Ingegneria delle Telecomunicazioni

Information Theory @ Digital Communications

MIMO Wireless Communications

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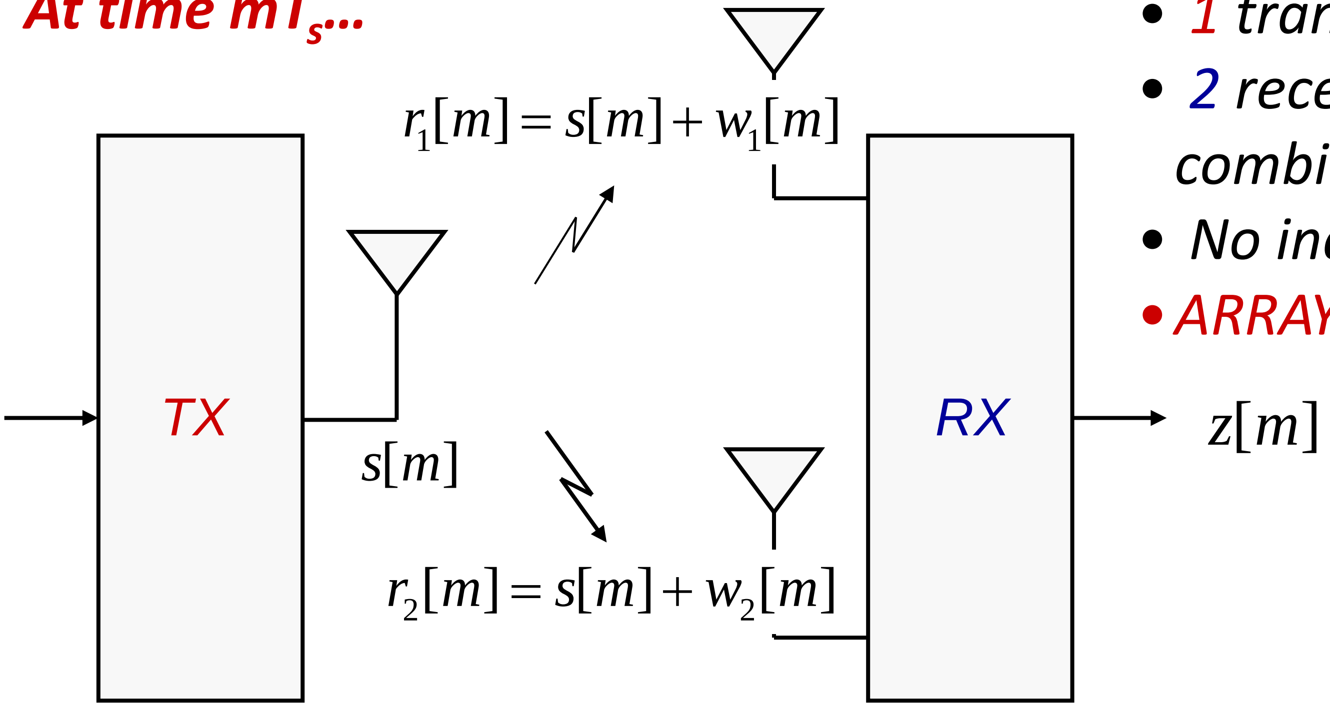
Multiantenna Systems 1: Reception Diversity

At time mT_s…



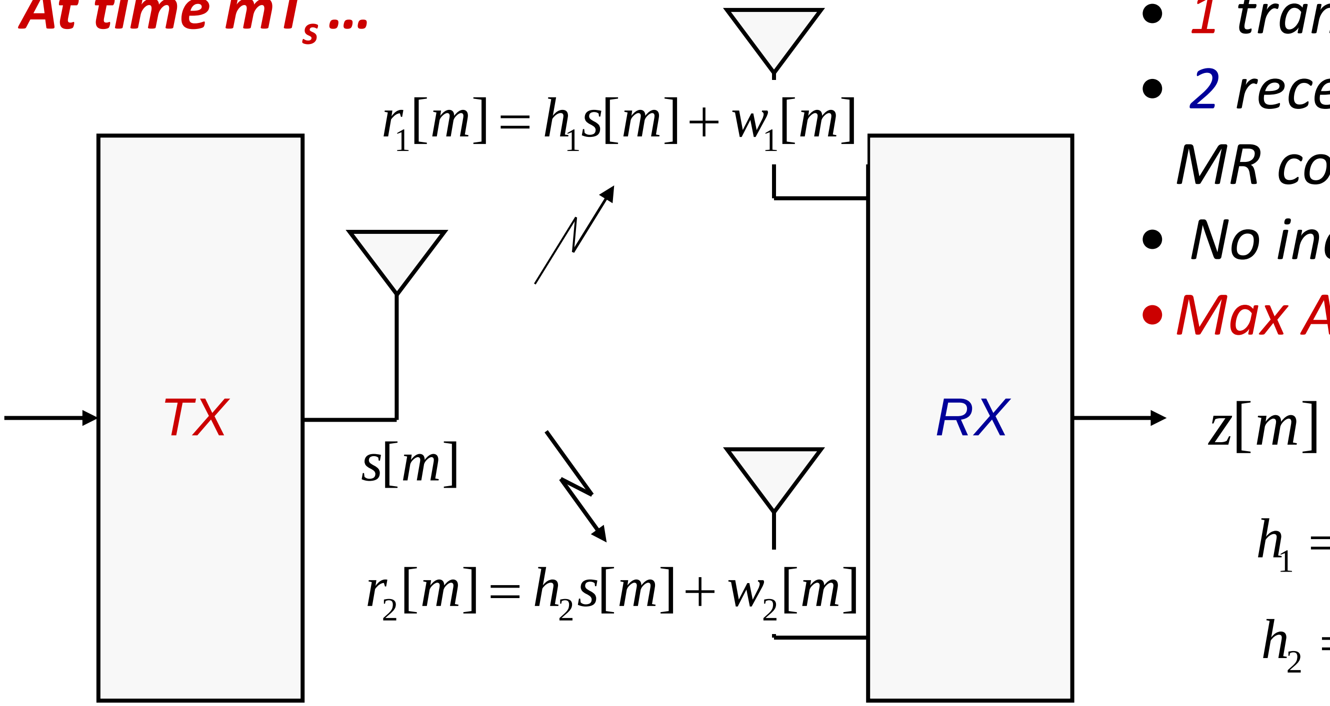
- *1 transmitting antenna*
- *2 receiving antennas with combining*
- *No increase in bit-rate*
- *ARRAY Gain = 3 dB*

$$z[m] = \frac{r_1[m] + r_2[m]}{2} = s[m] + \frac{w_1[m] + w_2[m]}{2} = s[m] + w[m]$$

The noise variance is reduced by a factor 2

Multiantenna Systems 1: Reception Diversity

At time mT_s …



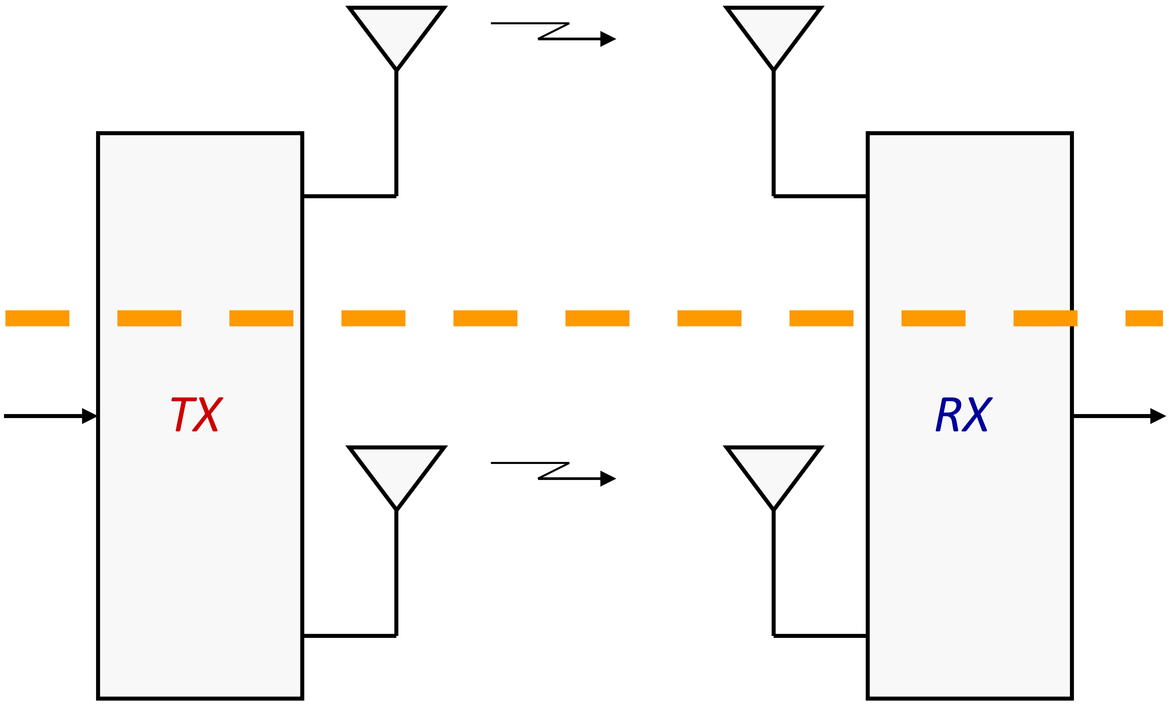
- *1 transmitting antenna*
- *2 receiving antennas with MR combining*
- *No increase in bit-rate*
- *Max ARRAY Gain = 3 dB*

$$h_1 = |h_1| e^{j\angle h_1}$$

$$h_2 = |h_2| e^{j\angle h_2}$$

$$z[m] = \frac{h_1^* r_1[m] + h_2^* r_2[m]}{|h_1|^2 + |h_2|^2} \Rightarrow SNR = SNR_1 + SNR_2$$

