursive delay estimation

- We know that the current estimate is correct (optimum) *iff* the feedback signal $e(\hat{\tau}[k-1])$ is zero, so that the estimate is maximum-likelihood.
- BUT, the feedback signal is in general NOT 0 - we can use this feedback signal (aka *discriminator output*) to recursively update the current estimate and find a better one:

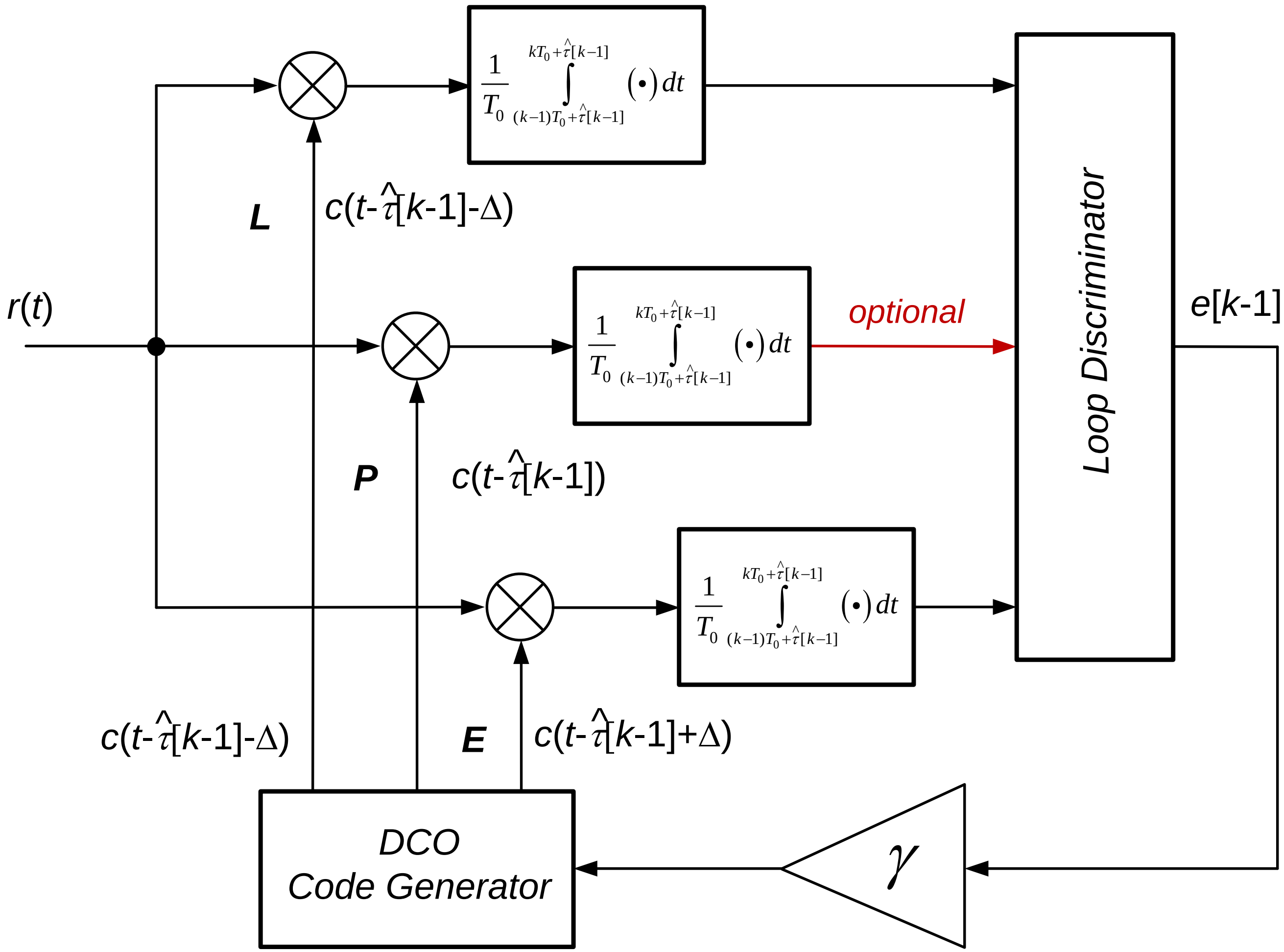
$$\hat{\tau}[k] = \hat{\tau}[k-1] - \gamma e[k-1]$$

where, for short,

$$e[k-1] \triangleq e(\hat{\tau}[k-1])$$

- This is the equation of a *negative feedback control loop* that finds an equilibrium at steady state whenever *e*[*k*-1]=0 $\rightarrow$ finds the (optimum) ML delay estimate !
- The constant γ is a *stepsize* (the *gain* of the feedback loop) that dictates the magnitude of the feedback correction term

