MSc Course in *Cybersecurity*

Digital Signals

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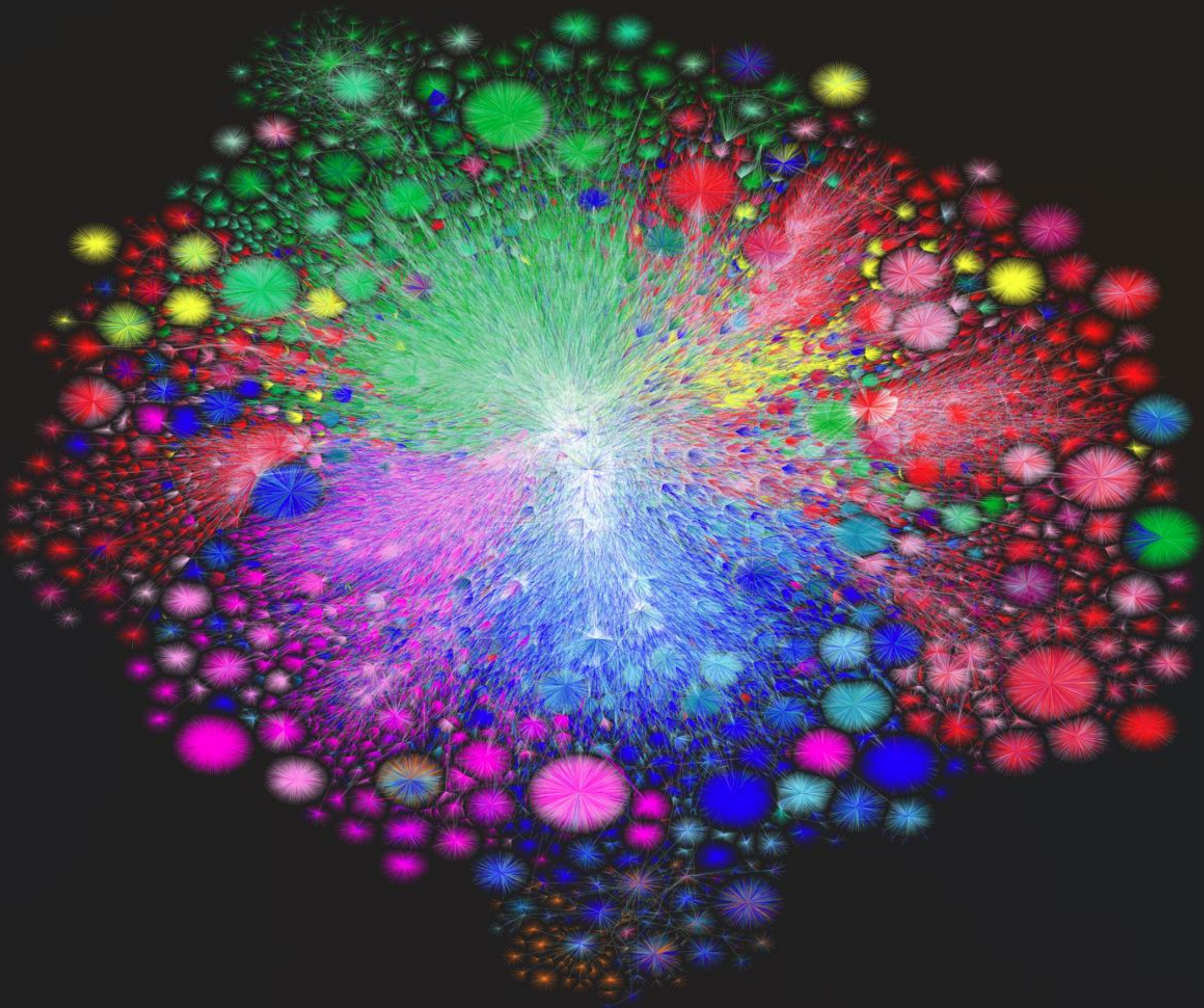
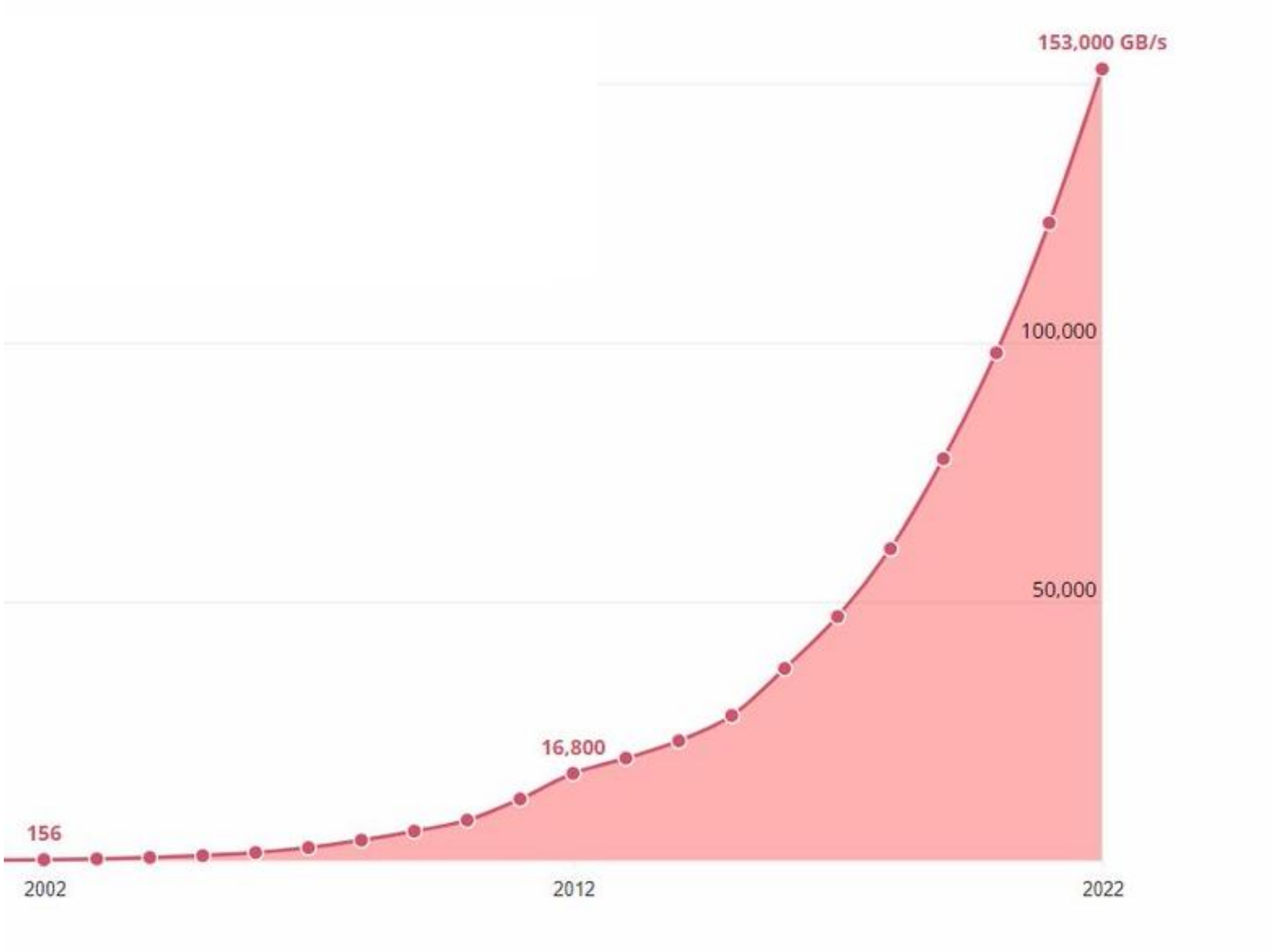
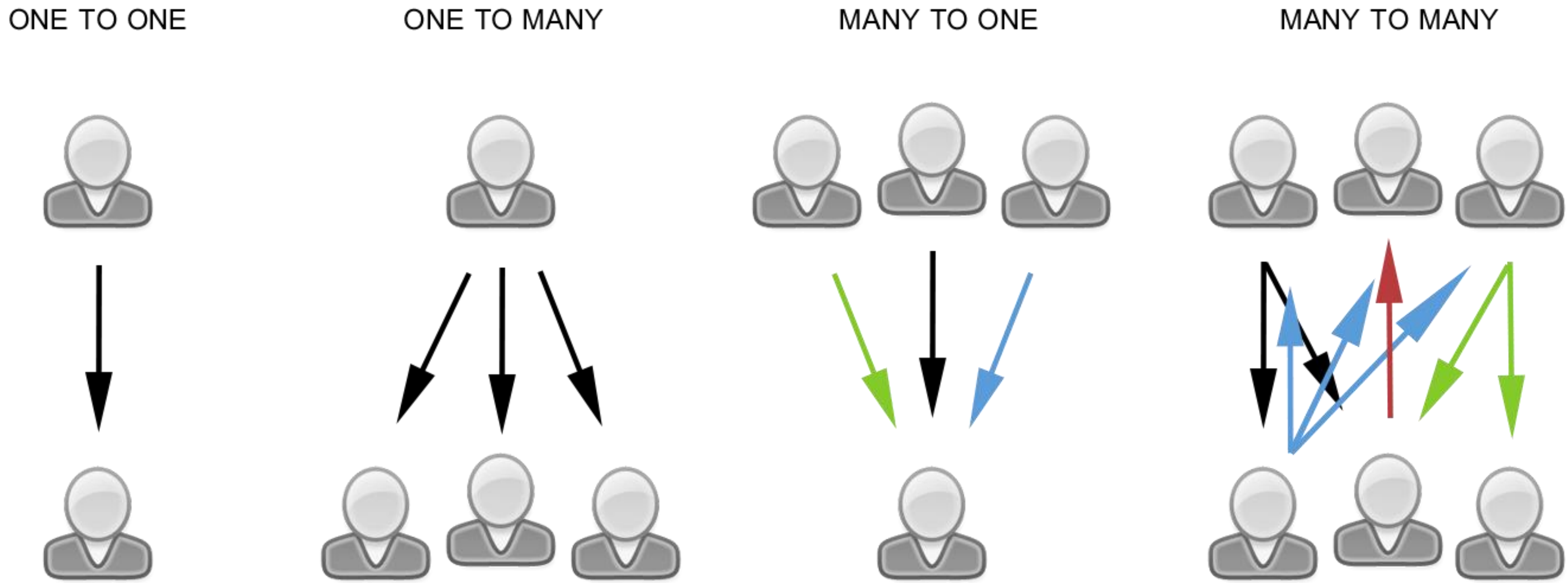


https://www.opte.org/

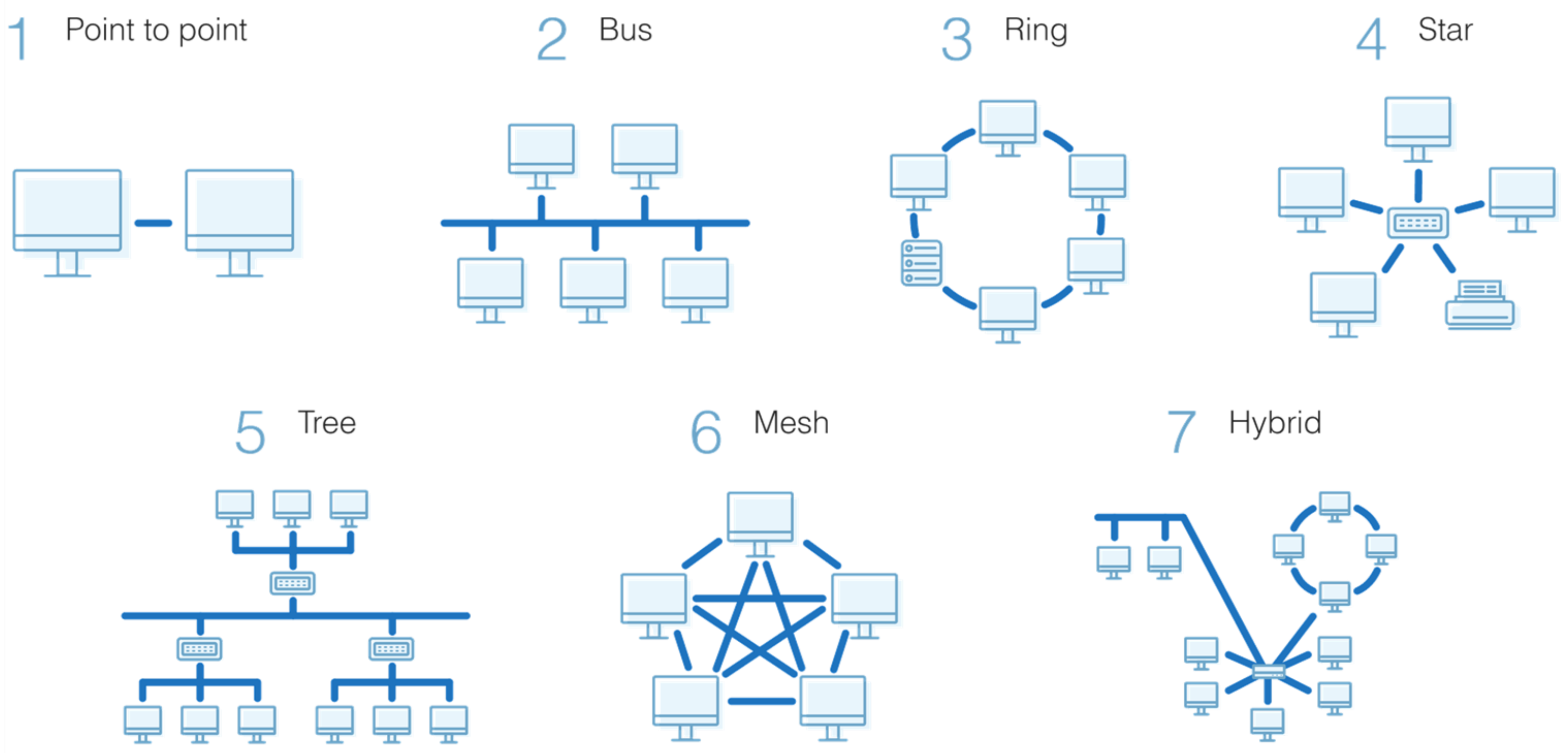
Global Internet Traffic



Communications…



Communication Networks Architecture



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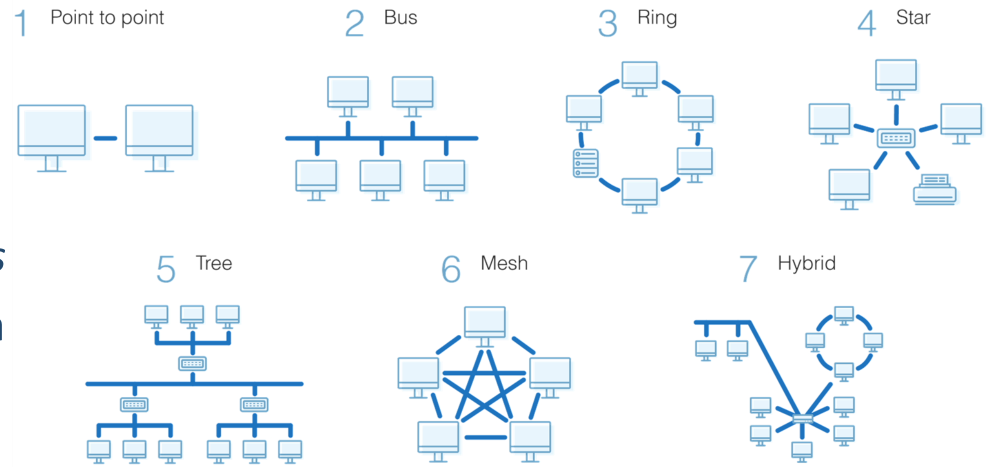
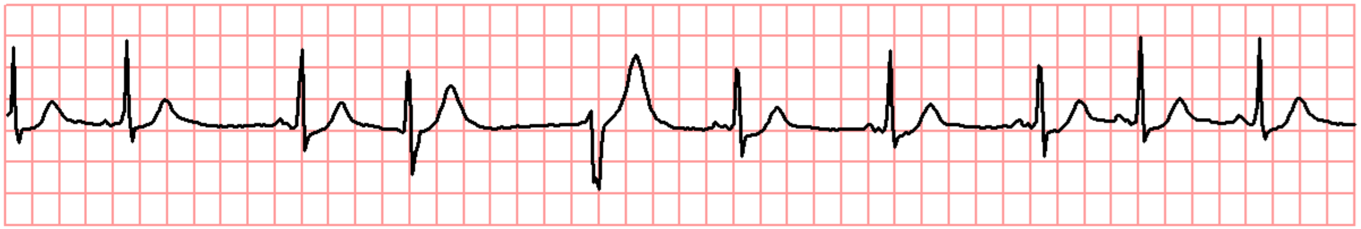
Point-to-Point Multimedia Digital Communications