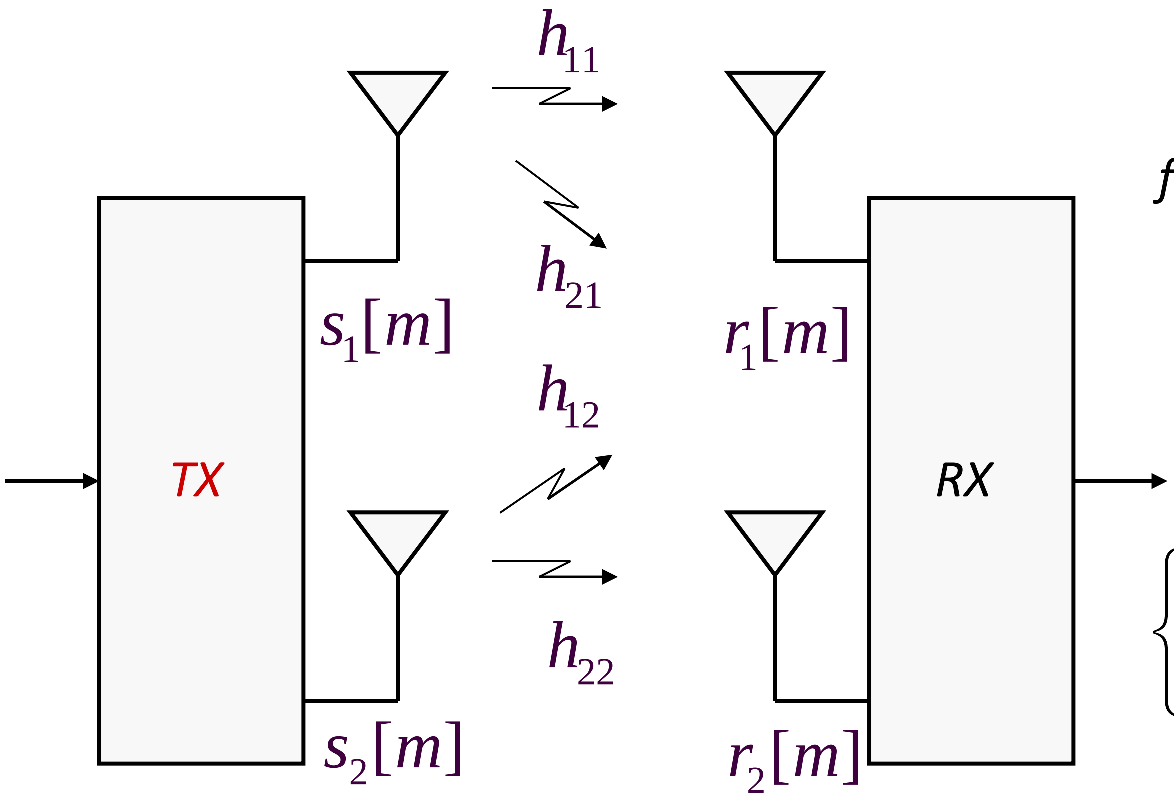
Multiantenna Systems 2: Independent Channels



- *2 transmitting antennas*
- *2 receiving antennas*
- *No mutual interference*
- *No array gain*
- *MULTIPLEXING GAIN x 2*

The bit-rate is DOUBLED

MIMO Channel Modeling

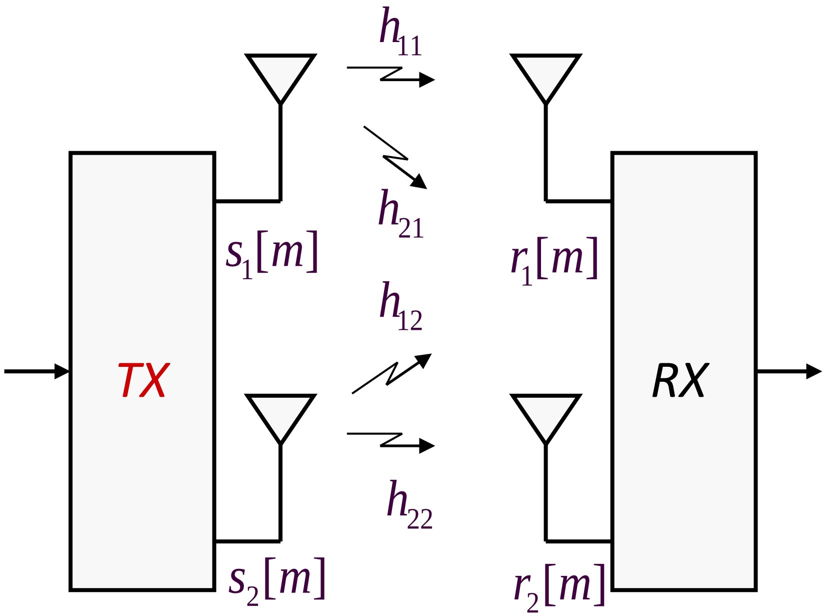


2 x 2 simple example with narrowband signals on a frequency-flat channel at time mT_s, neglecting noise

$$\begin{cases} r_1[m] = h_{11}s_1[m] + h_{12}s_2[m] \\ r_2[m] = h_{21}s_1[m] + h_{22}s_2[m] \end{cases}$$

The terms h_{ik} are just complex coefficients that modify the amplitude/phase of the transmitted symbols because the channel is non-frequency-selective !

MIMO-OFDM 1/2



How can we possibly have a narrowband channel at the high signaling rates of modern communications?

$$\begin{cases} r_1[m] = h_{11}s_1[m] + h_{12}s_2[m] \\ r_2[m] = h_{21}s_1[m] + h_{22}s_2[m] \end{cases}$$

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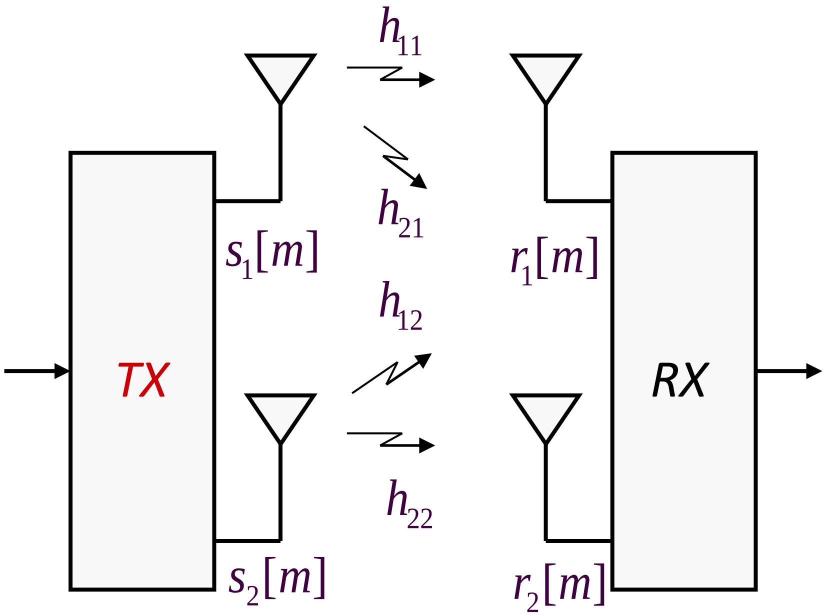
MIMO-OFDM 2/2



The flat MIMO channel is actually experienced on each single frequency k/T_{MC} of the subcarriers raster in OFDM communications !

