Ingegneria delle Telecomunicazioni
Satellite Communications

21. TOMTOM and beyond - GNSS Receiver Design

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His(Her) Majesty TomTom GO, 2004

- GPS receiver
- Digital Maps
- Navigation SW
- User-Friendly GUI
- Voice Directions
- Touch-screen Display
Modern receivers perform all of the signal processing functions with DSP components and algorithms.
• I/Q BaseBand Sampling with IF Conversion

• I/Q BaseBand Sampling with Direct Baseband Conversion (aka ZERO-IF Architecture)
Form RF to Digital - II

- IF BandPass Sampling with Digital IF

- Direct RF Sampling with Digital IF

![Diagram of RF to Digital conversion]

- Analog section:
  - RF Low-Noise Amplifier
  - IF Amplifier
  - LO

- Digital section:
  - ADC
  - Digital Section

Symbols:
- f_RF: RF frequency
- f_LO: Local Oscillator frequency
- f_IF: Intermediate Frequency
- f_ADC: ADC output frequency
- x[n]: Digital signal sample
Band-Pass Sampling

- **Pass-band, real signal**
  - *Pass-band bandwidth* $BW$
  - *carrier frequency* $f_{RF}$

- **Nyquist-sampling condition:**
  $$f_s \geq 2B = 2\left(f_{RF} + \frac{BW}{2}\right)$$

- **For navigation signals in L-band, this is impractical!**
  $$f_{RF} \in [1\text{GHz}, 2\text{GHz}] \quad \Rightarrow \quad f_s > 2 \div 4\text{GHz}$$

- **Solution:** *band-pass sampling* at reduced rate:
  $$f_s \approx BW$$

**HERMITIAN SYMMETRIC**

$$X(-f) = X(f)^*$$
Direct Sampling Front-End Architecture

- Front-end architecture of direct-sampling receiver

  - RX signal from antenna is sampled **directly after LNA** and RF BP filter
  - no RF-to-IF downconversion stage / all-digital processing
  - **Digital** IF $f_{IF,D}$ resulting from the interplay between $f_{RF}$ and $f_s$ (later on)
  - **Digital** I/Q oscillator to convert the received signal at $f_{IF,D}$ to I/Q base-band
  - low-pass filter to isolate the channel of interest
    - filter bandwidth is BW
  - base-band decimation
    - base-band signal may be oversampled for many applications
Direct Sampling Requirements

- Any A/D converter is characterized by the *aperture time*, i.e., the time over which the signal must be stable (constant) to compute the digital output
  - it must be short enough to have correct signal sampling (constant level)

- In the frequency domain, this means that the input bandwidth of the A/D converter must be *wide enough* to accommodate the whole signal bandwidth
 Band Pass Sampling: Nyquist Zones

- **Digital sampling → replica of the signal spectrum**
  - correct, Nyquist sampling only in NY-1
  - pass-band sampling in NY-2 and above
  - fundamental replica of the signal spectrum centred upon a **new (digital) carrier frequency** $f_{IF,D}$

- **Local carrier $f_{IF}$:**

  $$0 \leq f_{IF} < f_s$$

  ***Shouldn’t it be $f_s/2$??***
• case I: fundamental replica in NY-1 zone:

- fundamental replica
- band inversion

• case II: fundamental replica in NY-2 zone:

- fundamental replica
- band inversion
In general, the resulting Digital IF is derived from the relation $f_{IF,D} = f_{RF} - mf_s$ where $m$ is integer and is such that $f_{IF,D}$ lies within the digital bandwidth $-f_s/2 \leq f < f_s/2$. To sum up,

$$f_{IF,D} = f_{RF} - mf_s \quad \Rightarrow \quad f_{IF,D} = \left|f_{RF}\right| f_s - f_s / 2$$

where the $-f_s/2$ terms has the purpose of re-mapping the “modulus-$f_s$” function to $-f_s/2, f_s/2$ instead of $0, f_s/2$ as in the mathematical definition.
Digital IF frequency of the fundamental replica:

\[ f_{IF} = \left| f_{RF} + \frac{f_s}{2} \right|_{f_s} - \frac{f_s}{2} = \| f_{RF} \|_{f_s} \]  