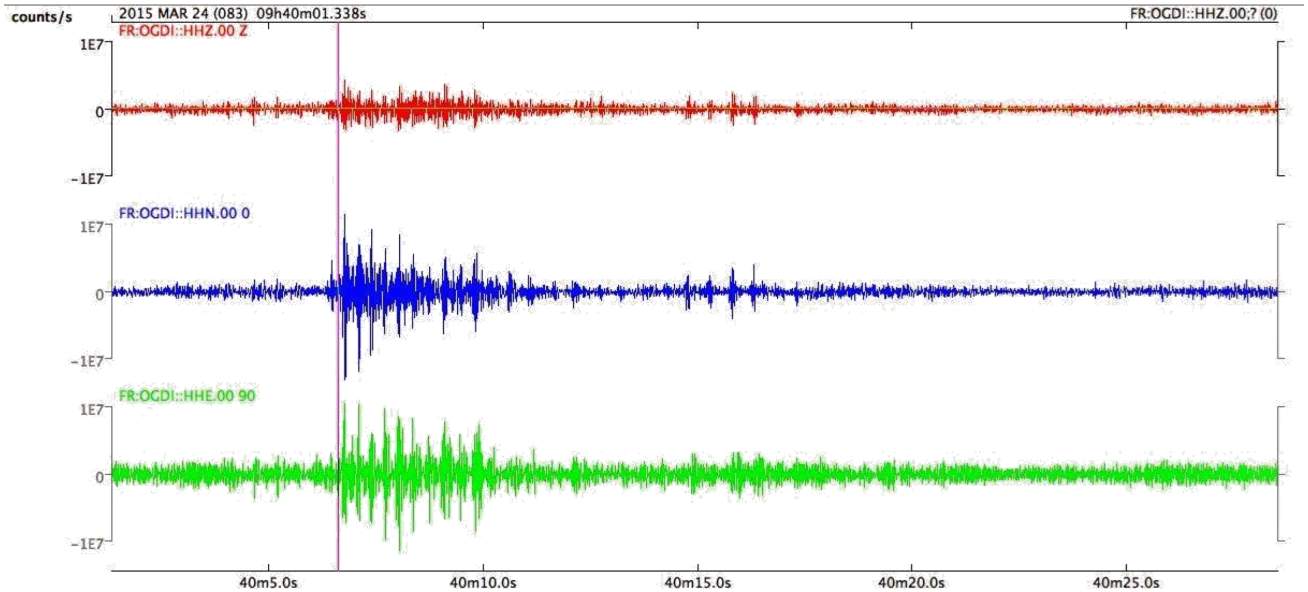


- Any network topology is made of a collection of point-to-point digital *links*
- The links are implemented exploiting a *physical medium*
- Different physical media require different technologies
  – Wireless
  – Copper
  – Fibers
- Whatever the medium, any link transfers *digital information* carrying *multimedia messages* (audio, images, video, text, pure data)
- How can everything be reduced to the same *digital format* ?

Signals…

A signal is *any variable physical quantity* that carries a certain amount of *information*, that makes the signal itself «interesting»





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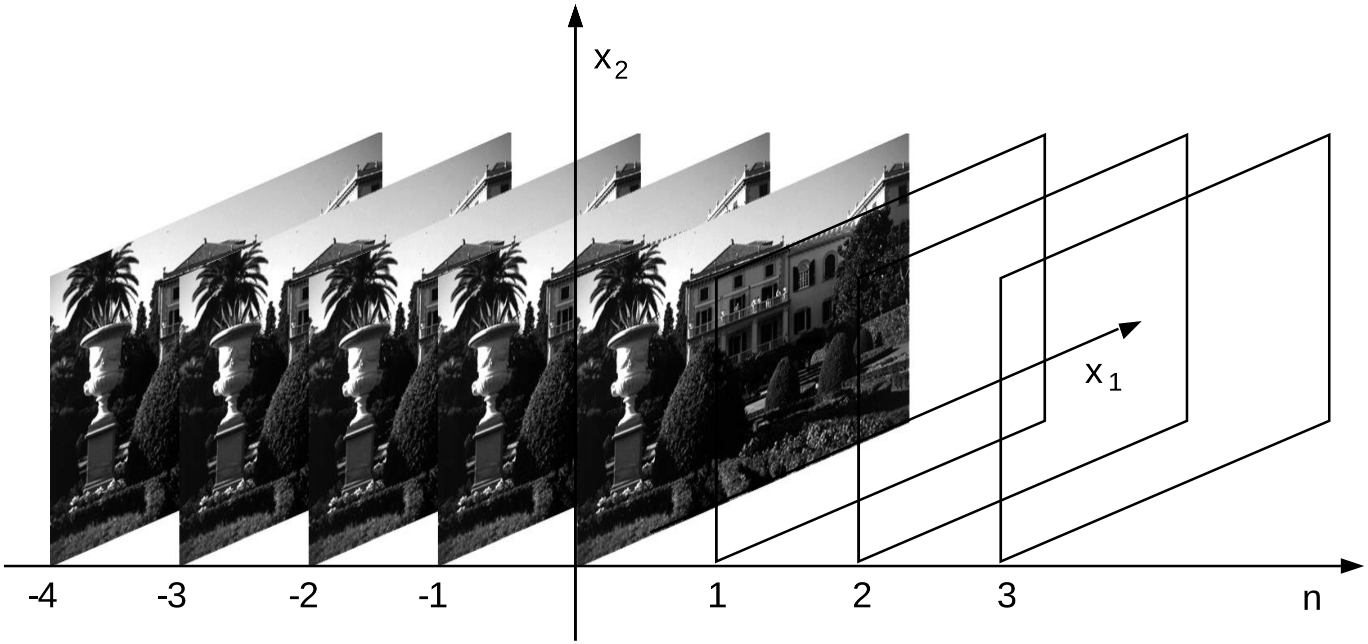
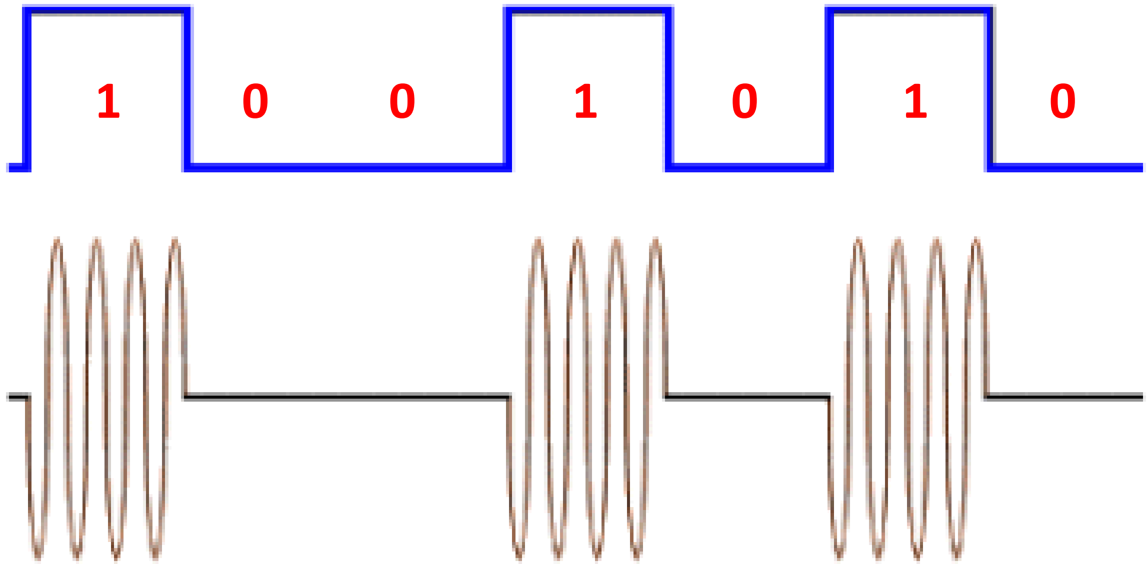
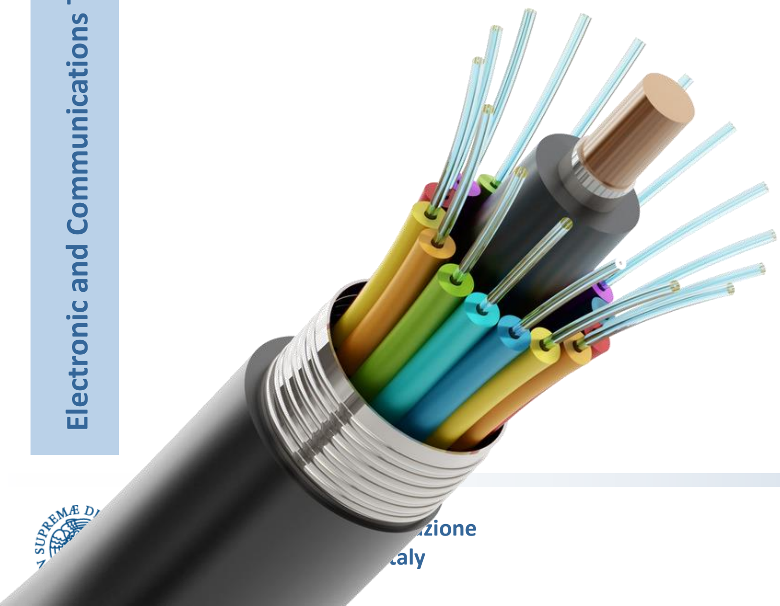