At time mT_{MC}...

$$
\begin{cases}
r_{1,k}[m] = H_{11}\left(\dfrac{k}{T_{MC}}\right) s_{1,k}[m] + H_{12}\left(\dfrac{k}{T_{MC}}\right) s_{2,k}[m] \\
r_{2,k}[m] = H_{21}\left(\dfrac{k}{T_{MC}}\right) s_{1,k}[m] + H_{22}\left(\dfrac{k}{T_{MC}}\right) s_{2,k}[m]
\end{cases}
$$

$$k = 0, \ldots, N-1$$

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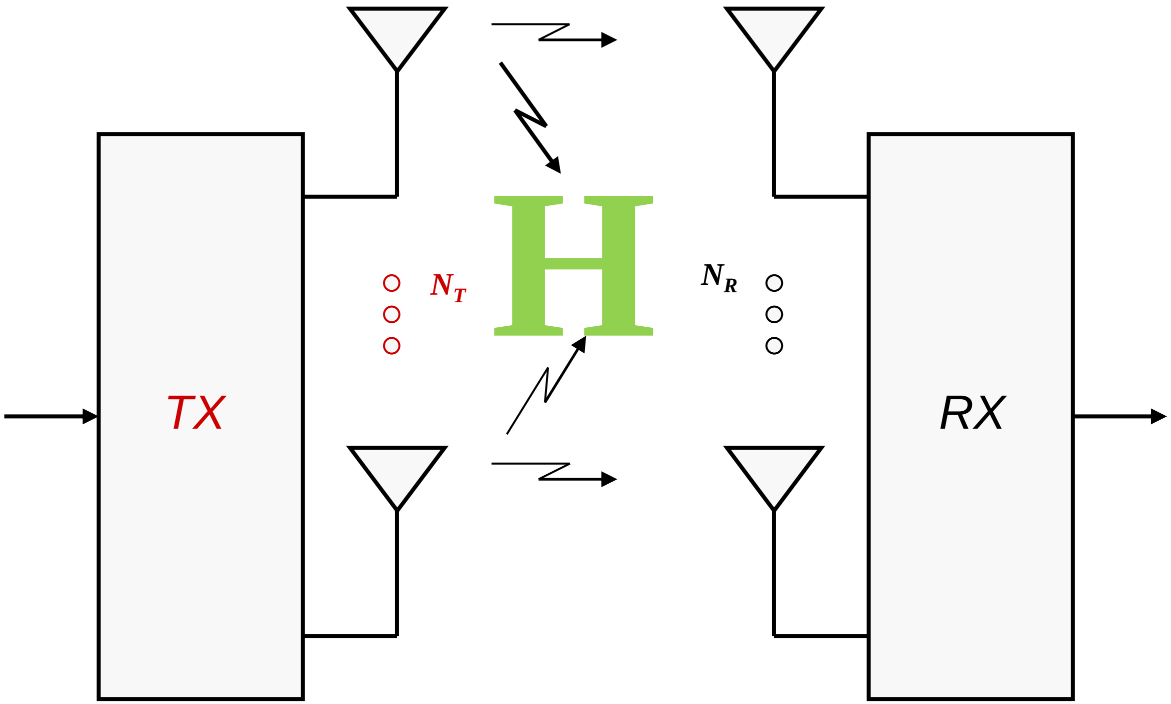
General MIMO (Multiantenna) System



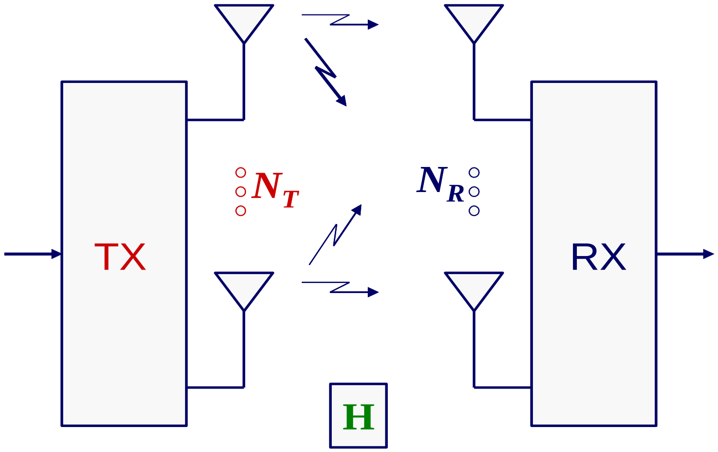
- N_T *transmitting antennas*
- N_R *receiving antennas*
- *low-rate channels with flat-fading*
- N_R *x* N_T *channel matrix* $\mathbf{H}$

The channels are "different" in SPACE

The complex-valued MIMO channel at time *m*

- *Independent (equi-power Rayleigh) Channels*
- *Spatially White equi-power noise*
- *Stationary channel (long coherence time)*

$$\mathbf{r}[m] = \mathbf{H}\mathbf{s}[m] + \mathbf{w}[m]$$



$$\mathbf{w}[m] = \begin{bmatrix} w_1[m] \\ w_2[m] \\ \vdots \\ w_{N_R}[m] \end{bmatrix}$$

White noise vector

$$\mathbf{s}[m] = \begin{bmatrix} s_1[m] \\ s_2[m] \\ \vdots \\ s_{N_T}[m] \end{bmatrix}$$

Transmitted symbols vector

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix}$$

Channel Matrix

$$\mathbf{r}[m] = \begin{bmatrix} r_1[m] \\ r_2[m] \\ \vdots \\ r_{N_R}[m] \end{bmatrix}$$

Received (soft) symbols vector

Simple Examples

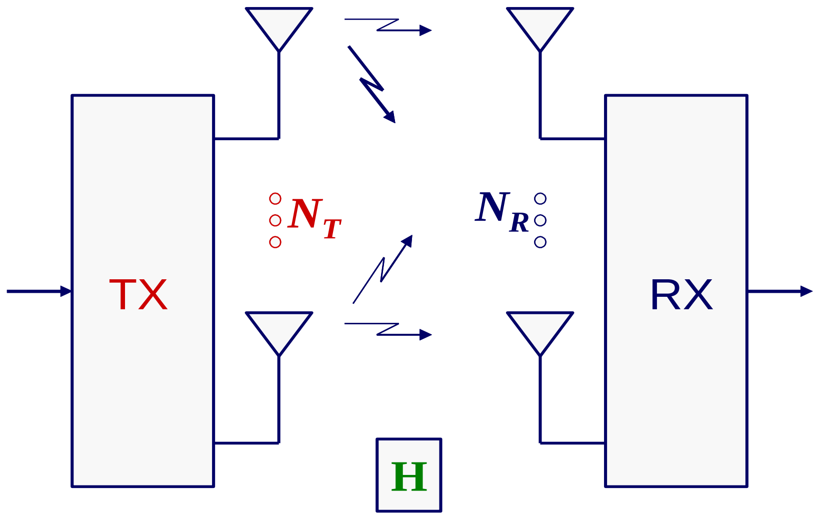
- **Diagonal matrix ($N_R=N_T$): independent channels, max. multiplexing gain, no array gain**

- **N_R x 1 matrix: conventional diversity reception, max. array/diversity gain, no multiplexing gain**

Wi-Fi6 Access Point

MIMO Reception (spatial MUX)

$$\mathbf{r}[m] = \mathbf{H}\mathbf{s}[m] + \mathbf{w}[m]$$



$$N_T = N_R$$

$$\mathbf{z}[m] = \hat{\mathbf{H}}^{-1}\mathbf{r}[m] = \hat{\mathbf{H}}^{-1}\left(\mathbf{H}\mathbf{s}[m] + \mathbf{w}[m]\right) = \mathbf{s}[m] + \hat{\mathbf{H}}^{-1}\mathbf{w}[m] = \mathbf{s}[m] + \mathbf{w}'[m]$$

- *Need to know (estimate) the channel matrix* $\mathbf{H}$
- $\mathbf{H}$ *has to be non-singular*
- *Possible noise enhancement→need to reduce constellation size*

The MIMO Channel

- **ARRAY/DIVERSITY gain by combination of multiple RX copies of the same signal with *independent* noise components (improves *SNR*)**

- **MULTIPLEXING gain by using multiple *independent* channels with multiple TX/RX antennas (improves *rate*) but increases noise and increases bit-rate less than expected**

SNR/Rate relation through capacity theorem

Computation of MIMO Capacity - 1

Compute the SVD of H: $\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^{\dagger}$

Eigenvectors of $\mathbf{H}\mathbf{H}^{\dagger}$ $\quad \mathrm{diag}\{d_i\}$ $\quad$ *Eigenvectors of* $\mathbf{H}^{\dagger}\mathbf{H}$

N_R $\mathbf{H}$ N_T = *Unitary Matrix* $\mathbf{U}\mathbf{U}^{\dagger} = \mathbf{I}_{N_R}$ N_R $\quad$ 0 0 $\mathbf{D}$ $\quad$ *Unitary Matrix* $\mathbf{V}\mathbf{V}^{\dagger} = \mathbf{I}_{N_T}$ N_T

$N_R > N_T$ *, uplink-like*

are the N_T $\sqrt{\mathrm{eigen}(\mathbf{H}^{\dagger}\mathbf{H})}$ *and the channels are DECOUPLED*