The Delay-Lock "Loop" (DLL)



Early/Late Correlation Values

Received signal $r(t) = \sqrt{2C} \cdot c(t-\tau) + w(t)$

$$e^{\pm}(\hat{\tau}[k-1]) = \int_{(k-1)T_0}^{kT_0} r(t)c(t-\hat{\tau}[k-1] \pm \Delta)dt$$

Early/Late Correlations

$$= \sqrt{2C} \int_{(k-1)T_0}^{kT_0} c(t-\tau)c(t-\hat{\tau}[k-1] \pm \Delta)dt + \int_{(k-1)T_0}^{kT_0} w(t)c(t-\hat{\tau}[k-1] \pm \Delta)dt$$

$$= \sqrt{2C}\, R_c\left(\hat{\tau}[k-1] - \tau \mp \Delta\right) + W^{\pm}[k-1]$$

Code Autocorrelation Function

Gaussian RVs

The S-curve

- Let us disregard noise for the moment, and let us define the estimation *error*

$$\varepsilon[k] \triangleq \hat{\tau}[k] - \tau$$

- The feedback signal (discriminator output) is therefore

$$e[k-1] = e^{+}(\hat{\tau}[k-1]) - e^{-}(\hat{\tau}[k-1])$$
$$= \sqrt{2C} \cdot \left[R_c\left(\varepsilon[k-1] - \Delta\right) - R_c\left(\varepsilon[k-1] + \Delta\right) \right] = e(\varepsilon[k-1])$$

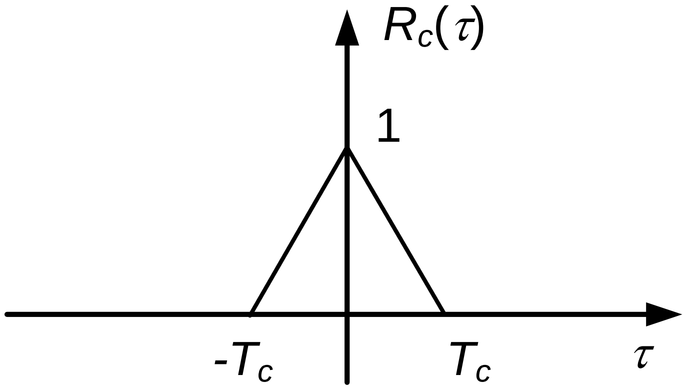
that depends on the estimation *error* $\varepsilon[k\text{-}1]$ – it is an *estimation error detector*

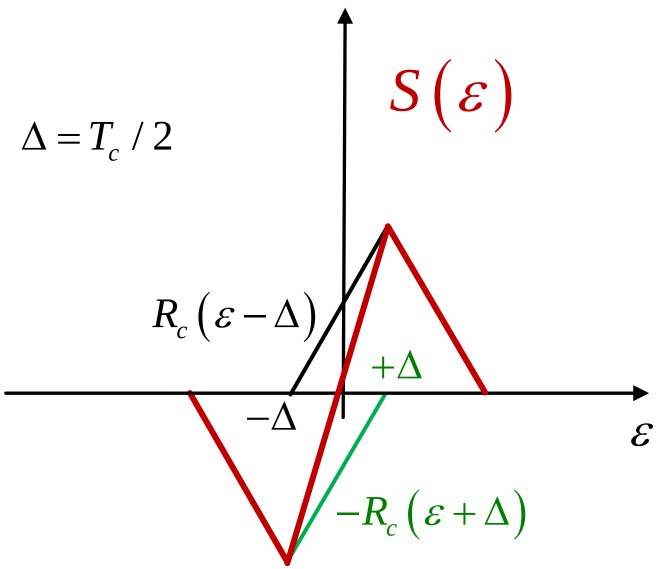
- The value of *e* as a function of a *fixed* estimation error ε (and without noise) is called the *discriminator characteristics* or *S-curve* of the estimator (loop)

$$S(\varepsilon) = e[k-1]\Big|_{\varepsilon[k-1]=\varepsilon} = \sqrt{2C} \cdot \left[R_c\left(\varepsilon - \Delta\right) - R_c\left(\varepsilon + \Delta\right) \right]$$