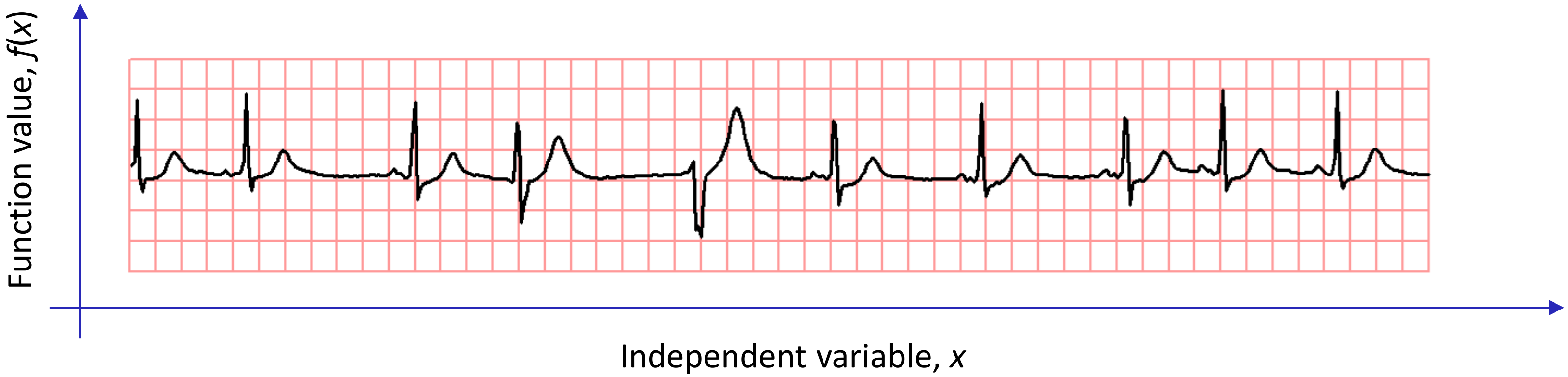
Signals…



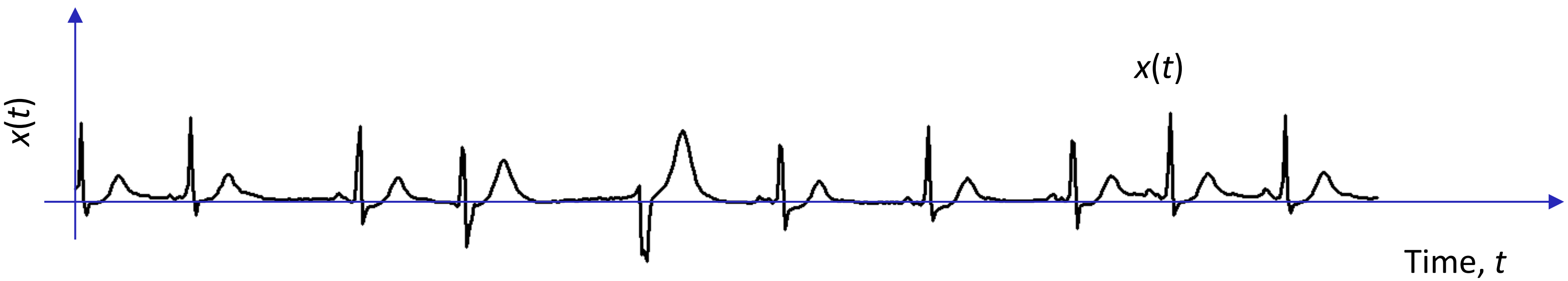
194 THz Frequency – 1.55 µm wavelength



Mathematical representation of a signal

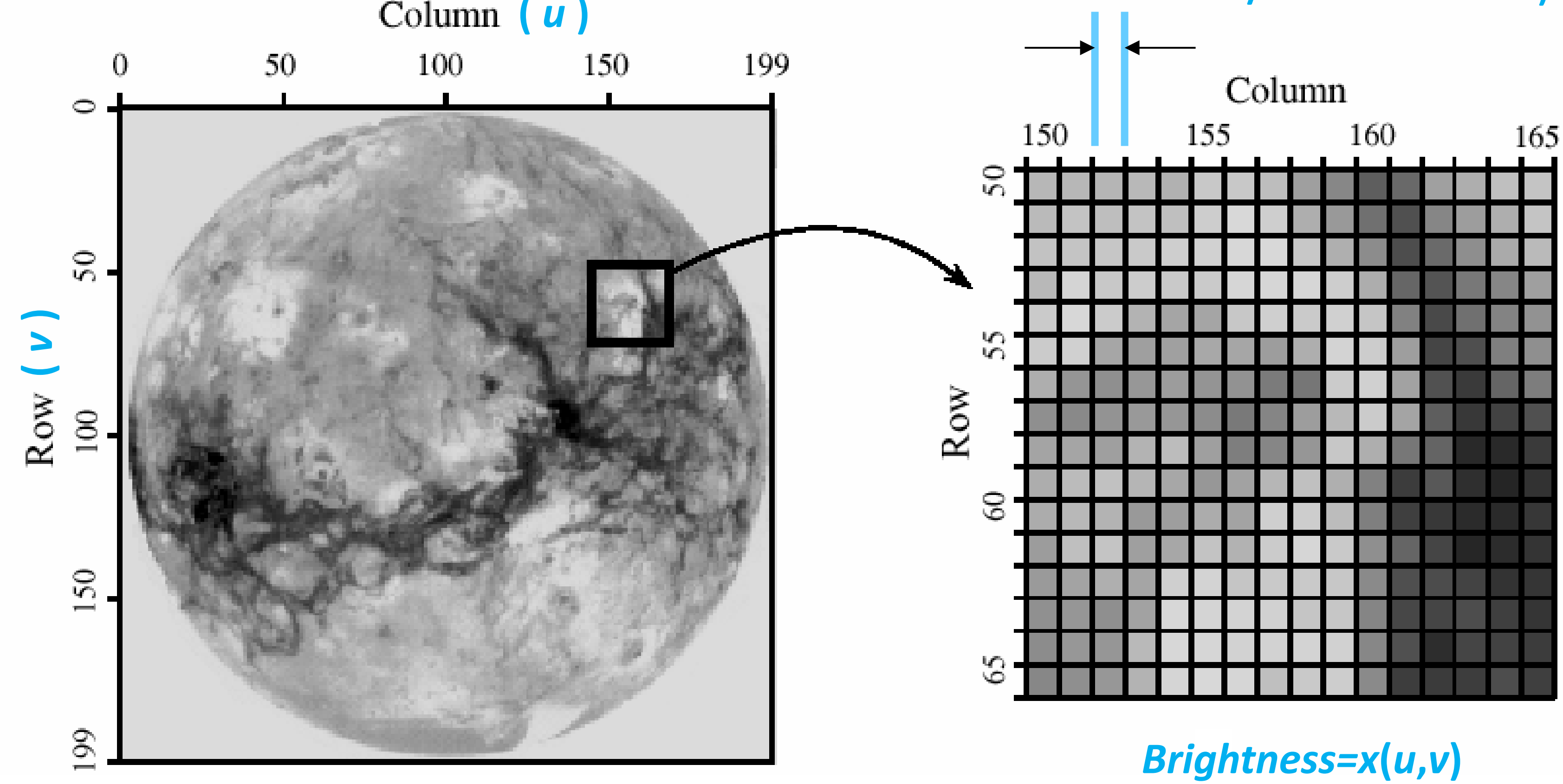


- The "signal" is the value of a mathematical function of a certain independent variable
- Here (ECG), the independent variable is *time*
- We will reserve the letter *f* for the *frequency*, and we will call the signal *x, y*, *z*



B/W Satellite Image for Weather Forecast

Sampling interval or RESOLUTION (e.g. 300 dots/inch=0.0847 mm)

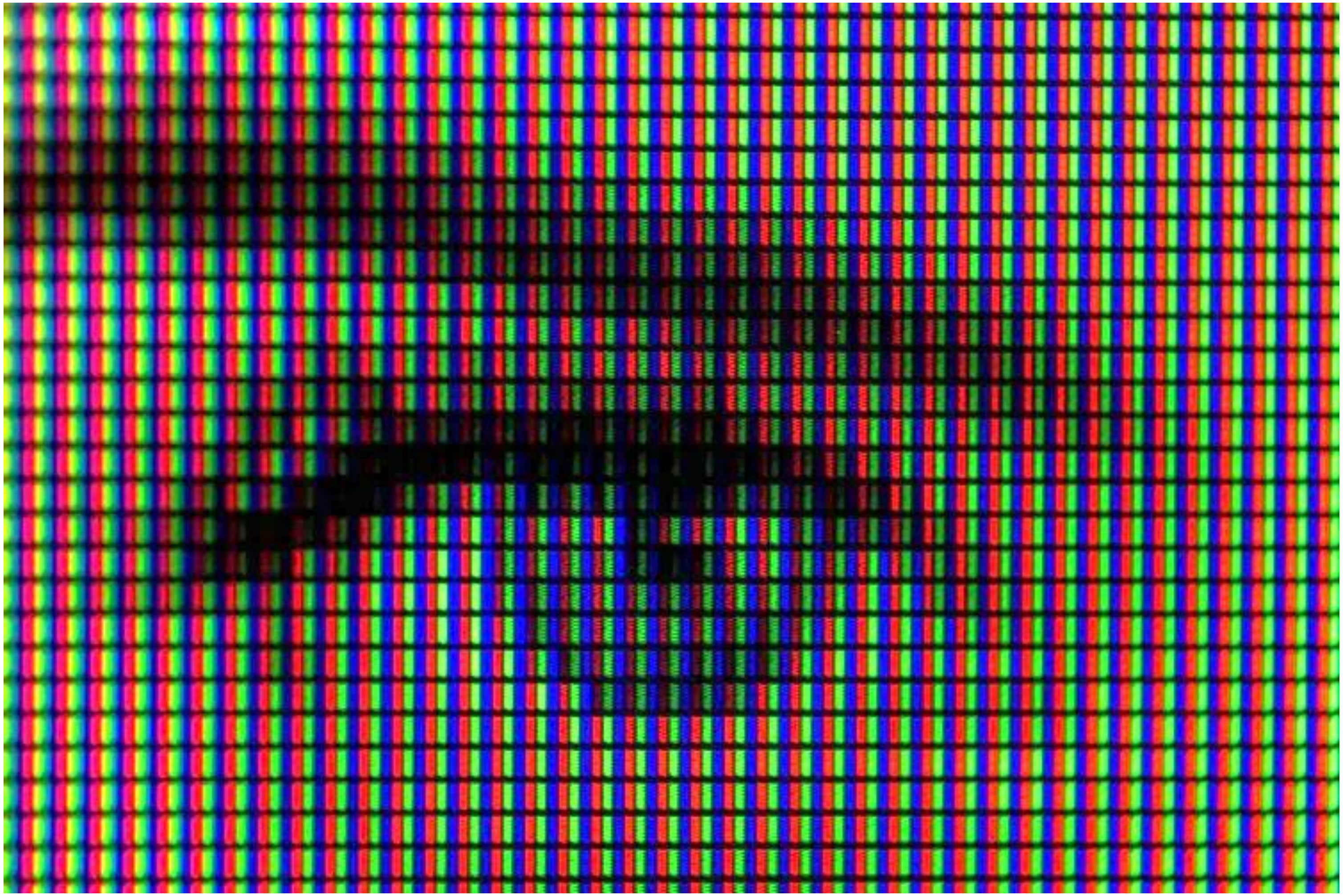


Brightness=x(u,v)

Creating PIXELS: «Sampling» the values of the image along the two u-v SPACE dimensions

Color Image…

… = 3 x monochromatic RGB images



B/W Original image: digital, but you cannot perceive it

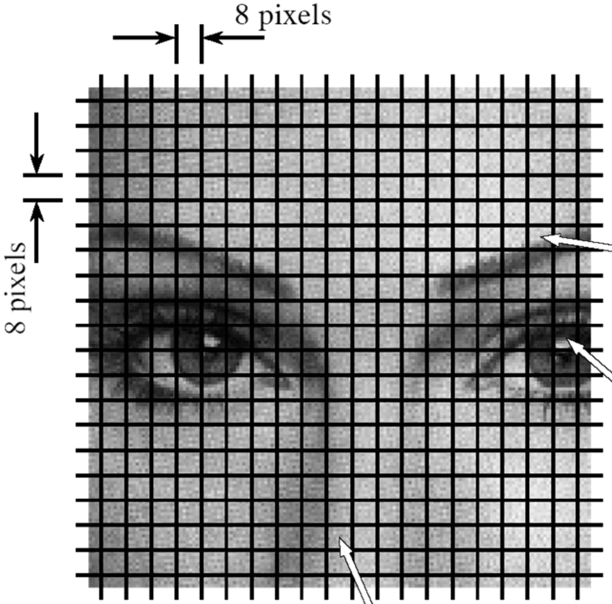
Sampling: (too) Low resolution

Quantization: Different numbers of bits/sample

4 bits/pixel

2 bits/pixel

Image segments (JPEG)

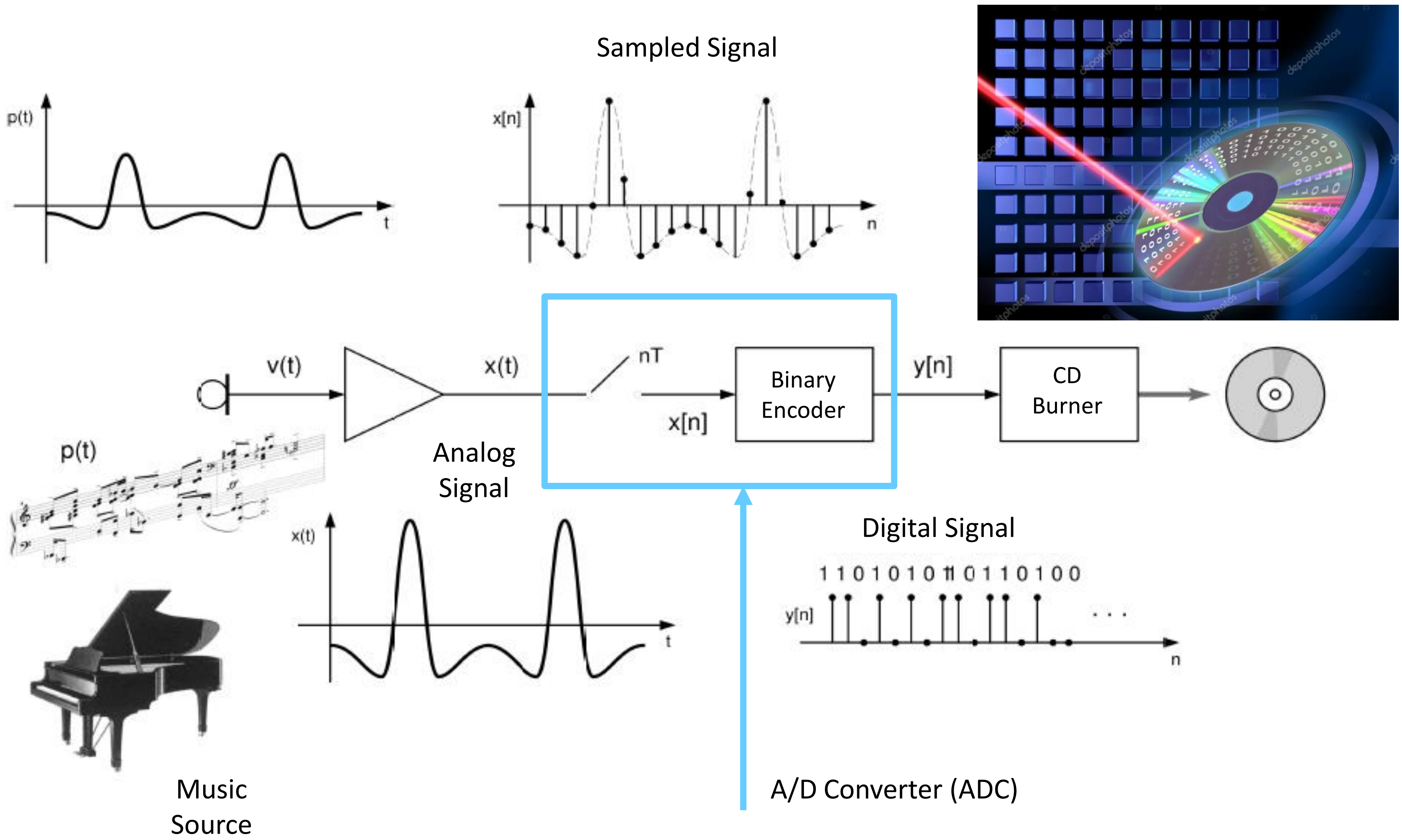


231	224	224	217	217	203	189	196
210	217	203	189	203	224	217	224
196	217	210	224	203	203	196	189
210	203	196	203	182	203	182	189
203	224	203	217	196	175	154	140
182	189	168	161	154	126	119	112
175	154	126	105	140	105	119	84
154	98	105	98	105	63	112	84

42	28	35	28	42	49	35	42
49	49	35	28	35	35	35	42
42	21	21	28	42	35	42	28
21	35	35	42	42	28	28	14
56	70	77	84	91	28	28	21
70	126	133	147	161	91	35	14
126	203	189	182	175	175	35	21
49	189	245	210	182	84	21	35

154	154	175	182	189	168	217	175
154	147	168	154	168	168	196	175
175	154	203	175	189	182	196	182
175	168	168	168	140	175	168	203
133	168	154	196	175	189	203	154
168	161	161	168	154	154	189	189
147	161	175	182	189	175	217	175
175	175	203	175	189	175	175	182