Computation of MIMO Capacity - 2

$$\mathbf{r}[m] = \mathbf{H}\mathbf{s}[m] + \mathbf{w}[m]$$

$$= \mathbf{U}\mathbf{D}\mathbf{V}^{\dagger}\mathbf{s}[m] + \mathbf{w}[m] = \mathbf{U}\mathbf{D}(\mathbf{V}^{\dagger}\mathbf{s}[m]) + \mathbf{w}[m] = \mathbf{U}\mathbf{D}\mathbf{s}'[m] + \mathbf{w}[m]$$

deterministic invertible transformation NOT changing capacity

$$\mathbf{r}'[m] = \mathbf{U}^{\dagger}\mathbf{r}[m] = \mathbf{U}^{\dagger}\left(\mathbf{U}\mathbf{D}\mathbf{s}'[m] + \mathbf{w}[m]\right) = \mathbf{D}\mathbf{s}'[m] + \mathbf{w}'[m]$$

Same as above… the equivalent channel boils down to

$$r_i'[m] = d_i s'[m] + w_i'[m], \; i = 1, \ldots, N_T$$

Computation of MIMO Capacity - 3

$$r'[m] = d_i s'[m] + w'[m], \;\; i = 1,\ldots, N_T$$

I am left with an equivalent set of N_T independent (parallel) channels whose capacity is evaluated via the Shannon formula for the AWGN channel

$$c_i = \log\left(1 + SNR_k\right) \quad , \quad SNR_i = \frac{d_i^{\,2} S_i}{2\sigma_i^2} \quad , \quad i = 1,\ldots, N_T$$

$$C = \sum_{i=1}^{N_T} \log_2\left(1 + SNR_i\right) = \sum_{i=1}^{N_T} \log_2\left(1 + \frac{d_i^{\,2} S_2}{2\sigma_w^2}\right)$$

MIMO Capacity – no TX Feedback

On each antenna, $S_2/2 = P_k = P_{TOT}/N_T$

$$C = \sum_{i=1}^{N_T} \log_2\left(1+\frac{d_i^2 P_{TOT}}{\sigma^2 N_T}\right) = \log_2 \prod_{i=1}^{N_T}\left(1+\frac{d_i^2 P_{TOT}}{\sigma^2 N_T}\right)$$

BUT

$$d_i^2 = \mathrm{eigen}_i(\mathbf{H}^\dagger\mathbf{H}) \Rightarrow 1+\frac{d_i^2 P_{TOT}}{\sigma^2 N_T} = \mathrm{eigen}_i\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^\dagger\mathbf{H}\right)$$

$$\boxed{C = \log_2 \prod_{i=1}^{N_T} \mathrm{eigen}_i\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^\dagger\mathbf{H}\right) = \log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^\dagger\mathbf{H}\right)}$$

MIMO Capacity – no TX Feedback

On each antenna, $S_2/2 = P_k = P_{TOT}/ N_T$

$$C = \sum_{i=1}^{N_T} \log_2\left(1+\frac{d_i^2 P_{TOT}}{\sigma^2 N_T}\right) = \log_2 \prod_{i=1}^{N_T}\left(1+\frac{d_i^2 P_{TOT}}{\sigma^2 N_T}\right)$$

BUT

$$d_i^2 = \mathrm{eigen}_i(\mathbf{H}\mathbf{H}^\dagger) \Rightarrow 1+\frac{d_i^2 P_{TOT}}{\sigma^2 N_T} = \mathrm{eigen}_i\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^\dagger\mathbf{H}\right)$$

$$\boxed{C = \log_2 \det\left(\mathbf{I}_{N_T} + \frac{E_s}{N_0}\frac{1}{\left(\|\mathbf{H}\|_2\right)^2}\mathbf{H}^\dagger\mathbf{H}\right) \quad \text{bit/c.u.} \quad N_T \le N_R}$$

Computation of MIMO Capacity – 1 Again

Compute the SVD of H: $\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^{\dagger}$

Eigenvectors of $\mathbf{HH}^{\dagger}$ $\qquad \mathrm{diag}\{d_i\}$ $\qquad$ *Eigenvectors of* $\mathbf{H}^{\dagger}\mathbf{H}$

N_R $\quad \mathbf{H} \quad$ = $\quad$ *Unitary Matrix* $\mathbf{UU}^{\dagger} = \mathbf{I}_{N_R}$ $\mathbf{HH}^{\dagger}$ $\quad$ 0 $\quad$ 0 $\quad \mathbf{D}$ $\quad$ *Unitary Matrix* $\mathbf{VV}^{\dagger} = \mathbf{I}_{N_T}$ $\quad N_T$

$N_T \qquad N_R$

$N_R > N_T$, *uplink-like*

are the N_T $\sqrt{\mathrm{eigen}(\mathbf{H}^{\dagger}\mathbf{H})}$ *and the channels are DECOUPLED*

An Example

$$\mathbf{H} = \begin{bmatrix} 0.1 & -0.09 \\ -0.09 & 0.1 \end{bmatrix} \qquad SNR_n = S_2 / 2\sigma^2 = 45\ \text{dB}$$

$$\begin{array}{ccc} \mathbf{U} & \mathbf{D} & \mathbf{V}^\dagger \end{array}$$

$$\mathbf{H} = \begin{bmatrix} -0.7071 & 0.7071 \\ -0.7071 & -0.7071 \end{bmatrix} \begin{bmatrix} 0.01 & 0 \\ 0 & 0.19 \end{bmatrix} \begin{bmatrix} -0.7071 & -0.7071 \\ 0.7071 & -0.7071 \end{bmatrix}$$

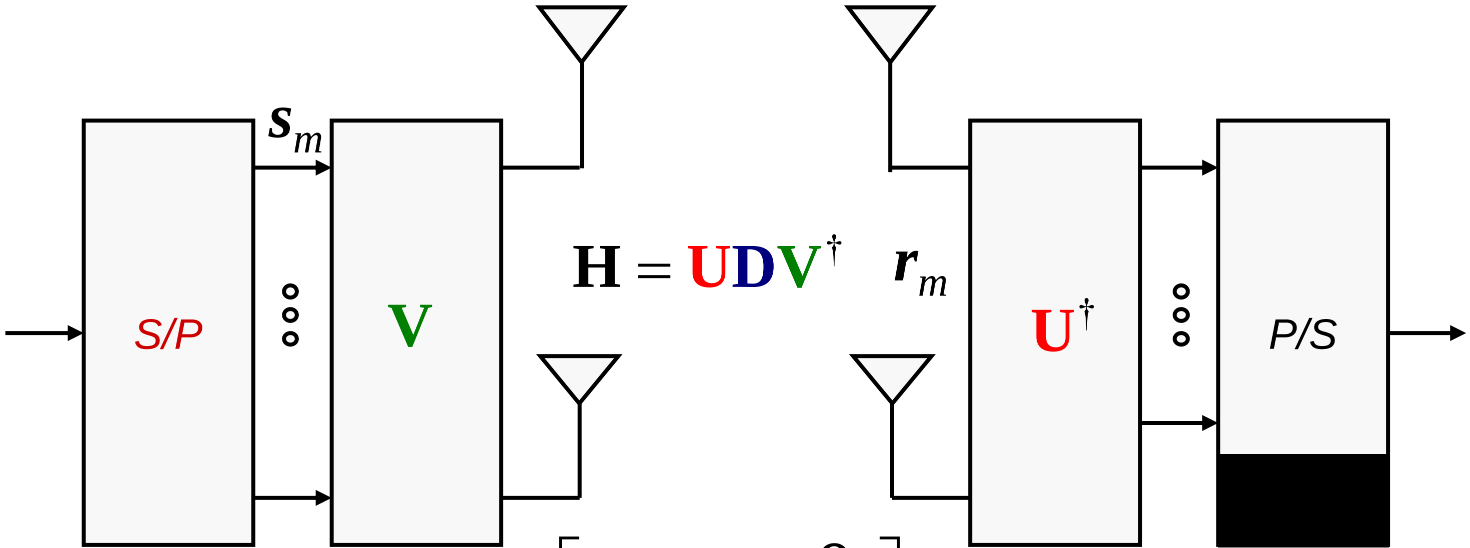
$$c_{MIMO} = \log_2\left(1 + 10^{-4}\text{SNR}_n\right) + \log_2\left(1 + 0.0361\text{SNR}_n\right) = 2.057 + 10.16 = 12.22\ \ \text{bit/c.u.}$$

With the same OVERALL TX power:

$$c_{1-to-1} = \log_2\left(1 + 0.02\text{SNR}_n\right) = 9.31\ \ \text{bit/c.u.}$$

MIMO Capacity – TX Feedback - 1

If we want to actually “experience” the parallel channels, we have to pre-code $\mathbf{s}_m$ *by* $\mathbf{V}$ *before transmitting, and then post-process* $\mathbf{r}_m$ *by* $\mathbf{U}^{\dagger}$ *disregarding the excess* N_R-N_T *useless channels – CSI is needed at the TX !*



$$D=\begin{bmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_{N_T} \end{bmatrix}$$