The S-curve

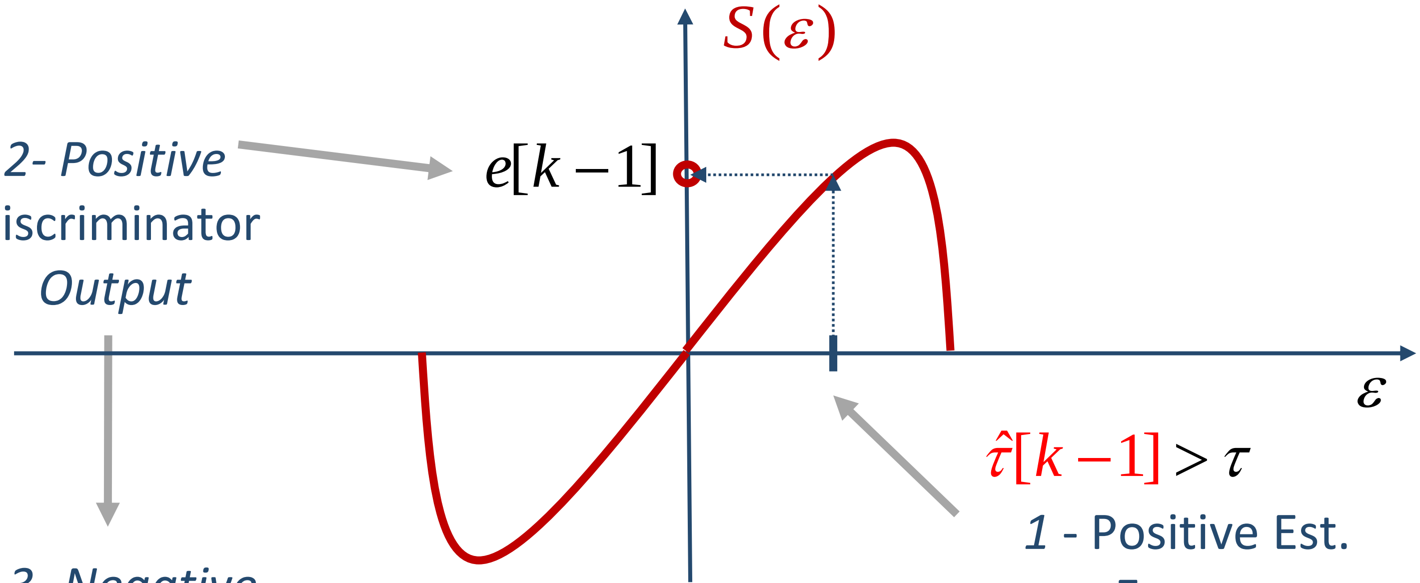
$$S(\varepsilon) = e[k-1]\Big|_{\varepsilon[k-1]=\varepsilon} = \sqrt{2C}\cdot\left[R_c\left(\varepsilon-\Delta\right)-R_c\left(\varepsilon+\Delta\right)\right]$$





Negative Feedback Loop

$$\hat{\tau}[k] = \hat{\tau}[k-1] - \gamma_\tau e[k-1]$$



2- Positive Discriminator *Output*

$\hat{\tau}[k-1] > \tau$

1 - Positive Est. Error ε

3- Negative Correction *Term*

$-\gamma_\tau e[k-1] < 0$

4 – The estimate is reduced

$\hat{\tau}[k] < \hat{\tau}[k-1]$

Loop with Noise

- In addition to the useful term, the discriminator output also contains *noise*

$$e[k-1]=e^{+}(\hat{\tau}[k-1])-e^{-}(\hat{\tau}[k-1])$$
$$=\sqrt{2C}\cdot\left[R_c\left(\varepsilon[k-1]-\Delta\right)-R_c\left(\varepsilon[k-1]+\Delta\right)\right]$$
$$+\left[W^{+}[k-1]-W^{-}[k-1]\right]=S\left(\varepsilon[k-1]\right)+W[k-1]$$

so the successive estimates $\hat{\tau}[k]$ will be *noisy* as well

$$\hat{\tau}[k]=\hat{\tau}[k-1]-\gamma\left(S\left(\varepsilon[k-1]\right)+W[k-1]\right)$$