Digital Music Recording



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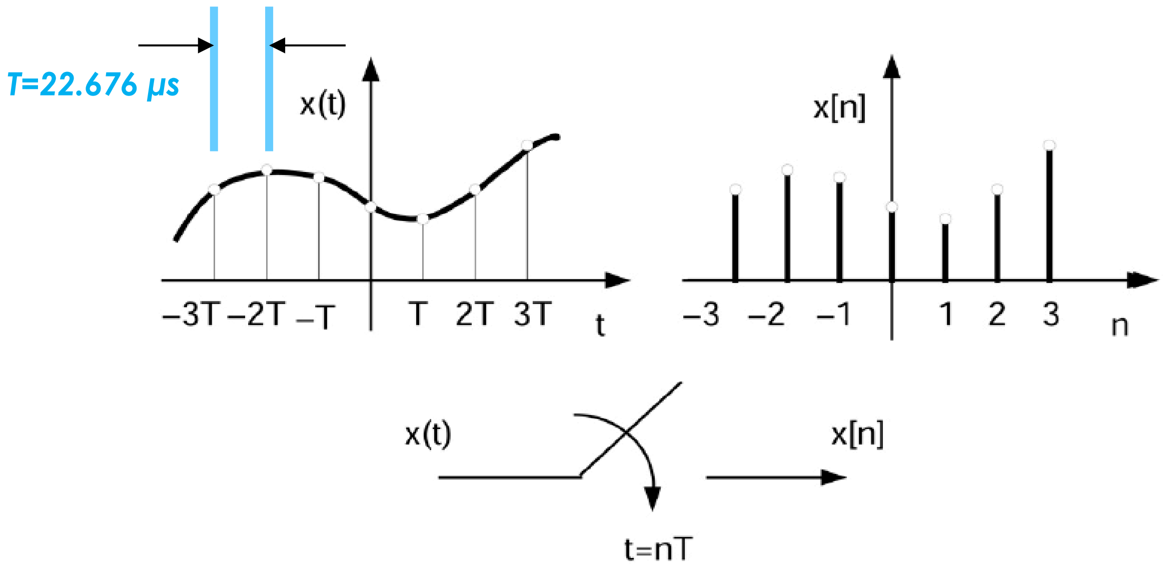
Sampling an analog signal



Sampling an analog signal $x(t)$ means collecting one after the other the sequence $x[n]$ of values of that signal at the time instants nT, i.e, taken at the rate f_c=1/T that is called the *sampling frequency* (Hz or samples/s)

Audio recording: f_c=44.1 kHz, T=22.676 μs

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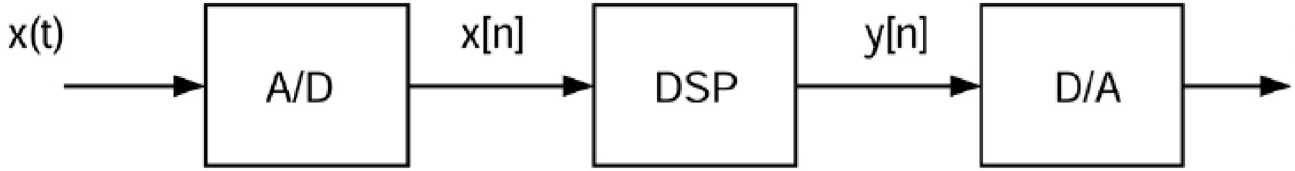
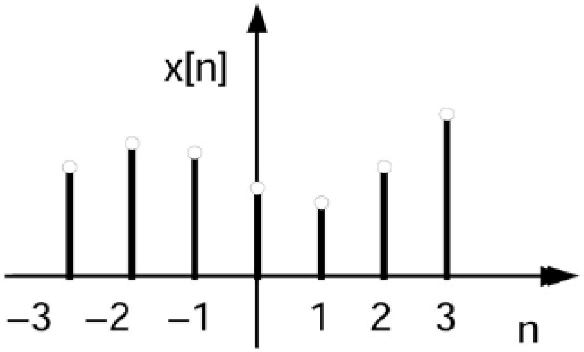
ADC & DAC

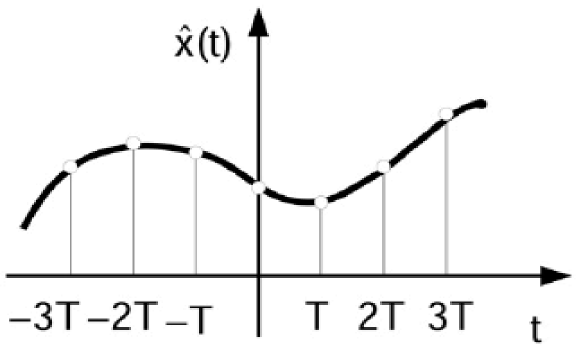
Signal processing is performed on the digital values extracted from the signal itself after A / D conversion, and results in the execution of an appropriate program by a digital processor. This structure is extremely flexible, in the sense that different processing functions can be realized simply by changing the processing program (software) without having to modify the physical structure (hardware) of the system.

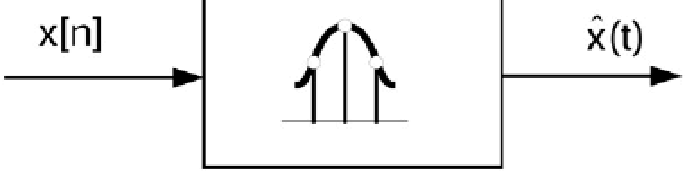


Sampling (ADC) & Interpolation (DAC)

(Time continuous) interpolation of a time-discrete signal (sequence)







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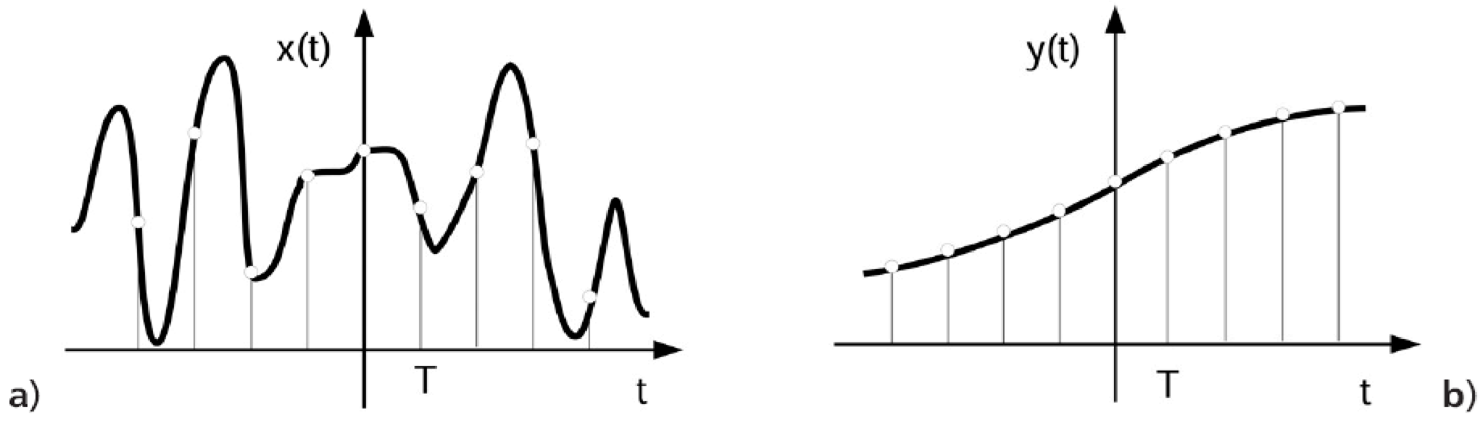
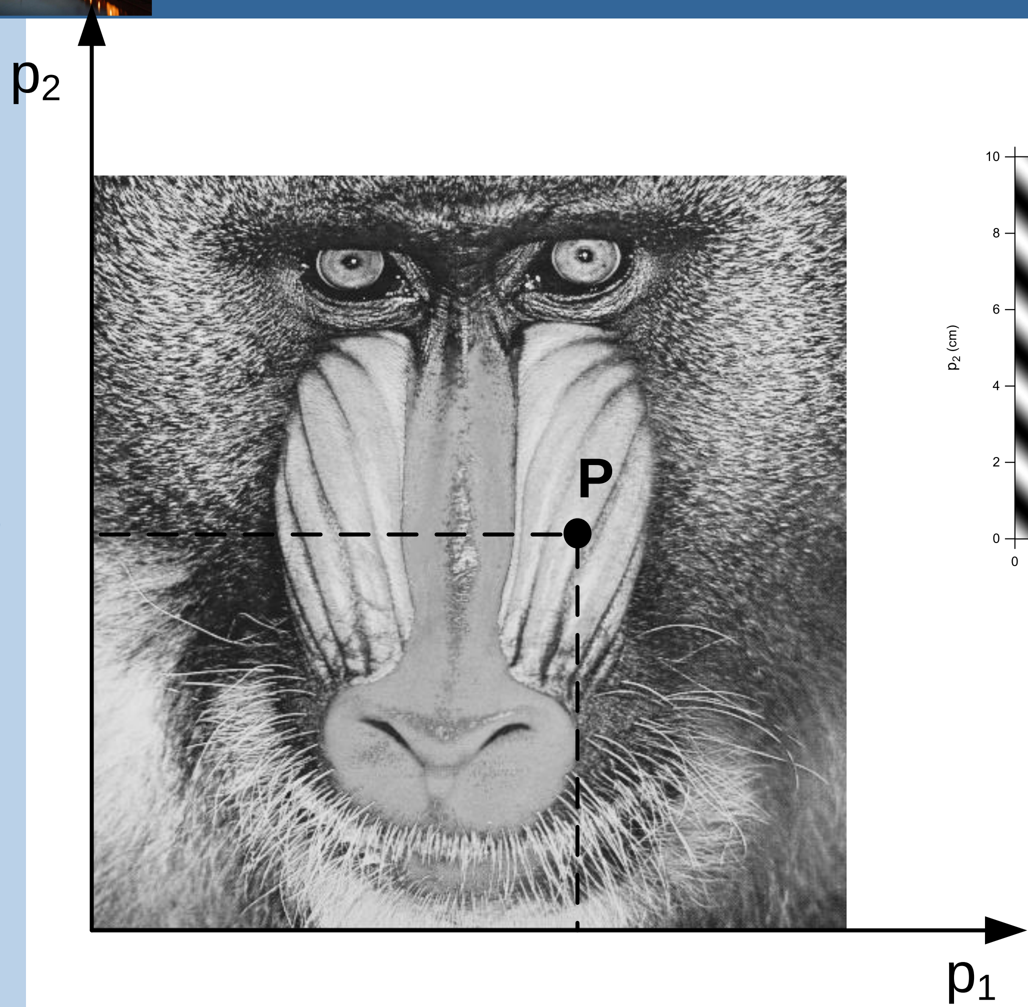
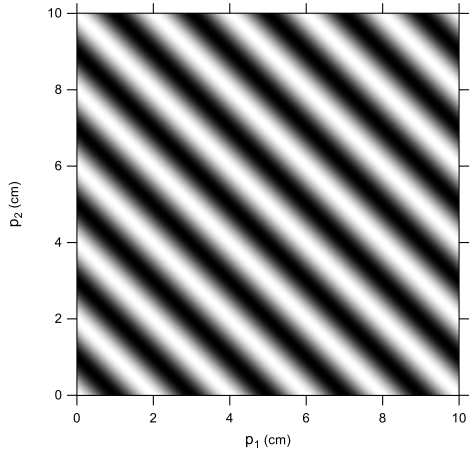
«Fast» and «Slow» Signals



The sampling period is adequate for y(t), but it is clearly too large for x(t). In this sense, the sampling frequency must be commensurate with the "speed" (i.e., with the rate of change) of the signal – how can we quantify this feature?

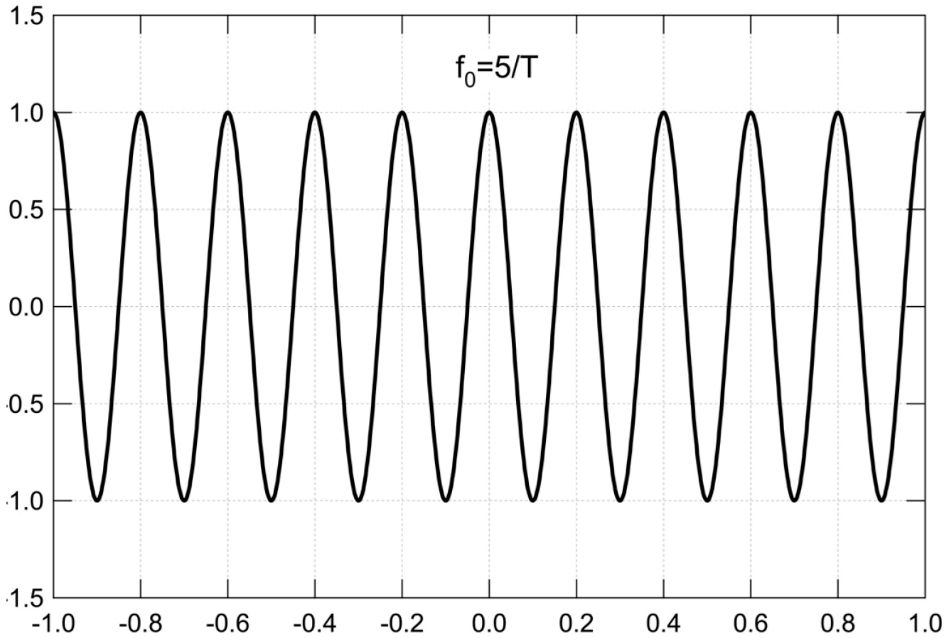
«Fast» and «Slow» Signals





ONE (Simple) Sinusoidal Signal

$$x(t) = \cos\left(2\pi t \cdot \frac{5}{T}\right)$$



PERIODIC signal: the repetition period is

T_0=5/*T*

The oscillation frequency f_0 is the inverse of the period:

f_0=1/T_0

The amplitude A is the peak value (maximum value) of the signal, in the example A=1

In general,

$$x(t) = A\cos(2\pi f_0 t + \vartheta)$$

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ONE (Simple) Sinusoidal Signal

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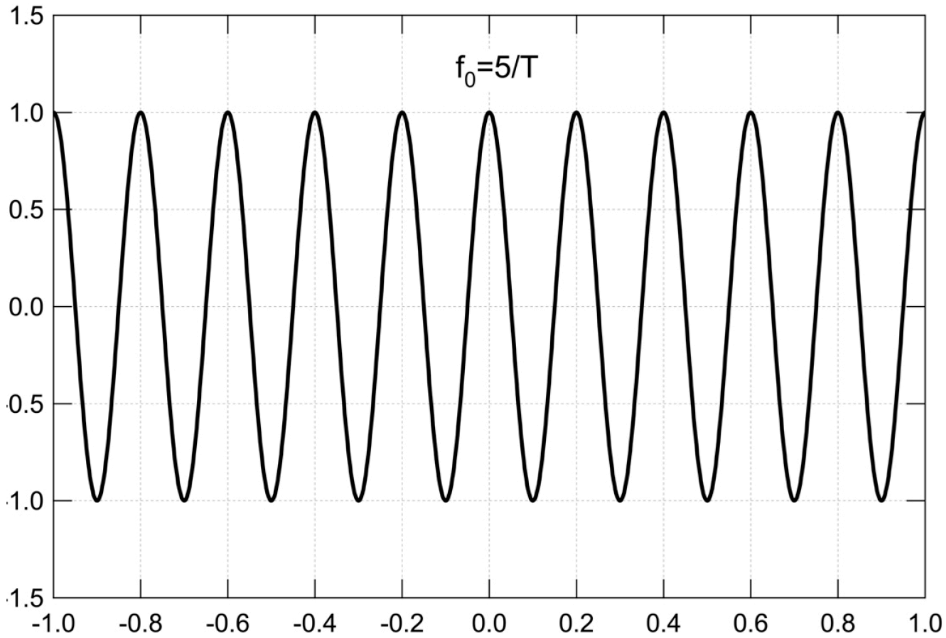
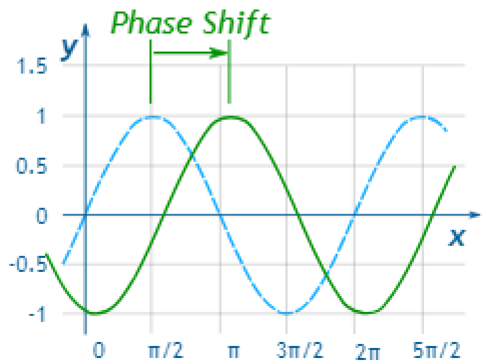