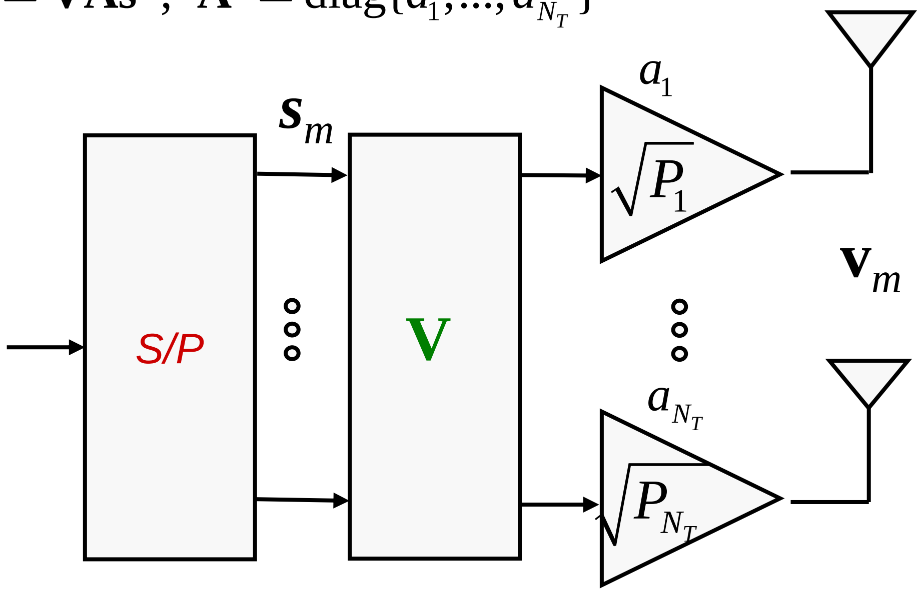
MIMO Capacity – TX Feedback - 2

Now, if the ***TX has knowledge*** *of* $\mathbf{H}$ $(\mathbf{D}$ *and* $\mathbf{V})$ *then it can allocate different power on the different «virtual channels»*

$$\mathbf{v} \triangleq \mathbf{VAs} \;,\; \mathbf{A} \triangleq \mathrm{diag}\{a_1,\ldots,a_{N_T}\}$$

Constraint:

$$\sum_{i=1}^{N_T} P_i = P_{TOT}$$



$$C = \sum_{i=1}^{N_T} \log_2\left(1 + \frac{d_i^2 P_i}{\sigma^2}\right)$$

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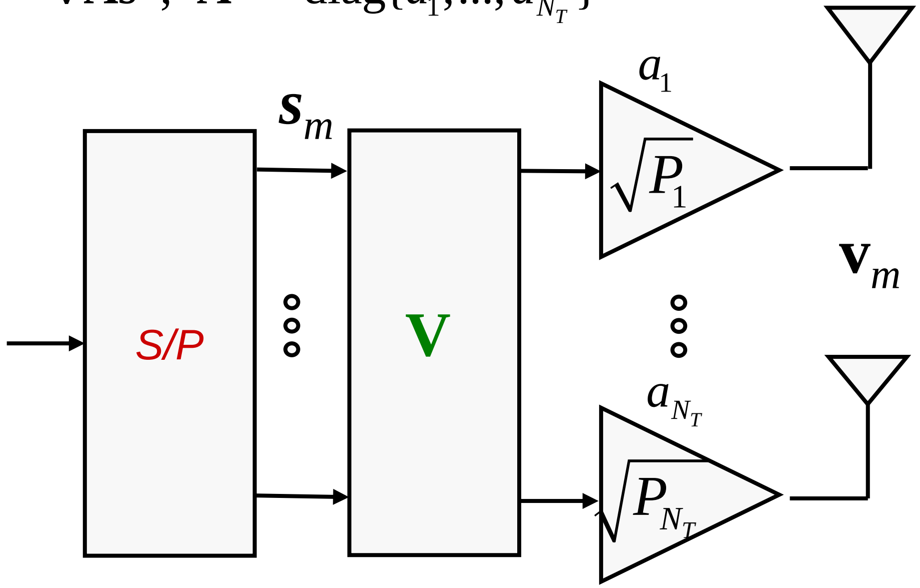
MIMO Capacity – TX Feedback - 2

*Now, if the **TX has knowledge** of* $\mathbf{H}$ ($\mathbf{D}$ *and* $\mathbf{V}$) *then it can allocate different power on the different «virtual channels» following a WATER FILLING criterion! (independent channels)*

$$\mathbf{v} \triangleq \mathbf{VAs} \;,\; \mathbf{A} \triangleq \mathrm{diag}\{a_1,\ldots,a_{N_T}\}$$

Constraint:

$$\sum_{i=1}^{N_T} P_i = P_{TOT}$$



$$C = \sum_{i=1}^{N_T} \log_2\left(1+\frac{d_i^2 P_i}{\sigma^2}\right) \;, \qquad P_i + \frac{\sigma^2}{d_i^2} = \bar{P} \quad \forall i$$

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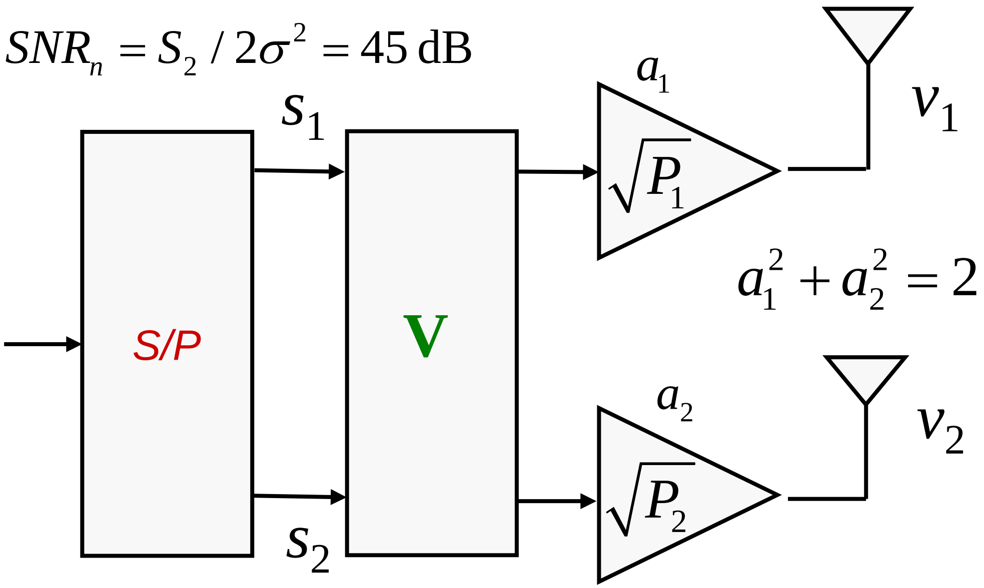
Another Example

$$SNR_n = S_2 / 2\sigma^2 = 45\ \text{dB}$$



$$\sigma_T^2 = \left(\frac{1}{d_1^2} + \frac{1}{d_2^2}\right)\sigma^2 = 10,028\sigma^2$$

$$\bar{P} = \frac{2S_2 + 10,028\sigma^2}{2} = S_2 + 5,014\sigma^2$$

$$\begin{cases} P_1 = \bar{P} - \sigma_1^2 = S_2 + 5,014\sigma_w^2 - 10,000\sigma_w^2 = S_2 - 4,986\sigma_w^2 \\ P_2 = \bar{P} - \sigma_2^2 = S_2 + 5,014\sigma_w^2 - 27.7\sigma_w^2 = S_2 + 4,986\sigma_w^2 \end{cases}$$

$$c_{MIMO,wf} = \log_2\left(\frac{d_1^2\bar{P}}{\sigma_w^2}\right) + \log_2\left(\frac{d_2^2\bar{P}}{\sigma_w^2}\right) = \log_2\left(0.5 + 2\cdot 10^{-4}\mathrm{SNR}_n\right) + \log_2\left(181 + 0.0722\mathrm{SNR}_n\right)$$

$$= 2.77 + 11.27 = 14.04\ \ \text{bit/c.u.} > c_{MIMO} = 12.22\ \ \text{bit/c.u.}$$

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Ergodic MIMO Capacity (Telatar)

$$C = \log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T} \mathbf{H}^\dagger \mathbf{H} \right)$$