- Large values of γ give a "fast" loop that is ready to react to large errors and drive back the estimate towards the correct value, but also enhance the effect of noise and provide a larger *error variance* σ_ε^2

Iterative Estimation ERROR Analysis

$$\hat{\tau}[k] = \hat{\tau}[k-1] - \gamma e[k-1]$$

Subtracting the true value τ we get the (recursive) ESTIMATION ERROR EQUATION

$$\varepsilon[k] = \varepsilon[k-1] - \gamma e[k-1]$$

still running at estimation time T_0

LOOP Noise

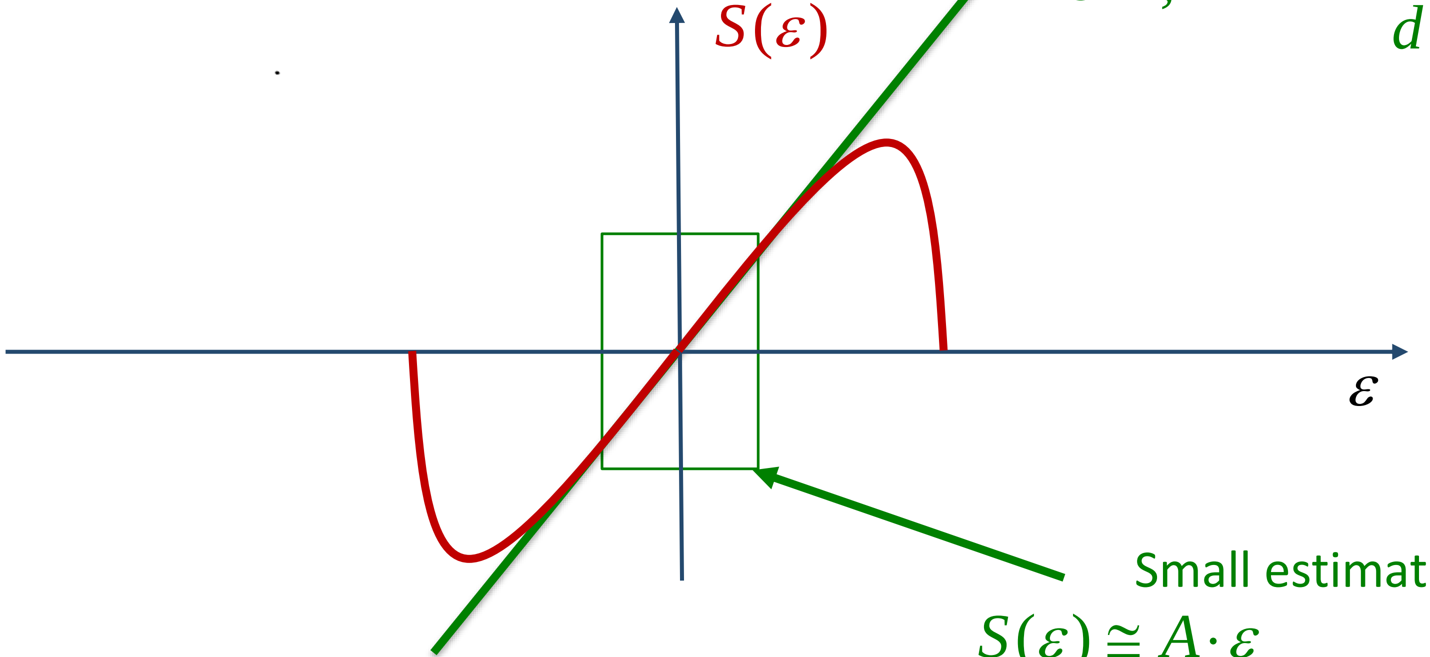
$$\varepsilon[k] = \varepsilon[k-1] - \gamma S\left(\varepsilon[k-1]\right) - \gamma W[k-1]$$