The phase-shift q indicates the initial point or initial value of the waveform at t=0:

$$x(0) = A\cos(\vartheta)$$

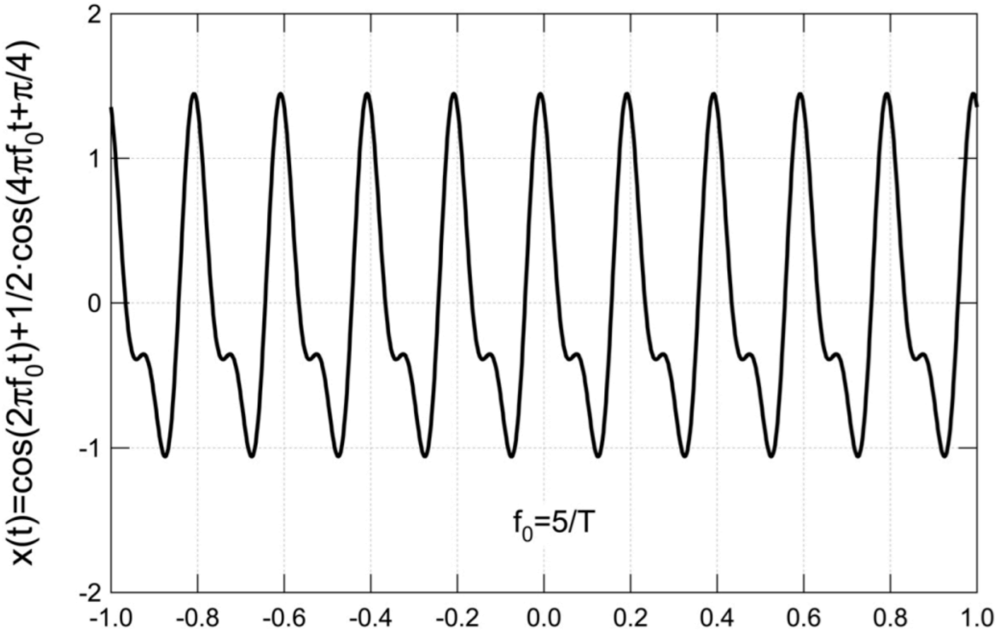


In general,

$$x(t) = A\cos(2\pi f_0 t + \vartheta)$$

TWO (Simple) Sinusoidal Signals

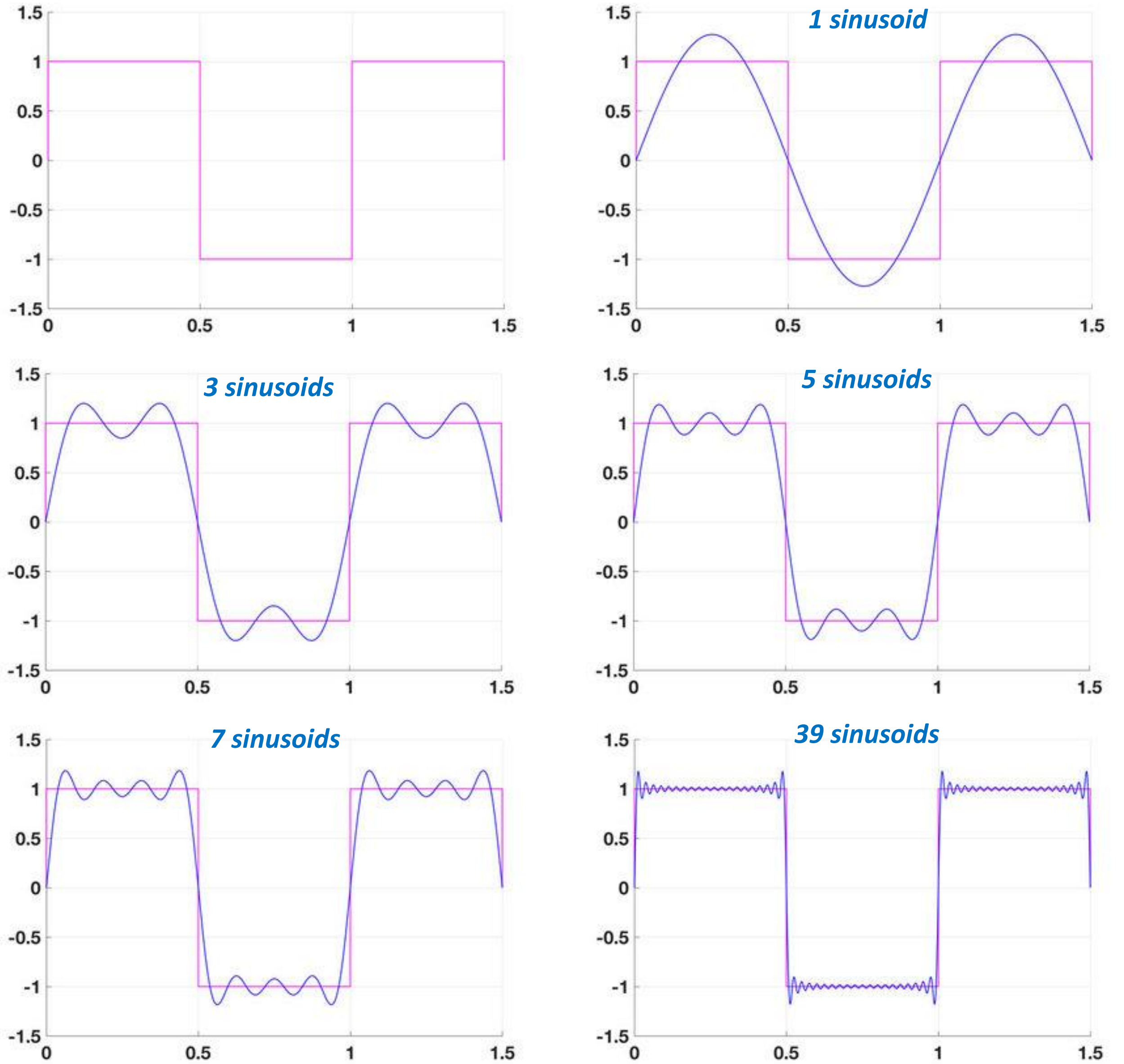
$$x(t) = \cos(2\pi 5t / T) + 0.5\cos(2\pi 10t / T + \pi / 4)$$



$$x(t) = A_1 \cos(2\pi f_0 t) + A_2 \cos(2\pi 2 f_0 t + \pi / 4)$$

VERY MANY (Simple) Sinusoidal Signals…

… can make a complicated (periodic) signal: the square wave – or: Fourier analysis for dummies



General Fourier Series

$$x(t) = A_1 \cos(2\pi f_0 t + \theta_1) + A_2 \cos(2\pi 2 f_0 t + \theta_2) + A_3 \cos(2\pi 3 f_0 t + \theta_3) +$$
$$\ldots + A_k \cos(2\pi k f_0 t + \theta_k) + \ldots$$

SYNTHESIS equation

General Fourier Series

$$x(t) = A_1 \cos(2\pi f_0 t + \theta_1) + A_2 \cos(2\pi 2 f_0 t + \theta_2) + A_3 \cos(2\pi 3 f_0 t + \theta_3) +$$
$$\ldots + A_k \cos(2\pi k f_0 t + \theta_k) + \ldots$$

SYNTHESIS equation

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General Fourier Series

$$A_k e^{j\vartheta_k} = \frac{1}{T_0}\int_0^{T_0} x(t)[\cos(2\pi k f_0 t) - j\sin(2\pi k f_0 t)]dt$$

ANALYSIS equation

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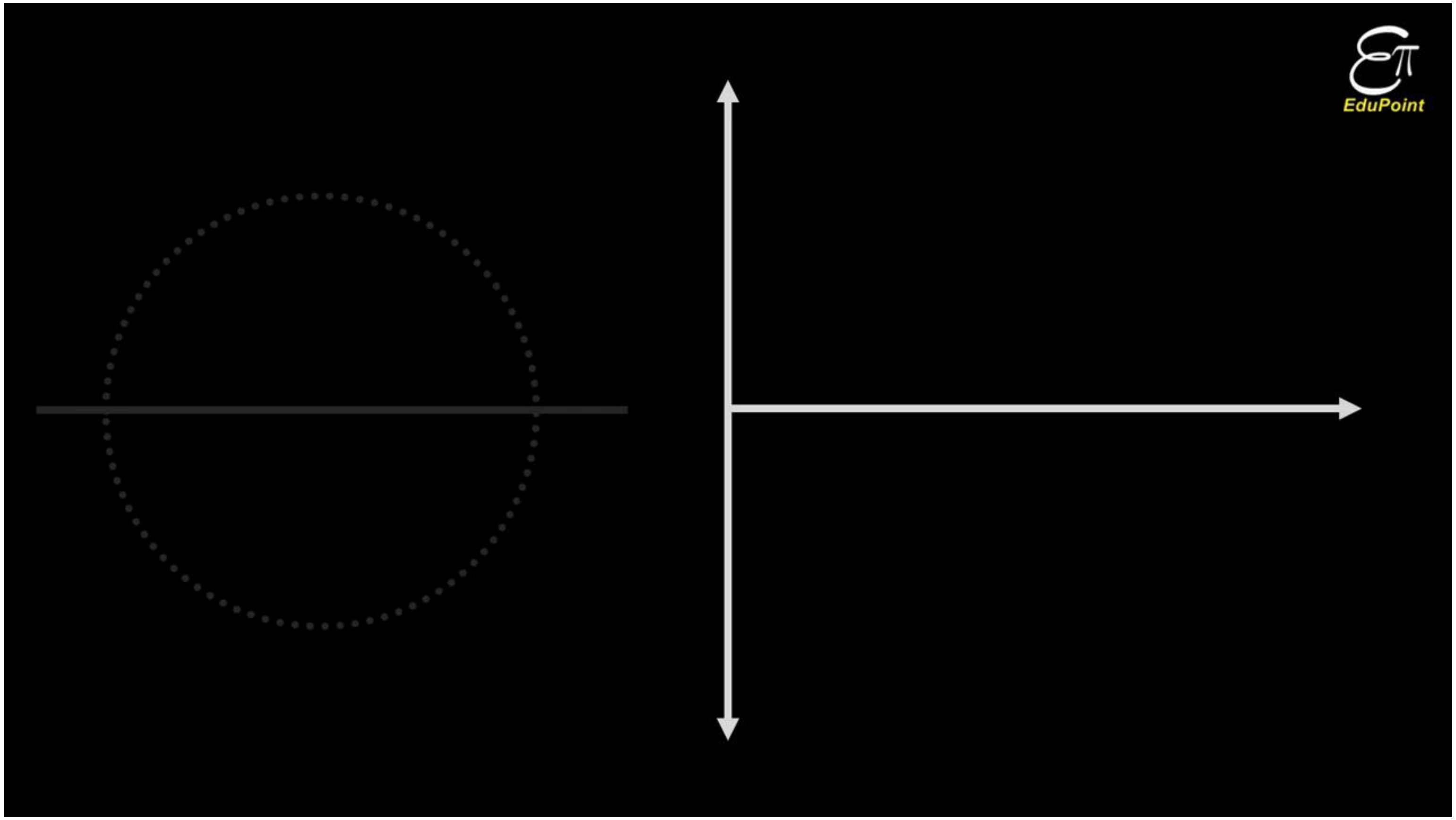
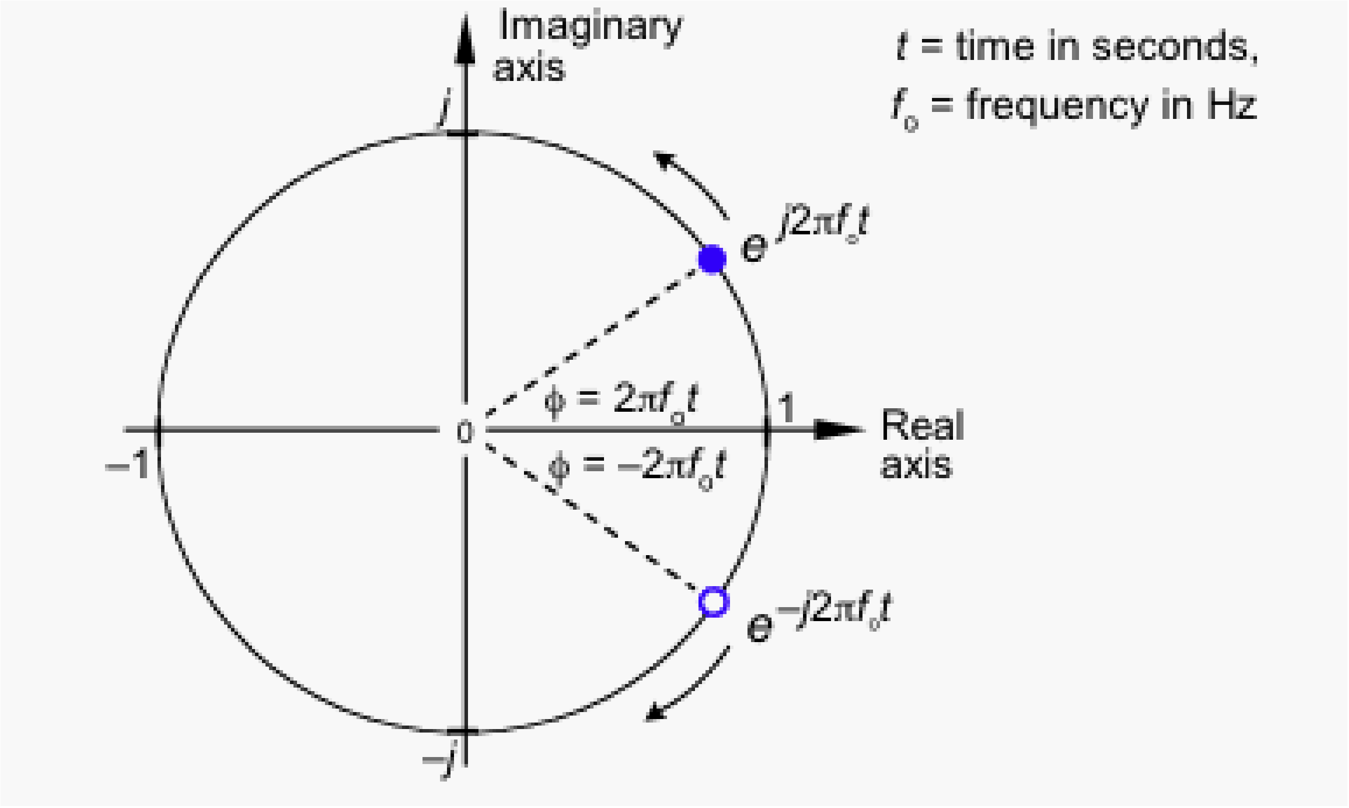
Positive and Negative (?) Frequencies 1/2