The entries h_{ik} of channel matrix are actually

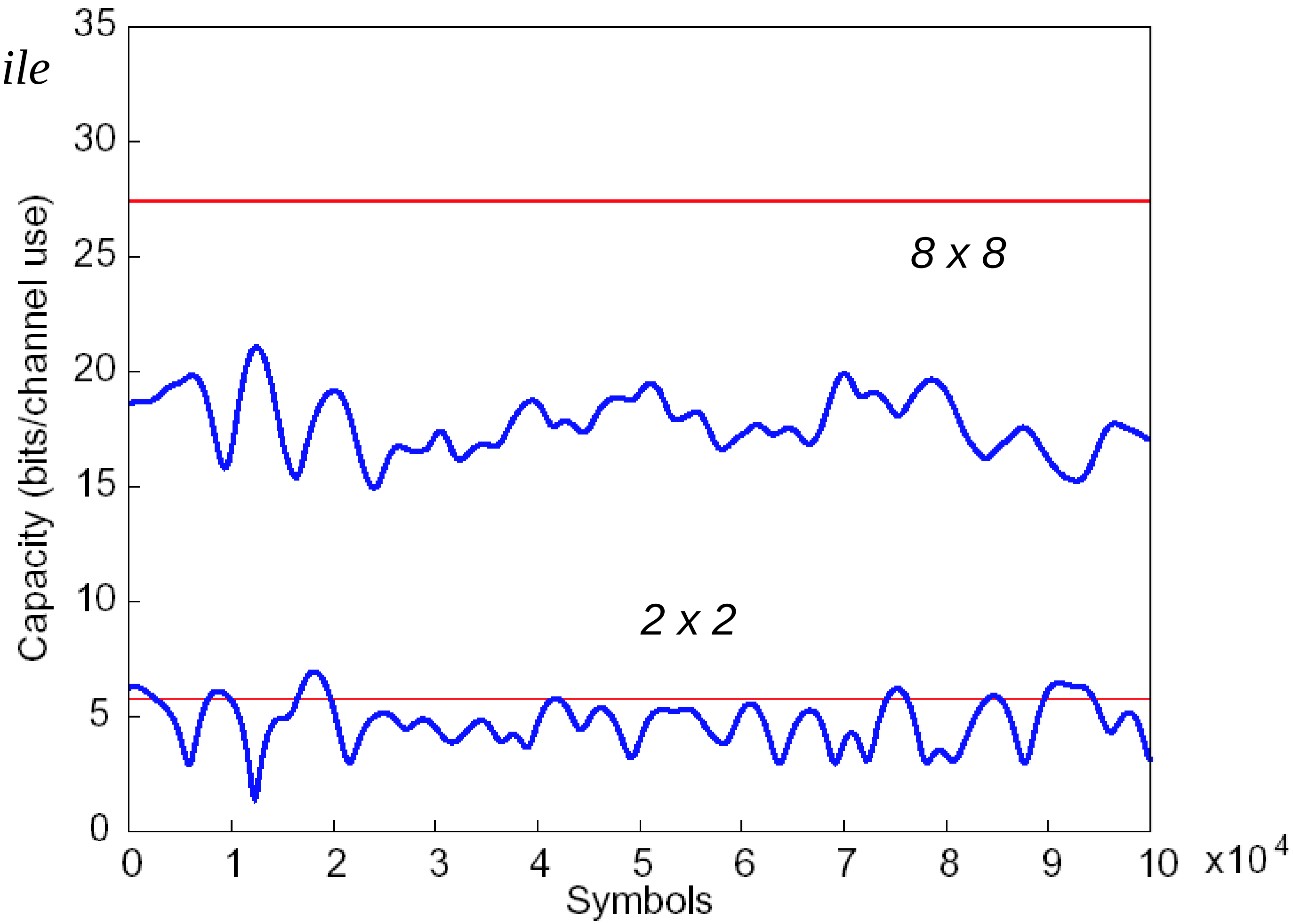
ZERO-MEAN GAUSSIAN (Rayleigh Fading) independent of each other

What we have computed up to now has to be averaged over the channel realizations:

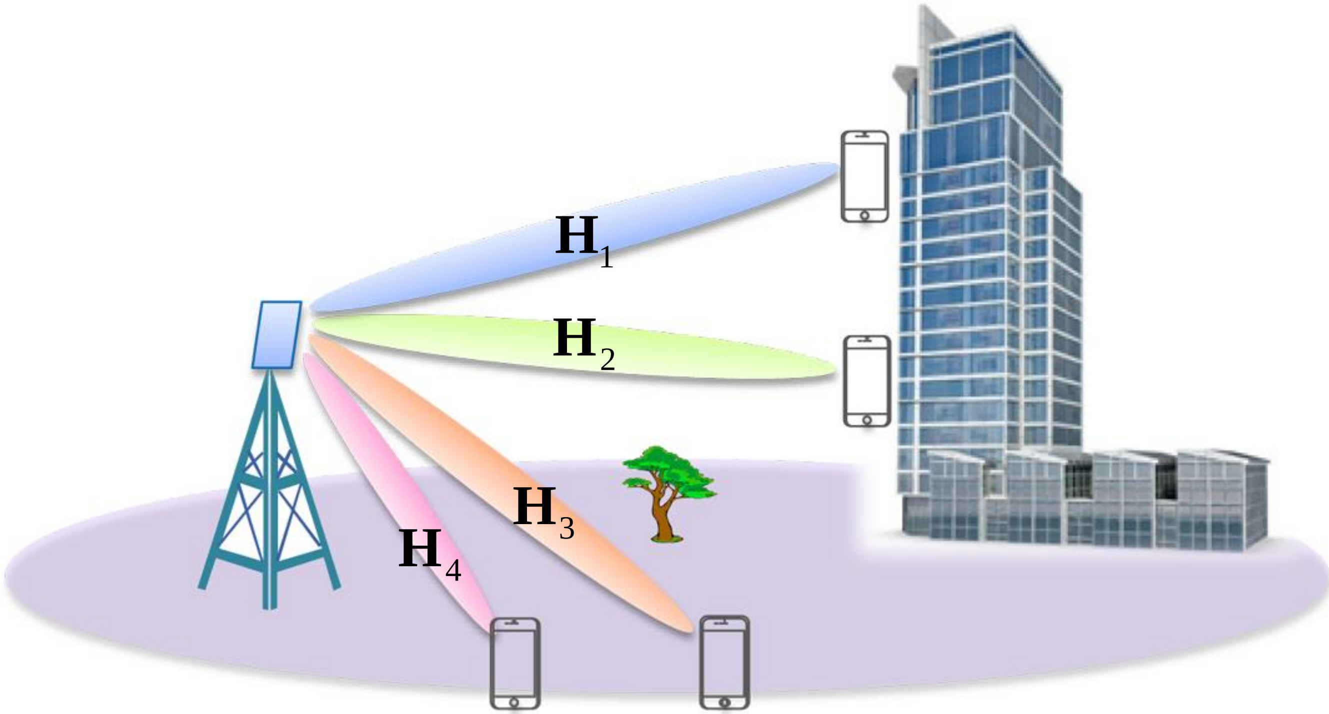
$$C_E = \mathrm{E}_{\mathbf{H}}\left\{ \log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T} \mathbf{H}^\dagger \mathbf{H} \right) \right\} \qquad N_R > N_T$$

An Example (Schlegel)

- v_{mobile} = 50 km/h - *mobile receiver speed*
- T_s = 1 μs – *sampling time*
- R = 50 m - *radius of circle containing the scatterers*
- L = 2 km – *separation of TX and RX*
- $d = 5\lambda$ - *separation of antenna elements*
- f_0 = 2.4GHz - *carrier frequency*
- E_s/N_0 = 10dB - *symbol signal to noise ratio*



MU MIMO / Space-Division Multiple Access (SDMA)



When the number of antennas is (much) larger than the number of users in the cell, we can also implement Multiple Access/Multiplexing by «synthesizing» multiple beams directed to spatially separated users: Multi-User MIMO (MU-MIMO) – Individual CSI is needed

Massive MIMO

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_R} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}\mathbf{H}^{\dagger}\right)\right\} \quad N_R < N_T$$

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^{\dagger}\mathbf{H}\right)\right\} \quad N_R > N_T$$

Lead to MASSIVE MIMO (very many antennas at the BTS or AP)

$N_T \rightarrow \infty$ *(Downlink):*

$$\left(\mathbf{H}\mathbf{H}^{\dagger}\right)_{i,\ell} \triangleq \sum_{p=1}^{N_T} h_{i,p} h_{\ell,p}^{*} = N_T \cdot \frac{1}{N_T}\sum_{p=1}^{N_T} h_{i,p} h_{\ell,p}^{*} \rightarrow N_T \mathrm{E}\{h_{i,p} h_{\ell,p}^{*}\} = N_T \delta[i-\ell]$$

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Massive MIMO

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_R} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}\mathbf{H}^{\dagger}\right)\right\} \qquad N_R < N_T$$

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^{\dagger}\mathbf{H}\right)\right\} \qquad N_R > N_T$$

Lead to MASSIVE MIMO (very many antennas at the BTS or AP)

$N_T \to \infty$ *(Downlink):* $\quad \mathbf{H}\mathbf{H}^{\dagger} \to N_T\,\mathbf{I}_{N_R}$

$$\left(\mathbf{H}\mathbf{H}^{\dagger}\right)_{i,\ell} \triangleq \sum_{p=1}^{N_T} h_{i,p}h_{\ell,p}^{*} = N_T \cdot \frac{1}{N_T}\sum_{p=1}^{N_T} h_{i,p}h_{\ell,p}^{*} \;\to\; N_T \mathrm{E}\{h_{i,p}h_{\ell,p}^{*}\} = N_T\delta[i-\ell]$$