LOOP Discriminator Function

Estimate Variance: Linearized Loop

$$\varepsilon[k]=\varepsilon[k-1]-\gamma e[k-1]$$

$$A\cdot\varepsilon \quad , \quad A \triangleq \left.\frac{dS(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0}$$



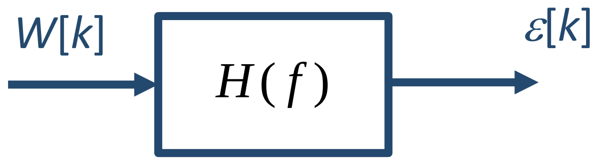
Small estimation error

$$S(\varepsilon)\cong A\cdot\varepsilon$$

$$\varepsilon[k]\cong\varepsilon[k-1]-\gamma\left(A\varepsilon[k-1]+W[k-1]\right) \quad , \quad W[k-1]=W^{+}[k-1]-W^{-}[k-1]$$

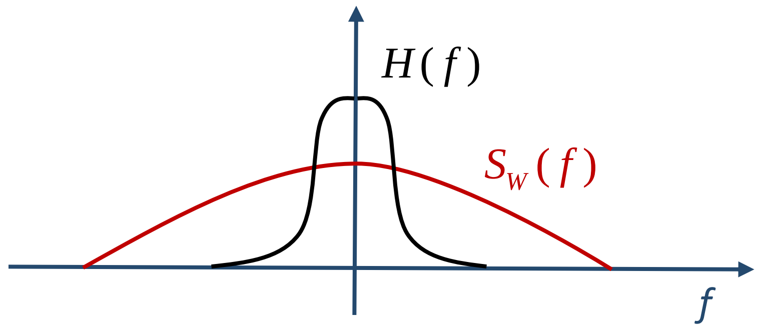
Linearized First Order Loop

$$\varepsilon[k] = (1-\gamma A)\varepsilon[k-1] - \gamma A \frac{W[k-1]}{A}$$



It is a (narrowband) FIRST-ORDER IIR Filter operating at a rate $1/T_0$

$$\sigma_\tau^2 = \sigma_\varepsilon^2 \cong 2B_L T_0 \left.\frac{S_w(f)}{A^2}\right|_{f=0}$$