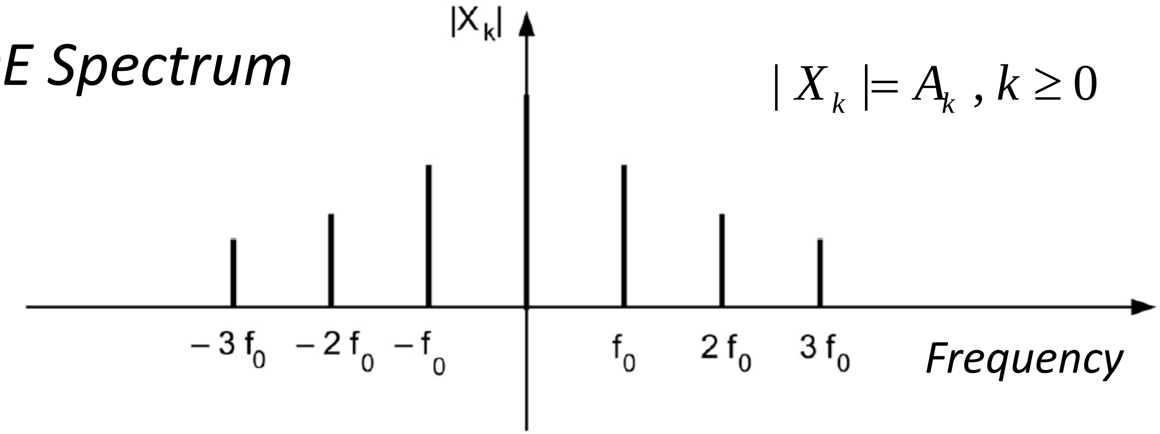
Positive and Negative (?) Frequencies 2/2



The frequency spectrum: frequency contents of a signal

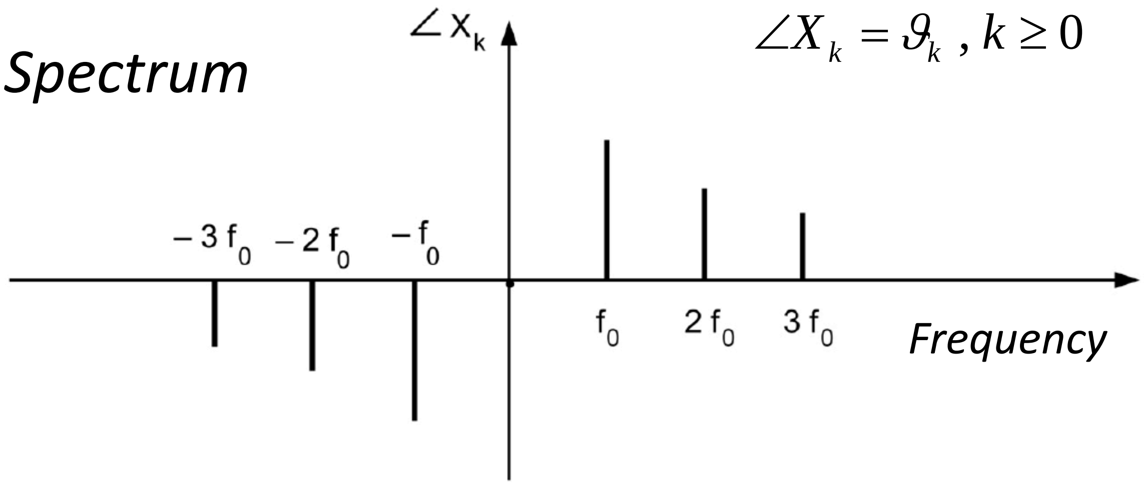
AMPLITUDE Spectrum

$$|X_k| = A_k \ , k \geq 0$$



PHASE Spectrum

$$\angle X_k = \vartheta_k \ , k \geq 0$$



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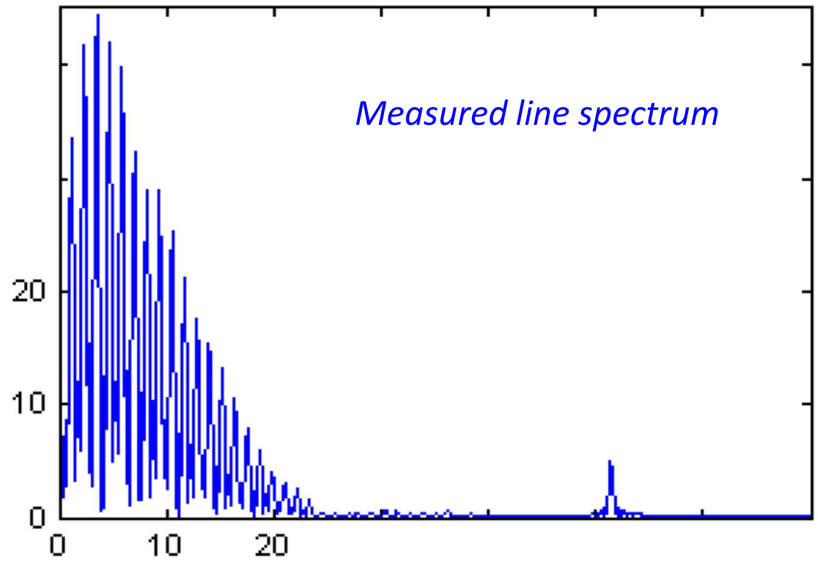
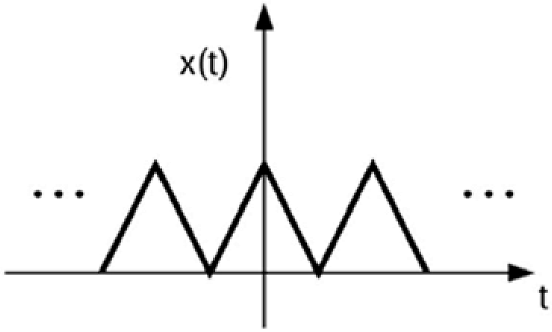
To sum-up: PERIODIC SIGNAL



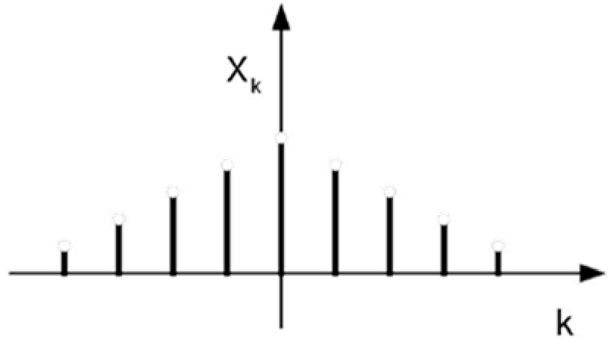
Frequency (Hz)

Signals and Spectra

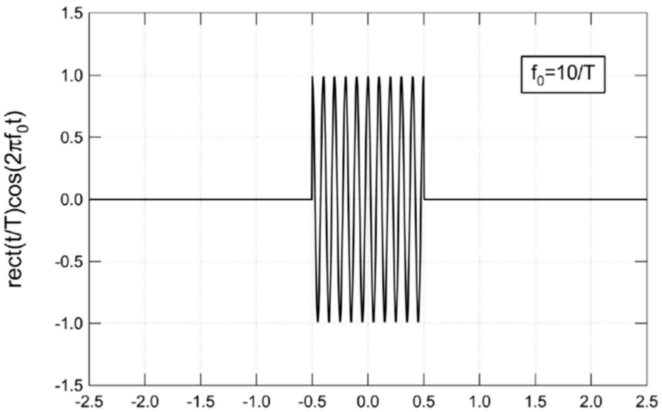
TIME | **FREQUENCY**



Periodic Signal



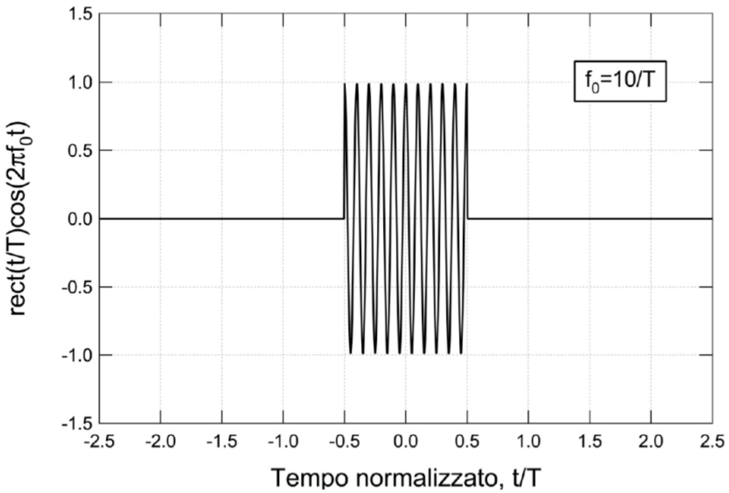
Line Spectrum



Aperiodic Signal

?

Time and Frequency



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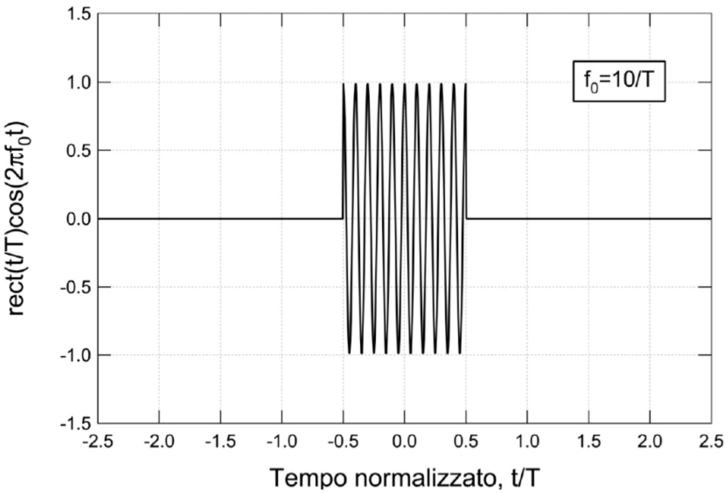
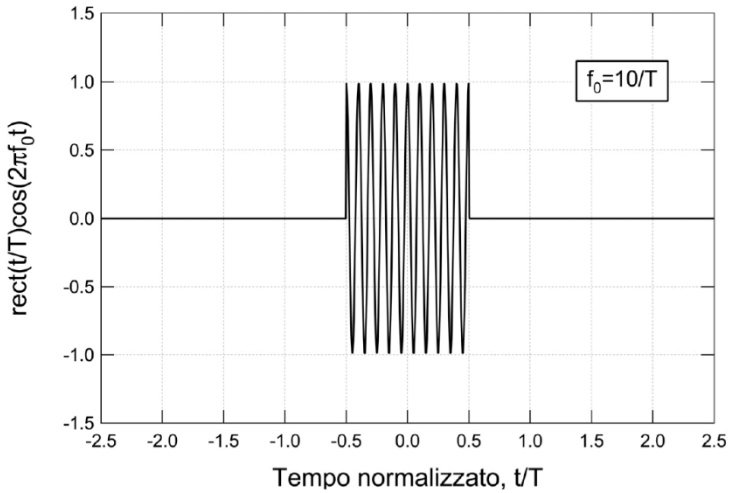
Time and Frequency