Massive MIMO

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_R} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}\mathbf{H}^{\dagger}\right)\right\} \qquad N_R < N_T$$

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^{\dagger}\mathbf{H}\right)\right\} \qquad N_R > N_T$$

Lead to MASSIVE MIMO (very many antennas at the BTS or AP)

$N_T \rightarrow \infty$ *(Downlink):* $\quad \mathbf{H}\mathbf{H}^{\dagger} \rightarrow N_T\, \mathbf{I}_{N_R}$

$N_R \rightarrow \infty$ *(Uplink):* $\quad \mathbf{H}\mathbf{H}^{\dagger} \rightarrow N_R\, \mathbf{I}_{N_T}$

Massive MIMO

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_R} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}\mathbf{H}^\dagger\right)\right\} \qquad N_R < N_T$$

$$C_E = \mathrm{E}_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{TOT}}{\sigma^2 N_T}\mathbf{H}^\dagger\mathbf{H}\right)\right\} \qquad N_R > N_T$$

Lead to MASSIVE MIMO (very many antennas at the BTS or AP)

$N_T \to \infty$ *(Downlink):* $C_E = N_R \log_2\left(1 + \frac{P_{TOT}}{\sigma^2}\right)$ *MUX gain only*

$N_R \to \infty$ *(Uplink):* $C_E = N_T \log_2\left(1 + \frac{P_{TOT}}{\sigma^2 / (N_R / N_T)}\right)$ *MUX and ARR gain*

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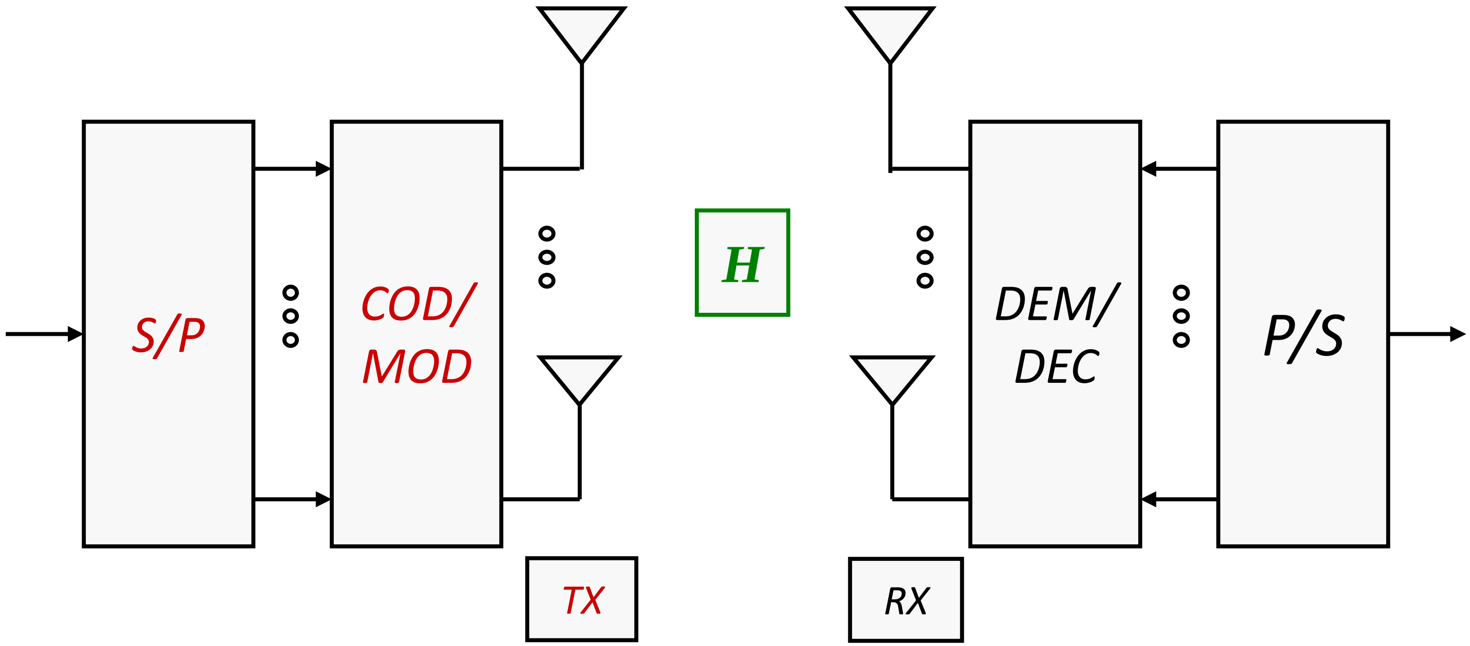
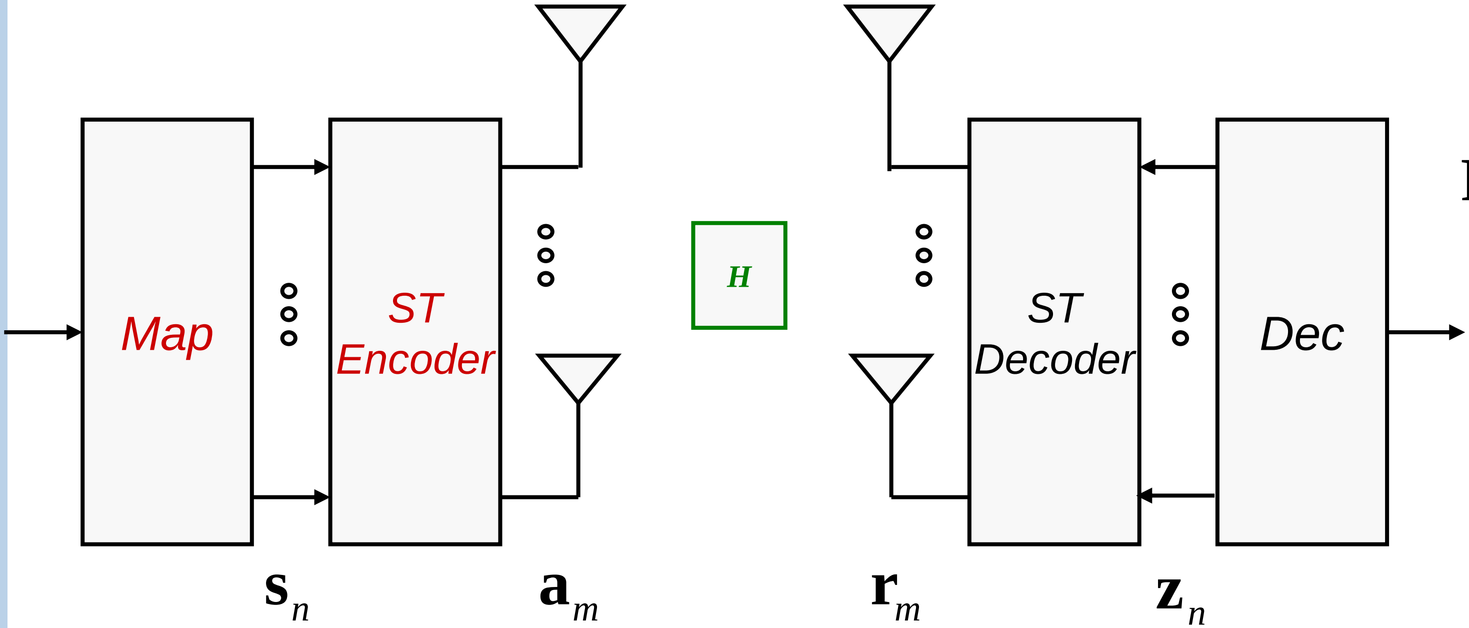
Time-, Space-, and Space-Time Coding



*Redundant (parity-check symbols) can be added as usual in **time** (interleaved with information symbols), or in **space**, by interleaving them on the TX antennas with information symbols, or BOTH: **space-time codes**.*

Space-Time Block Codes



$$\mathbf{R} = \mathbf{H}\mathbf{A} + \mathbf{W}$$

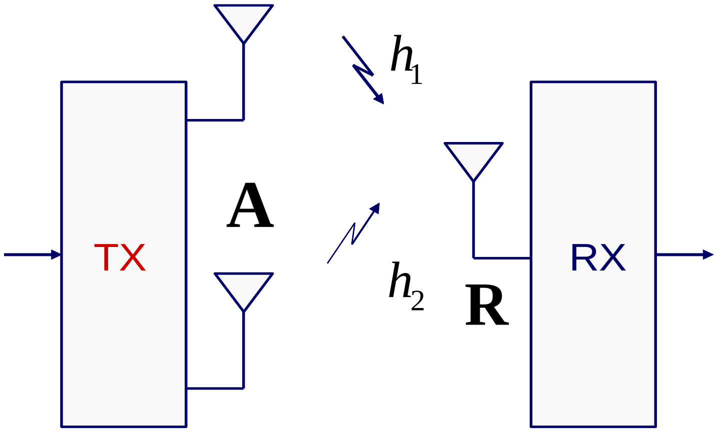
$$\mathbf{A} \triangleq \left[\mathbf{a}[0], \mathbf{a}[1], \ldots, \mathbf{a}[N-1]\right] = \begin{bmatrix} a_1[0] & a_1[1] & \cdots & a_1[N-1] \\ a_2[0] & a_2[1] & \cdots & a_2[N-1] \\ \vdots & \vdots & \ddots & \vdots \\ a_{N_T}[0] & a_{N_T}[1] & \cdots & a_{N_T}[N-1] \end{bmatrix}$$