with an equivalent (normalized) *noise bandwidth*

$$B_L T_0 = \frac{\gamma A}{2(2-\gamma A)} \cong \frac{\gamma A}{4}$$

For GPS C/A, $\Delta = T_c / 2$

$$A = \frac{2}{T_c}\sqrt{2C} \quad , \quad S_w(f) = \sigma_W^2$$

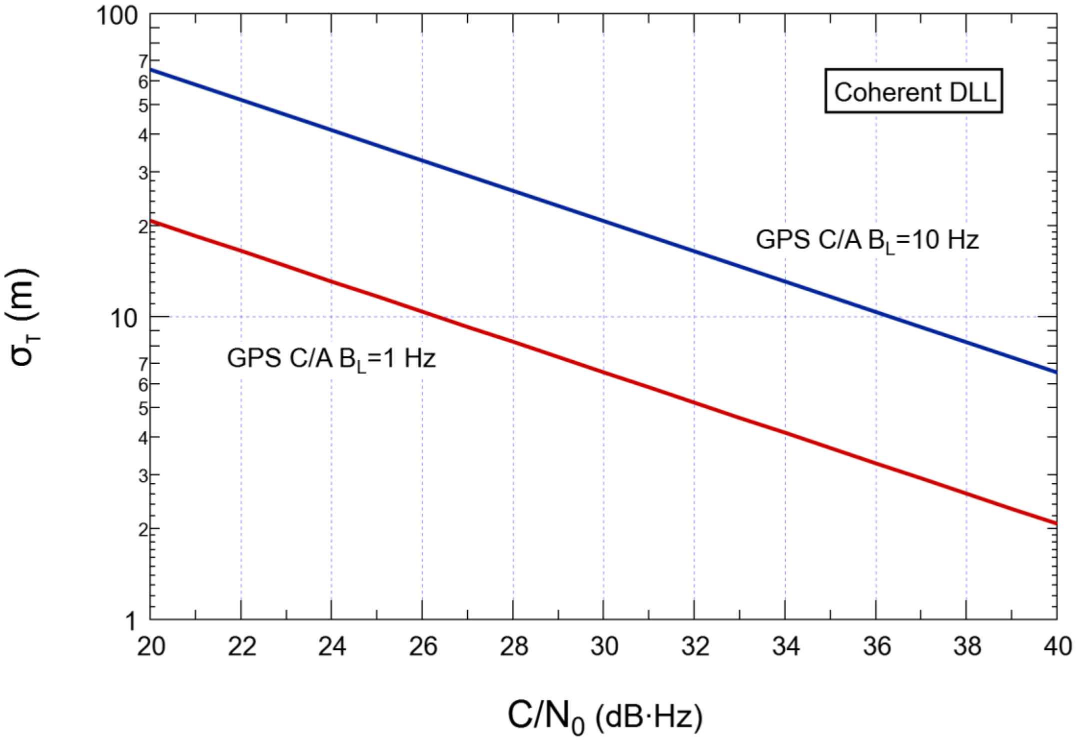
$$\sigma_W^2 = \mathrm{E}\left\{W^2\right\} = \mathrm{E}\left\{\left(W^+ - W^-\right)^2\right\} = \mathrm{E}\left\{\left(W^+\right)^2\right\} + \mathrm{E}\left\{\left(W^-\right)^2\right\} - 2\,\mathrm{E}\left\{W^+W^-\right\}$$

$$= \frac{N_0}{T_0} + \frac{N_0}{T_0} - 0$$

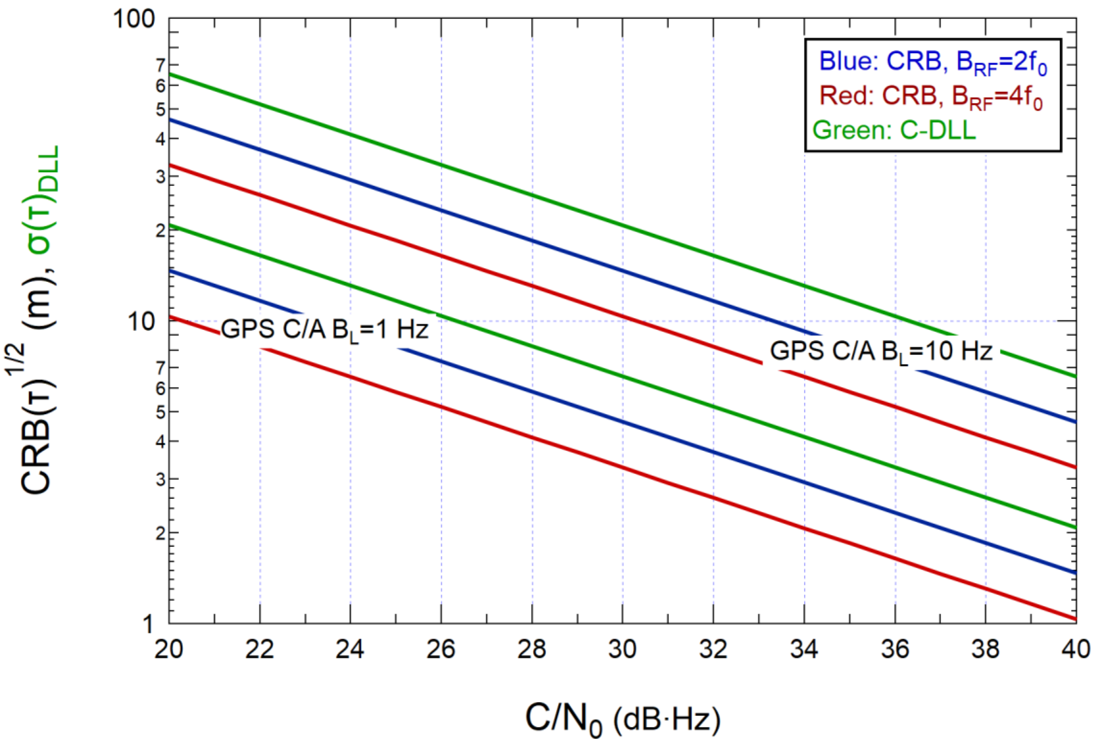
$$\sigma_\varepsilon^2 = 2B_L T_0 \frac{2N_0 / T_0}{2C} \frac{T_c{}^2}{4} = T_c{}^2 \frac{B_N}{2C / N_0}$$