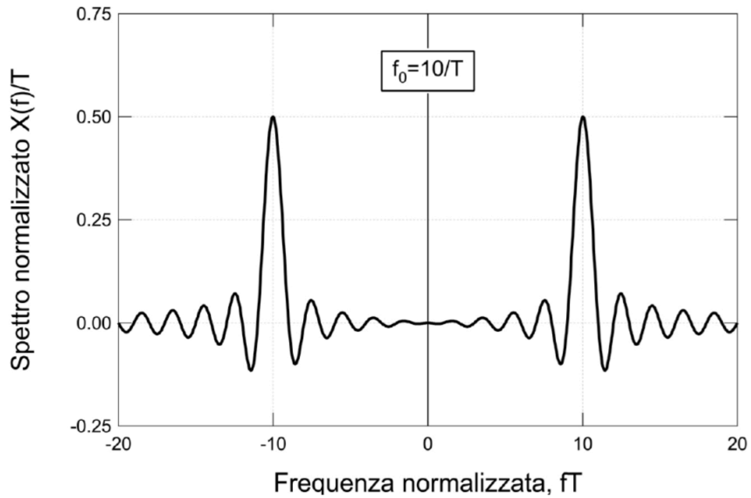


$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi ft} dt$$

ANALISYS *Equation*

Time and Frequency

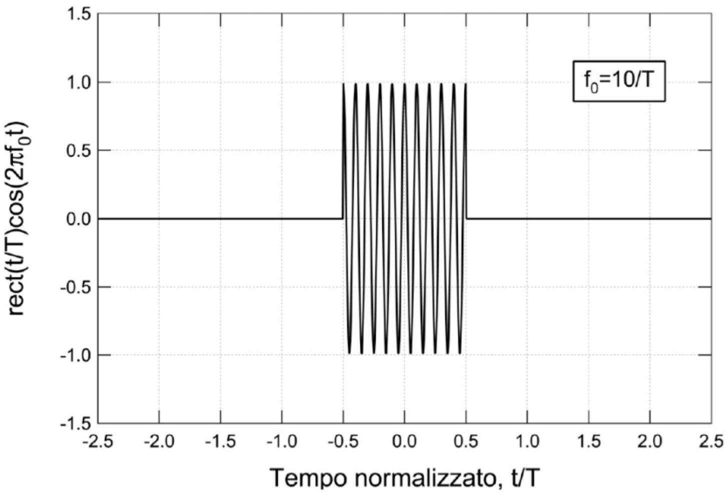


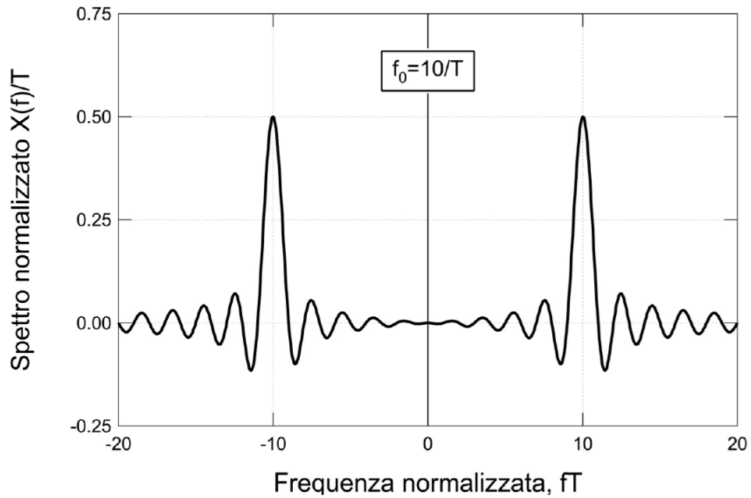


$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi ft} dt$$

ANALISYS *Equation*

Time and Frequency





$$x(t)=\int_{-\infty}^{+\infty} X(f)\,e^{j2\pi ft}df$$

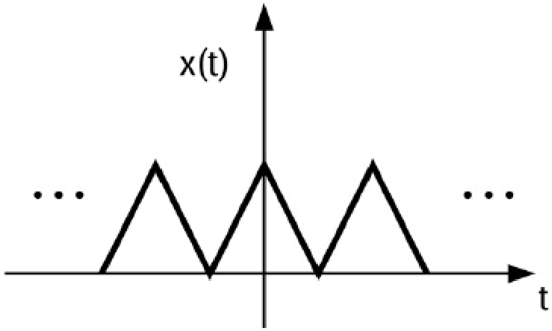
***SYNTHESIS** Equation*

$$X(f)=\int_{-\infty}^{+\infty} x(t)\,e^{-j2\pi ft}dt$$

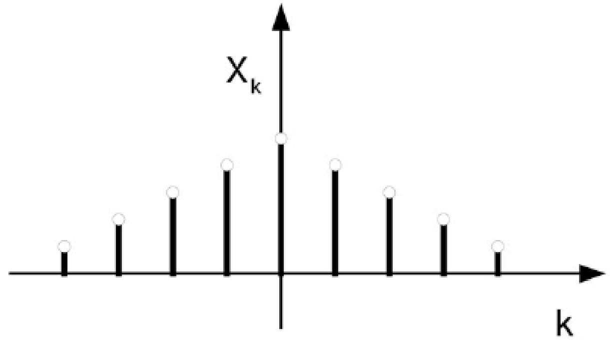
***ANALISYS** Equation*

Different kind of Spectra

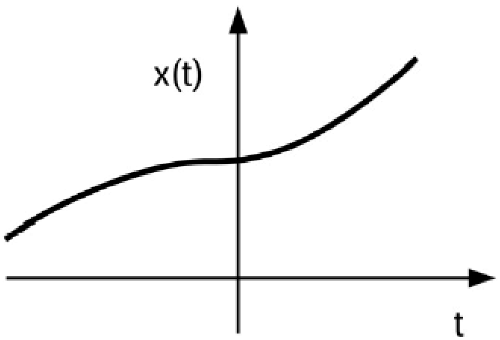
Time	Frequency



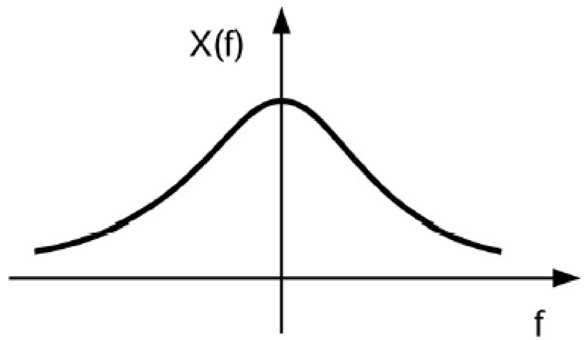
Periodic signal



Line (discrete) spectrum

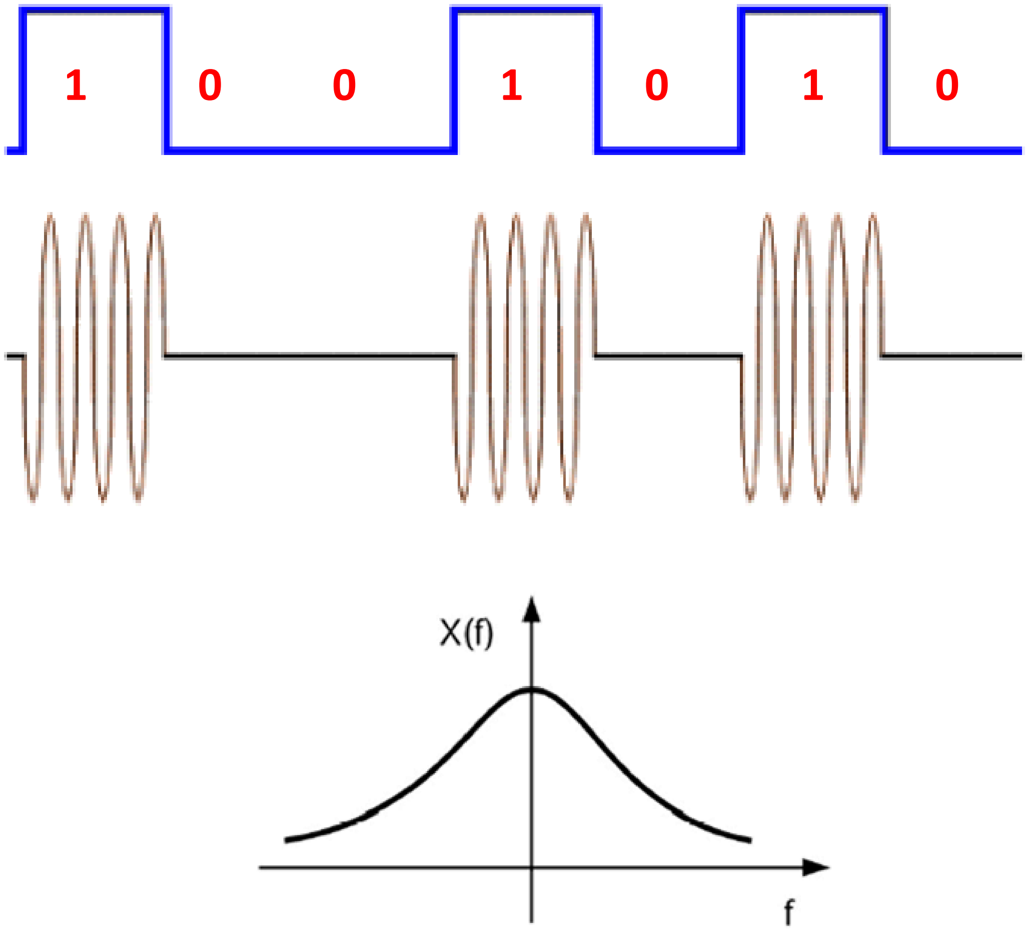


Non-periodic signal



Continuous spectrum

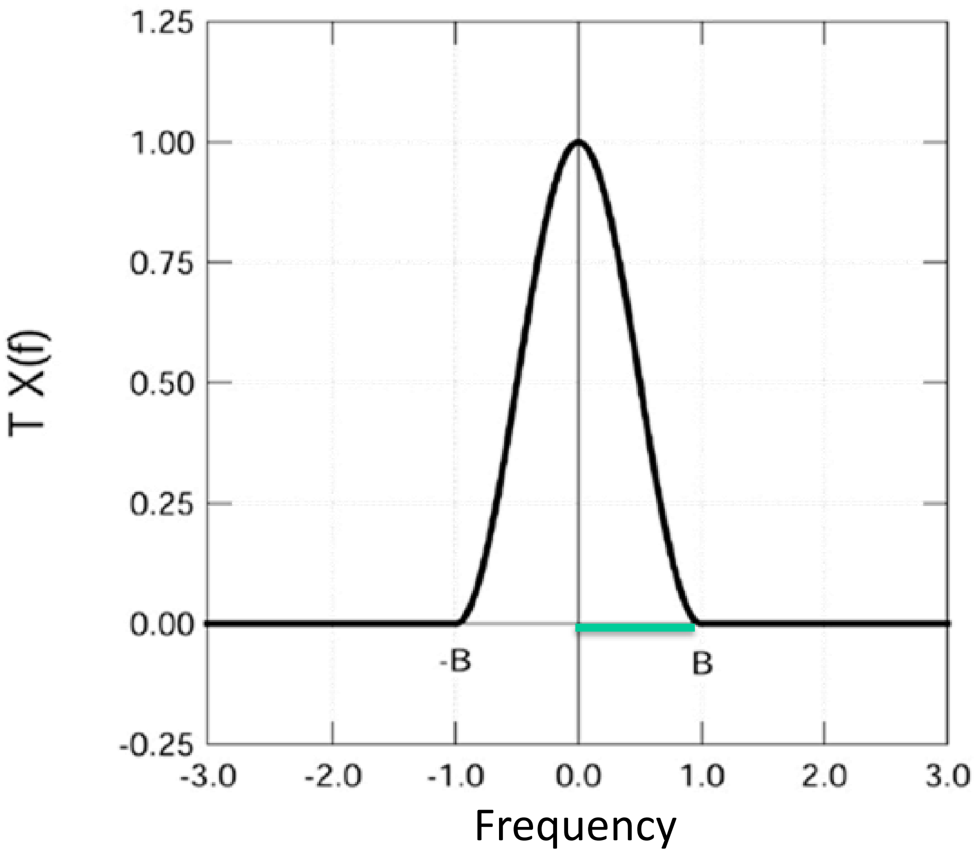
Non.-periodic, RANDOM Signals



Continuous POWER spectrum as well !

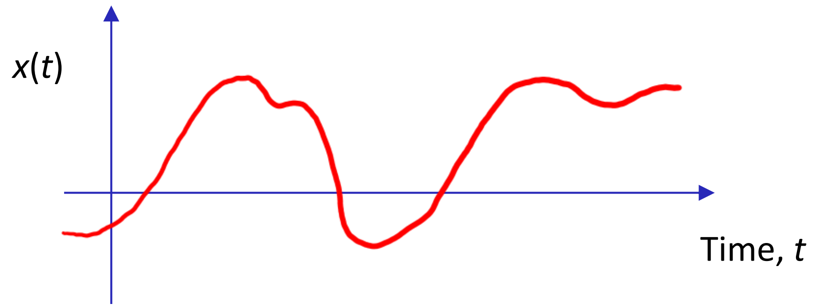
The BANDWIDTH of a signal

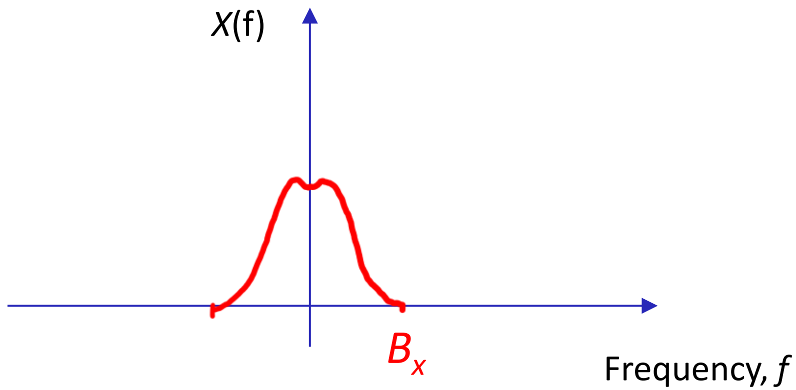
The bandwidth B is the width of the frequency interval (on positive frequencies) on which the signal spectrum is different from 0



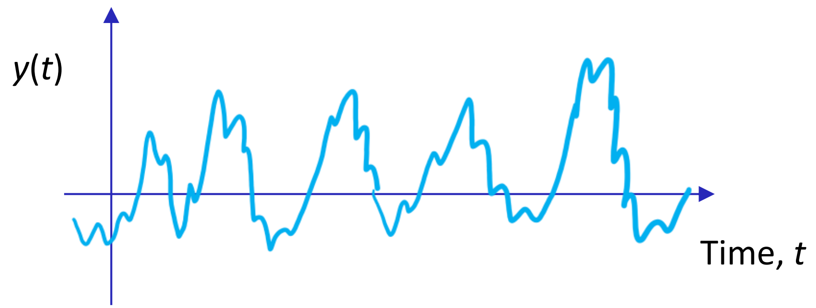
Wideband and Narrowband

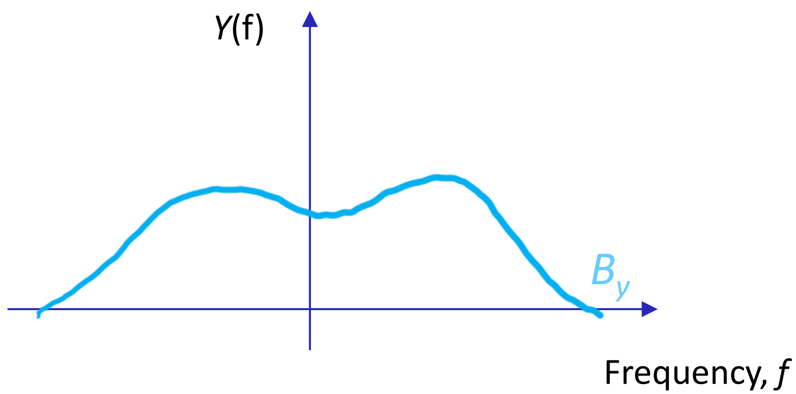
“Slow”, narrowband signal and its spectrum





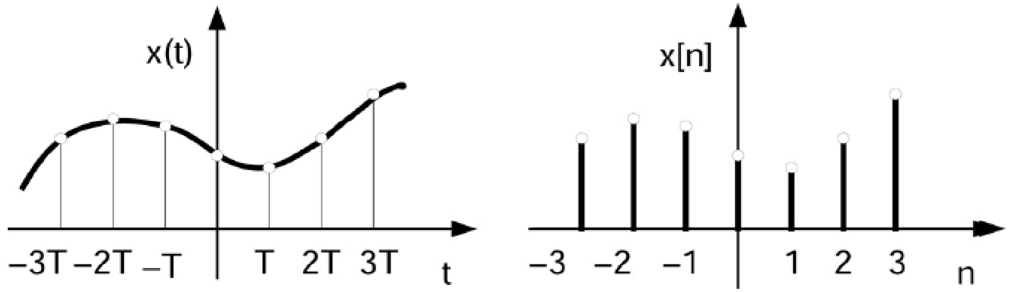
“Fast”, wideband signal and its spectrum