Space-Time Codeword

TIME

SPACE

The mother-of-all ST Block Codes (Alamouti)



$$\mathbf{A} = \begin{bmatrix} s_1[0] & s_2^*[0] \\ s_2[0] & -s_1^*[0] \end{bmatrix}$$

Designed for a N_R=1 x N_T=2 MIMO system ***(single RX antenna !!!)***

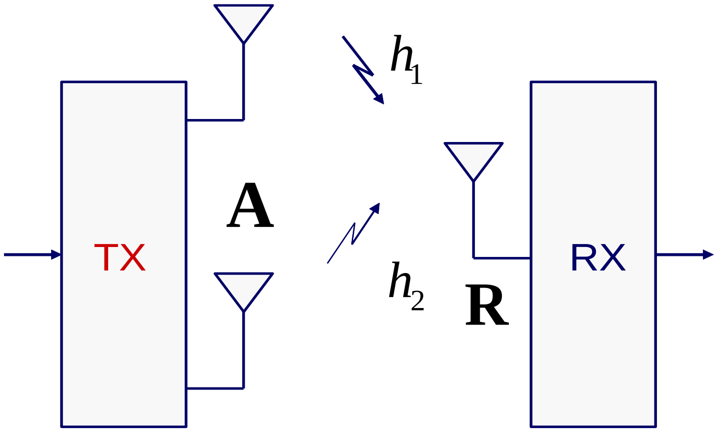
Coding rate: 1 in space, ½ in time, overall ½

BUT no bandwdith increase wrt 1 x 1 system with no coding

$$\mathbf{R} = \begin{bmatrix} r[0] & r[1] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} s_1[0] & s_2^*[0] \\ s_2[0] & -s_1^*[0] \end{bmatrix} + \begin{bmatrix} w[0] & w[1] \end{bmatrix}$$

Variance of noise terms: $2\sigma_1{}^2 = 2\sigma_2{}^2 = 2\sigma$

The mother-of-all ST Block Codes (Alamouti)



$$\mathbf{A} = \begin{bmatrix} s_1[0] & s_2^*[0] \\ s_2[0] & -s_1^*[0] \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} r[0] & r[1] \end{bmatrix}$$

Designed for a N_R=1 x N_T=2 MIMO system ***(single RX antenna !!!)***

Coding rate: 1 in space, ½ in time, overall ½

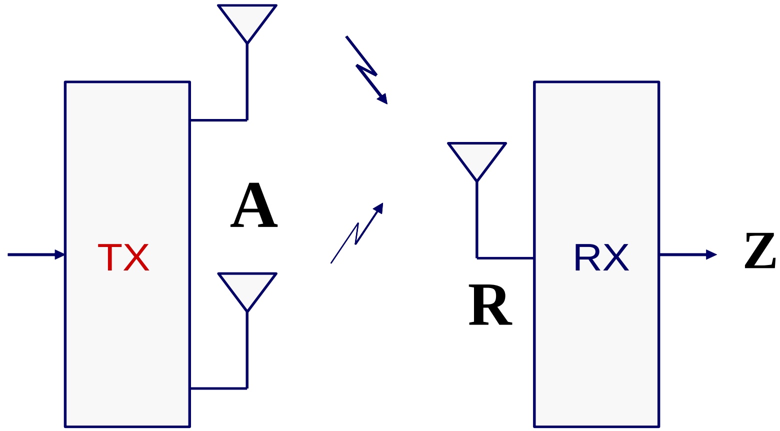
BUT no bandwidth increase wrt 1 x 1 system with no coding

$$\mathbf{R} = \begin{bmatrix} r[0] & r[1] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} s_1[0] & s_2^*[0] \\ s_2[0] & -s_1^*[0] \end{bmatrix} + \begin{bmatrix} w[0] & w[1] \end{bmatrix}$$

$$= \left[\left(h_1 s_1[0] + h_2 s_2[0] \right) + w[0] \quad \left(h_1 s_2^*[0] - h_2 s_1^*[0] \right) + w[1] \right]$$

Decoding the Alamouti code

The decoder needs (perfect) CSI to compute the sufficient statistics (soft symbols) $\mathbf{Z}$ for data detection



$$\mathbf{z}^T = \begin{bmatrix} z_1[0] & z_2[0] \end{bmatrix} = \begin{bmatrix} r[0] & r^*[1] \end{bmatrix} \begin{bmatrix} h_1^* & h_2^* \\ -h_2 & h_1 \end{bmatrix} = \begin{bmatrix} h_1^* r[0] - h_2 r^*[1] & h_2^* r[0] + h_1 r^*[1] \end{bmatrix}$$

$$= \begin{bmatrix} \left(|h_1|^2 + |h_2|^2\right) s_1[0] + w'[0] & \left(|h_1|^2 + |h_2|^2\right) s_2[0] + w'[1] \end{bmatrix}$$

where

$$w'[0] = w[0] h_1^* - w[1]^* h_2 \qquad w'[1] = w[0] h_2^* + w[1]^* h_1$$

Decoding the Alamouti code

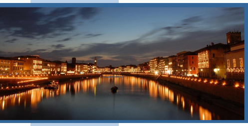
The decoder needs (perfect) CSI to compute the sufficient statistics (soft symbols) $\mathbf{Z}$ for data detection

$$\mathbf{z}^T = \left[z_1[0] \quad z_2[0] \right] = \left[r[0] \quad r^*[1] \right] \begin{bmatrix} h_1^* & h_2^* \\ -h_2 & h_1 \end{bmatrix} = \left[h_1^* r[0] - h_2 r^*[1] \quad h_2^* r[0] + h_1 r^*[1] \right]$$

$$= \left[\left(|h_1|^2 + |h_2|^2 \right) s_1[0] + n_1[0] \quad \left(|h_1|^2 + |h_2|^2 \right) s_2[0] + n_2[0] \right]$$

and the new I/Q noise variances are

$$\sigma_{n_1}{}^2 = \sigma_{n_2}{}^2 = \sigma^2 \left(|h_1|^2 + |h_2|^2 \right)$$



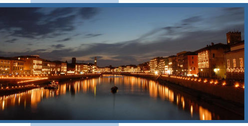
Where is the gain ?

- ***The signal power increases by*** $\left(|h_1|^2+|h_2|^2\right)^2$
- ***The noise power increases by*** $|h_1|^2+|h_2|^2$
- ***There are no CROSS TERMS between channels !***

$$SNR_1=\frac{S_2|h_1|^2}{2\sigma^2} \qquad SNR_1=\frac{S_2|h_2|^2}{2\sigma^2}$$

$$SNR_{Ala}=\frac{S_2\left(|h_1|^2+|h_2|^2\right)^2}{2\sigma^2\left(|h_1|^2+|h_2|^2\right)}=\frac{S_2\left(|h_1|^2+|h_2|^2\right)^2}{2\sigma^2}=SNR_1+SNR_2$$

- *JUST LIKE RX DIVERSITY ! (BUT what with the same TX power ?)*



With two RX antennas (2 x 2 code)

TIME

SPACE

$$\mathbf{R}=\begin{bmatrix} r_1[0] & r_1[1] \\ r_2[0] & r_2[1] \end{bmatrix}=\begin{bmatrix} h_{1,1}s_1[0]+h_{1,2}s_2[0] & h_{1,1}s_2^*[0]-h_{1,2}s_1^*[0] \\ h_{2,1}s_1[0]+h_{2,2}s_2[0] & h_{2,1}s_2^*[0]-h_{2,2}s_1^*[0] \end{bmatrix}+\mathbf{W}$$

$\mathbf{Z}_1$ *from RX antenna 1 only* $\qquad$ $\mathbf{Z}_2$ *from RX antenna 2 only*

$$\mathbf{z}^T=\begin{bmatrix} z_1[0] & z_2[0] \end{bmatrix}=\begin{bmatrix} r_1[0] & r_1^*[1] \end{bmatrix}\begin{bmatrix} h_{11}^* & h_{12}^* \\ -h_{12} & h_{11} \end{bmatrix}+\begin{bmatrix} r_2[0] & r_2^*[1] \end{bmatrix}\begin{bmatrix} h_{21}^* & h_{22}^* \\ -h_{22} & h_{21} \end{bmatrix}$$

RX diversity on top of TX diversity

Show that we have FOUR-FOLD array gain (but no mux gain)

$$SNR_{Ala}=SNR_{11}+SNR_{21}+SNR_{12}+SNR_{22}$$