(Symmetric Modulus Function)

Case I (NY-1):

\[ 0 \leq f_{IF} < \frac{f_s}{2} \]

- Simple frequency conversion

Case II (NY-2):

\[ -\frac{f_s}{2} \leq f_{IF} < 0 \]

- Frequency conversion AND band inversion, i.e., \( f_{IF} \) can be considered positive but the Q components is sign-changed

OK with \( f_{IF} \), but what is the relation of \( f_s \) with the (bandpass) signal BW?
• **Selection of sampling rate**
  
  - requirement: no aliasing of the signal bandwidth

\[
\begin{align*}
NY-1: & \begin{cases} 
    f_{IF} + \frac{BW}{2} \leq \frac{f_s}{2} \\
    f_{IF} - \frac{BW}{2} \geq 0
\end{cases} \\
NY-2: & \begin{cases} 
    f_{IF} + \frac{BW}{2} \leq 0 \\
    f_{IF} - \frac{BW}{2} \geq -\frac{f_s}{2}
\end{cases} \\
\Rightarrow & \begin{cases} 
    |f_{IF}| \geq \frac{BW}{2} \\
    |f_{IF}| \leq \frac{f_s}{2} - \frac{BW}{2}
\end{cases}
\end{align*}
\]

\[
\frac{BW}{2} \leq |f_{IF}| \leq \frac{f_s}{2} - \frac{BW}{2}
\]

• We can visualize the inequality on the Ladder diagram to actually find the allowed values of the digital carrier \(f_{IF}\) AND of the sampling frequency \(f_s\).
- **Case study:** $f_{RF} = 1\ GHz$ and $BW = 100\ MHz$
### Sample Design: GNSS Direct-Sampling Front-End

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<th>N.</th>
<th>RIMS V3 Req</th>
<th>Satellite System</th>
<th>Band</th>
<th>Lower/Upper</th>
<th>Signal</th>
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<th>ID Comb Carrier-BW</th>
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</table>

- **Lower-band:**
  - *SIS from 1 to 11 (from 1150 to 1310 MHz)*
  - \(BW_L = 160 \text{ MHz}; f_{RF,L} = 1230 \text{ MHz}\)

- **Upper-band**
  - *SIS from 12 to 18 (from 1550 to 1610 MHz)*
  - \(BW_U = 60 \text{ MHz}; f_{RF,U} = 1580 \text{ MHz}\)
GNSS Direct-Sampling Front-End: Ladder(s)

\[ \frac{f_s - 60}{2} \]
\[ \frac{f_s - 160}{2} \]

BOTH ladders (lower and upper bands) has to work at the same time!

327.5 MHz ≤ \( f_s \) < 328.5 MHz

375 MHz ≤ \( f_s \) < 383 MHz
Selection of sampling rate: Ladder diagram

- Requirement: no aliasing of the signal bandwidth

\[
\begin{align*}
\text{NY-1:} & \quad \begin{cases} 
    f_{IF} + \frac{BW}{2} \leq \frac{f_s}{2} \\
    f_{IF} - \frac{BW}{2} \geq 0 
\end{cases} \\
\text{NY-2:} & \quad \begin{cases} 
    f_{IF} + \frac{BW}{2} \leq 0 \\
    f_{IF} - \frac{BW}{2} \geq -\frac{f_s}{2} 
\end{cases} \\
\text{NY-1:} & \quad \begin{cases} 
    \frac{BW}{2} \leq f_{IF} \leq f_s - \frac{BW}{2} 
\end{cases}
\]

\[
\frac{BW}{2} \leq |f_{IF}| \leq \frac{f_s}{2} - \frac{BW}{2}
\]

Lowest-frequency solution (NY-2):

\[327.5 \text{ MHz} \leq f_s < 328.5 \text{ MHz}\]

\[f_s = 328 \text{ MHz}\]

Next solution (NY-1):

\[375 \text{ MHz} \leq f_s < 383 \text{ MHz}\]
- **Sampling frequency** $f_s = 328$ MHz