$$\sigma_\tau[m] = cT_c \sqrt{\frac{B_L}{2C / N_0}}$$

Coherent DLL Accuracy



Coherent DLL Accuracy



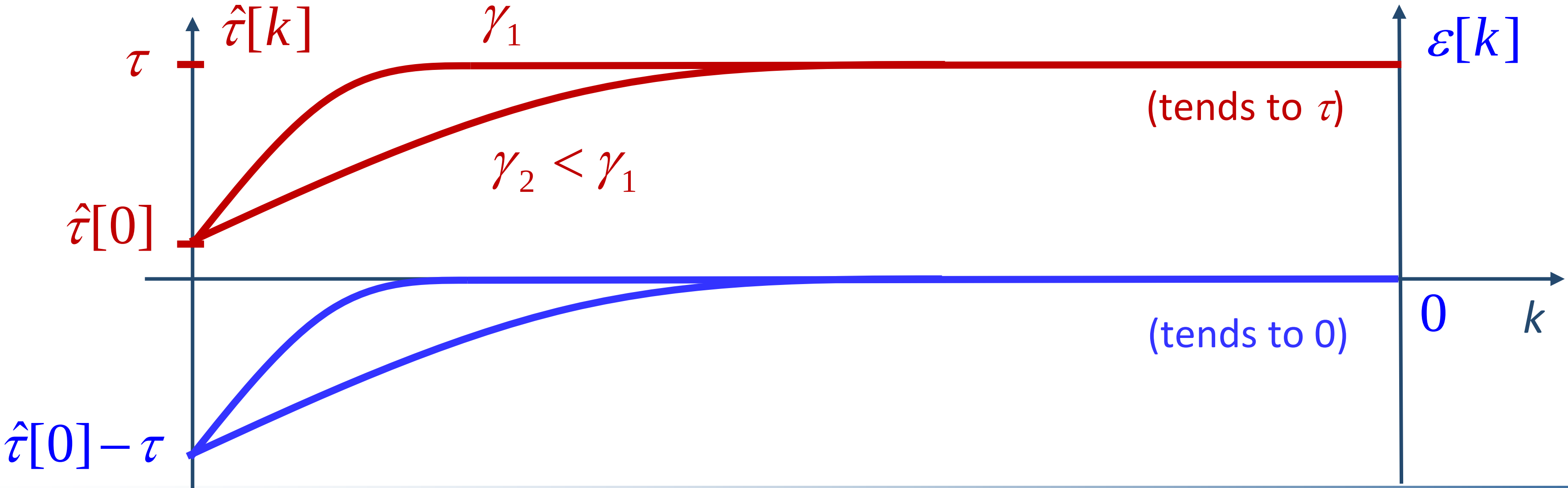
Visualizing the Loop Bandwdith

- Linearized loop error equation with no noise

$$\varepsilon[k] = (1-\gamma A)\varepsilon[k-1] \quad , \quad \gamma A \cong 4B_L T_0 \ll 1$$

The loop evolution starts from $\varepsilon[0] = \varepsilon_0 < T_c / 2$ as resulting from the acquisition procedure. Then from

$$\varepsilon[1] = (1-\gamma A)\varepsilon_0 \quad , \quad \varepsilon[2] = (1-\gamma A)^2 \varepsilon_0 \quad ,..., \quad \varepsilon[k] = (1-\gamma A)^k \varepsilon_0$$



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Visualizing the Loop Bandwdith

- Linearized loop error equation with noise

$$\varepsilon[k] = (1-\gamma A)\varepsilon[k-1] + \frac{\gamma}{A} W[k-1] \quad , \quad \gamma A \cong 4 B_N T_0 \ll 1$$

The loop bandwdith also impacts on the steady-state error variance (aka *jitter*)