  ![Spectrum Diagram]

  - **Upper-band:**
    - $[-90; -30]$ MHz
    - $f_{IF,U} = -60$ MHz
  - **Lower-band:**
    - $[-162; -2]$ MHz
    - $f_{IF,L} = -82$ MHz
Basic Modeling of the I/Q received signal

- $N_{sat}$ number of satellites in visibility (elevation larger than $\approx 10$ degrees)
- $i$ satellite identifier
- $C_i$ received signal power
- $\tau_i$ time of flight in the user time scale
- $\Delta f_i$ Doppler shift
- $\theta_i$ carrier phase

$$r(t) = \sum_{i=1}^{N_{sat}} \sqrt{2C_i} s_i(t - \tau_i) \exp\left[j\left(2\pi\Delta f_i t + \theta_i\right)\right] + w(t)$$

At the output of the RF to Digital Section:

$$x_{ADC}[n] = \sum_{i=1}^{N_{sat}} \sqrt{2C_i} s_i(nT_s - \tau_i) \exp\left[j\left(2\pi\Delta f_i nT_s + \theta_i\right)\right] + w[nT_s]$$
Modern receivers perform all of the signal processing functions with DSP components and algorithms.
Detailed Receiver Architecture

VE=Very Early, E=Early, L=Late, VL=Very Late correlations for the DLL
P=Prompt for Data Detection (may also be used for the DLL)
• Data detection performance depend on the bit-rate and on the kind of channel coding that is used on the different carriers/channels/systems
Super-simplified received signal

- $N_{sat} = 1$
- $i = 1$ satellite identifier
- $C_1 = 1/2$ received signal power
- $\tau_1 = \tau$ time of flight in the user time scale
- $\Delta f_i = 0$ Doppler shift
- $\theta_i = 0$ carrier phase
- Pilot signal only (no Q component needed)

$$r(t) = c(t - \tau) + w(t)$$

Let us concentrate on delay tracking...
Ranging Code Tracking

To maximize \( \int_{0}^{T_0} r(t)c(t - \tilde{\tau})dt \) we do:

\[
\frac{d}{d\tilde{\tau}} \int_{0}^{T_0} r(t)c(t - \tilde{\tau})dt = 0 \Rightarrow \int_{0}^{T_0} r(t) \frac{d}{d\tilde{\tau}} c(t - \tilde{\tau})dt = 0
\]

\[
\frac{d}{d\tilde{\tau}} c(t - \tilde{\tau}) \approx -\frac{c(t - \tilde{\tau} + \Delta) - c(t - \tilde{\tau} - \Delta)}{2\Delta} \Rightarrow \int_{0}^{T_0} r(t) \frac{c(t - \tilde{\tau} + \Delta) - c(t - \tilde{\tau} - \Delta)}{2\Delta} dt = 0
\]

\[
\int_{0}^{T_0} r(t)c(t - \tilde{\tau} + \Delta)dt - \int_{0}^{T_0} r(t)c(t - \tilde{\tau} - \Delta)dt = 0
\]

\( T_0 \) is the correlation (observation) time
Loop Discriminator and DLL Equation

\[ e(\tilde{\tau}) = \int_0^{T_0} r(t)c(t - \tilde{\tau} + \Delta)dt - \int_0^{T_0} r(t)c(t - \tilde{\tau} - \Delta)dt \]

Can be computed Real-Time on the k-th correlation interval \([(k-1)T_0, kT_0)\]

\[ e(\tilde{\tau}[k]) = \int_{(k-1)T_0}^{kT_0} r(t)c(t - \tilde{\tau}[k] + \Delta)dt - \int_{(k-1)T_0}^{kT_0} r(t)c(t - \tilde{\tau}[k] - \Delta)dt \]

\( k \) runs at the correlation time \( T_0 \)
Derivation of the DLL – Coherent Detection

\[ r(t) = \sqrt{2P_e} \ c(t - \Delta) + w(t) \quad [\text{PILOT}] \]

Assume that, at time \((k-1)T_0\), we have a trial estimate \(\hat{\Delta}[k-1]\) of the delay. Then,

\[ e[k-1] = \frac{1}{T_0} \int_{(k-1)T_0}^{kT_0} r(t) c(t - \hat{\Delta}[k-1] + \Delta) \, dt \]

\[ \hat{\Delta}[k-1] = \text{Receiver estimate at time } (k-1)T_0 \]
The sequence of estimates at time $kT_0$ is

$$\hat{\gamma}_n = \hat{\gamma}_{n-1} + \frac{T}{2} (n-\gamma_{n-1} - e^{+ (n-1)} - e^{- (n-1)})$$

where $e^{+ (n-1)} = e^{+ (n-1)} - e^{- (n-1)}$.

The equation resembles that of a feedback control loop - let us see why.
\[ e^+ = \frac{1}{T_0} \int_{(k-\eta)T_0}^{kT_0} c(t-\eta) \hat{c}(t-\hat{2}[k-\eta] \pm \Delta) \, dt + \]

\[ + \frac{1}{T_0} \int_{(k-\eta)T_0}^{kT_0} w(t) \hat{c}(t-\hat{2}[k-\eta] \pm \Delta) \, dt \]

\[ = R_c \left( \hat{2}[k-\eta] - \eta \pm \Delta \right) + W^+ \hat{2}[k-\eta] \]

Gaussian RV
Error discrimination function

\[ \hat{e}_{[k-1]} - 2 \triangleq E_{[k-1]} \text{ estimation ERROR } e_{(k-1)T}. \]

\[ e(3[k-1]) = e^+ (3[k-1]) - e^- (3[k-1]) \]

\[ = R_c (3[k-1] - \Delta) - R_c (3[k-1] + \Delta) \]

\[ + (W^+ [k-1] - W^- [k-1]) \]

\[ \text{IF } \hat{e}_{[k-1]} = 2 \text{ then } e = 0 \text{ and } e(0) = 0 \]
\[ \hat{\mathbf{e}}[k] = \hat{\mathbf{e}}[k-1] - \gamma \cdot \mathbf{e}(3) (k-1) \]

where \( \gamma \) is a suitably sized stepsize. Subtracting

\[ \mathbf{e}[k] = \mathbf{e}[k-1] - \gamma \cdot \mathbf{e}(3) (k-1) \]

Runs at correlation time \( T_0 \) and provides a sequence of estimated errors every \( T_0 \) seconds.
The S-curve

\[ e(\varepsilon) = R_c(\varepsilon - \Delta) - R_c(\varepsilon + \Delta) + \text{NOISE} \]

\[ \Delta = \frac{T_c}{2} \]
\[
\gamma \hat{Z}[k] \approx \gamma \hat{Z}[k-1] + \gamma e(\hat{s}(3))
\]

**Positive discriminator output**

\[-\gamma e(3) < 0\]

\[\hat{Z}[k] < \hat{Z}[k-1]\]

\[\hat{Z}[k] > 2\]

\[\gamma > 2\]

**Negative Feedback Loop**

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21. TOMTOM and beyond - GNSS Receiver Design
\[ \varepsilon[k] = A \cdot \varepsilon[k-1] - \varepsilon[k-3] \]

\[ T = T^+ - T^- \]

\[ A \cdot \varepsilon[k] = \frac{ds}{d\varepsilon} \]

Small estimation error
First Order Loop

$$\varepsilon[n] = (1-\gamma A)\varepsilon[n-1] - \gamma A \frac{W[n-1]}{A}$$

$$\sigma^2 \sim 2B_n \frac{S_w(f)}{A^2} \bigg|_{f=0}$$

$$\sigma_t[m] = c T_c \sqrt{\frac{B_N}{2C / N_0}}$$

$$B_n \tau_c = \frac{\gamma A}{2(2-\gamma A)} \Rightarrow \frac{\gamma A}{4}$$
Coherent DLL Accuracy

\[ \sigma_r (m) \]

\[ C/N_0 \text{ (dB·Hz)} \]

- GPS C/A \( B_L = 10 \text{ Hz} \)
- GPS C/A \( B_L = 1 \text{ Hz} \)