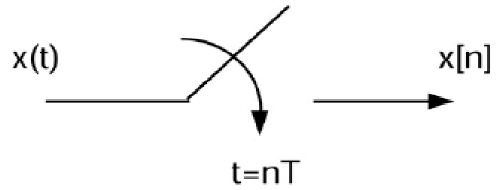




$B_x < B_y$

Sampling and Spectra



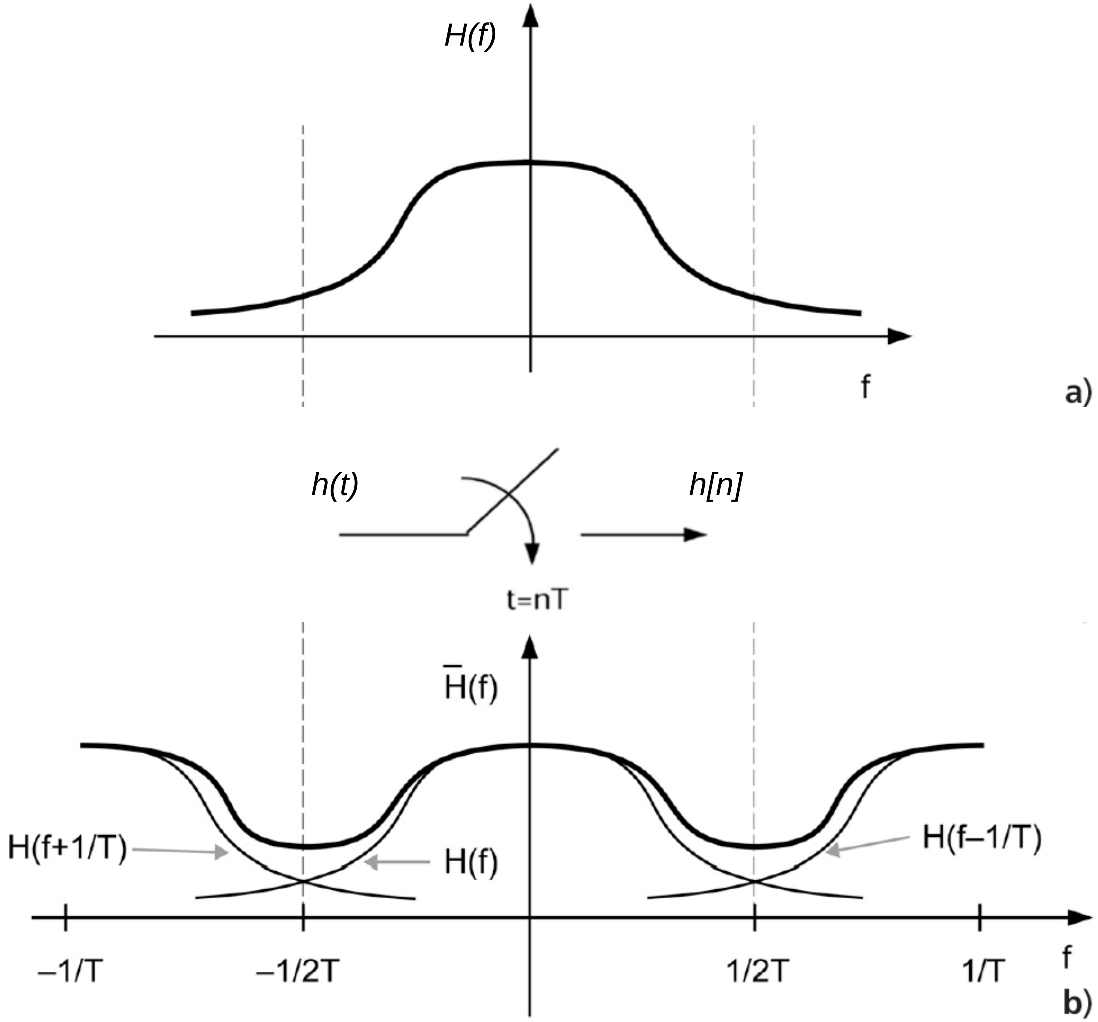


$$x[n] = x(nT)$$

Poisson's relation that gives the spectrum of the digital signal

$$\overline{X}(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{T}\right)$$

Periodic spectrum of a sampled signal



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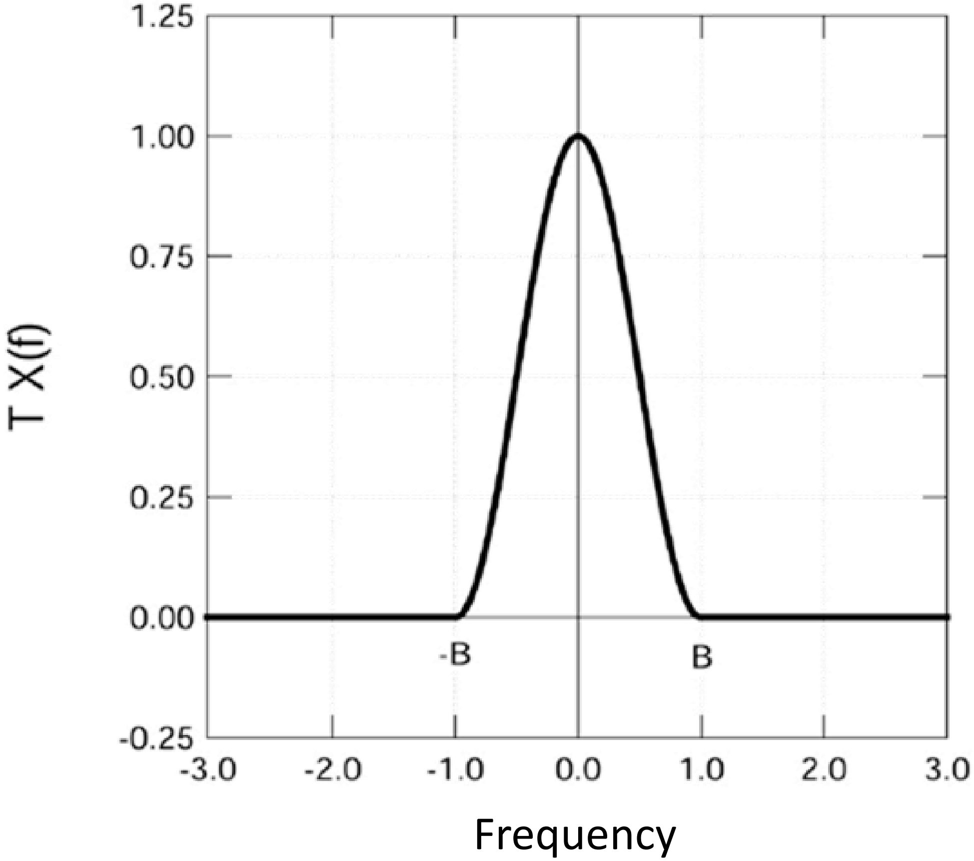
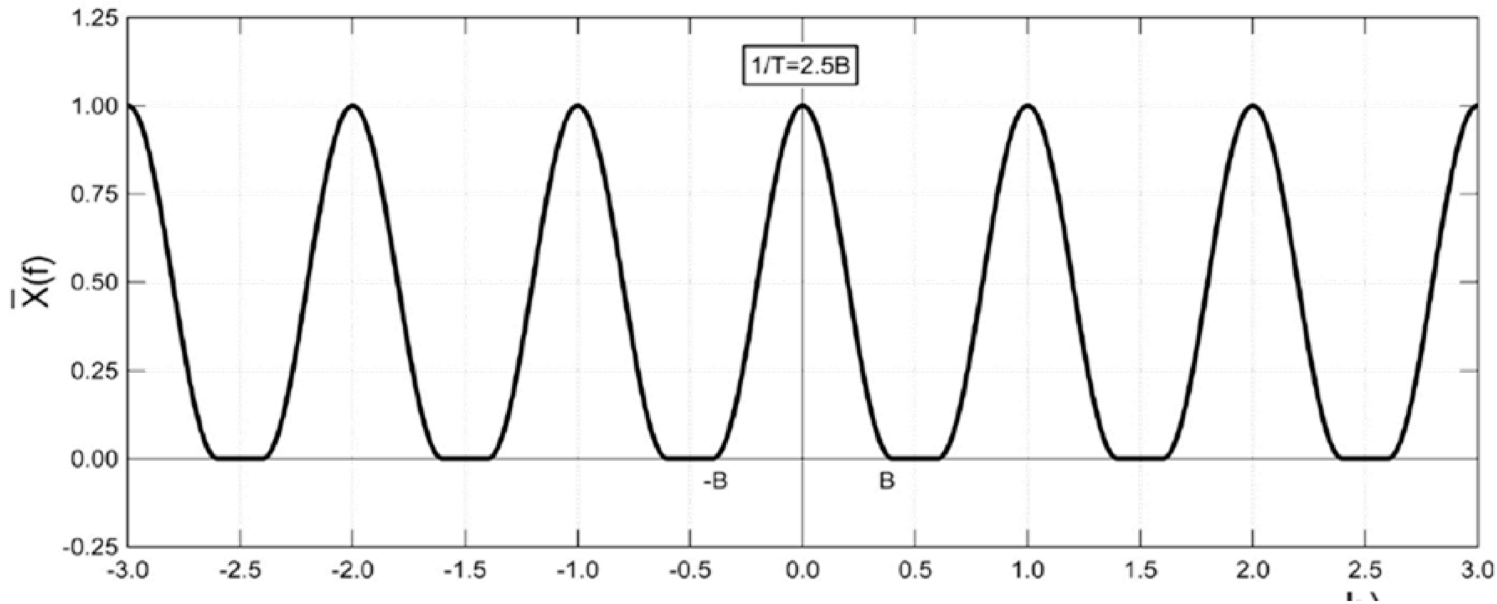
An example



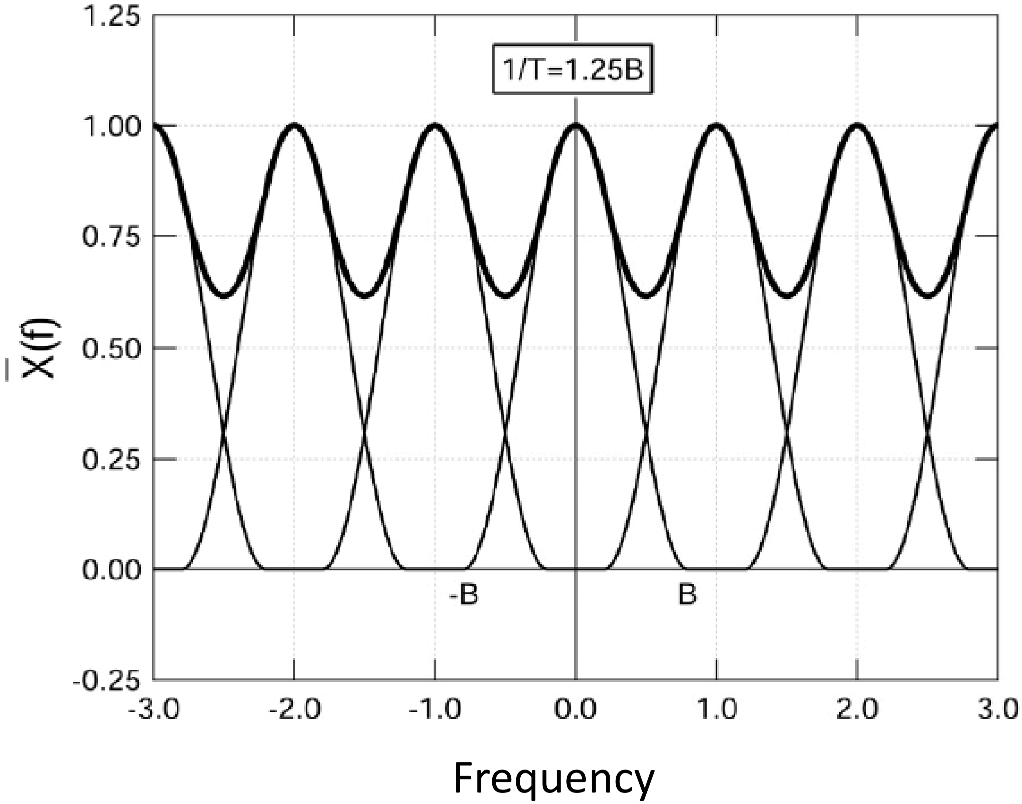
No Aliasing



Frequency

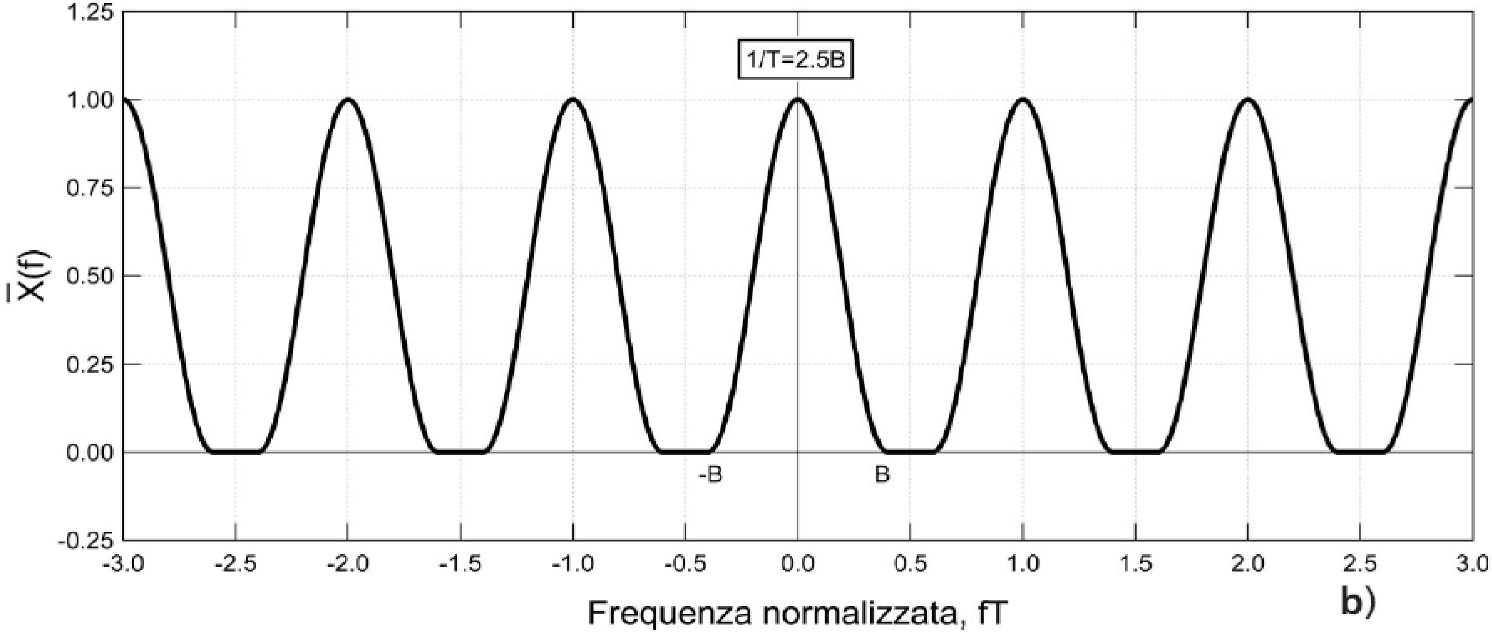
Using a sampling frequency f_c=5B/2

Aliasing Error !



Using a sampling frequency f_c=5B/4

Nyquist rule



Not to have aliasing, we have to make sure that

$$f_c \geq 2B$$

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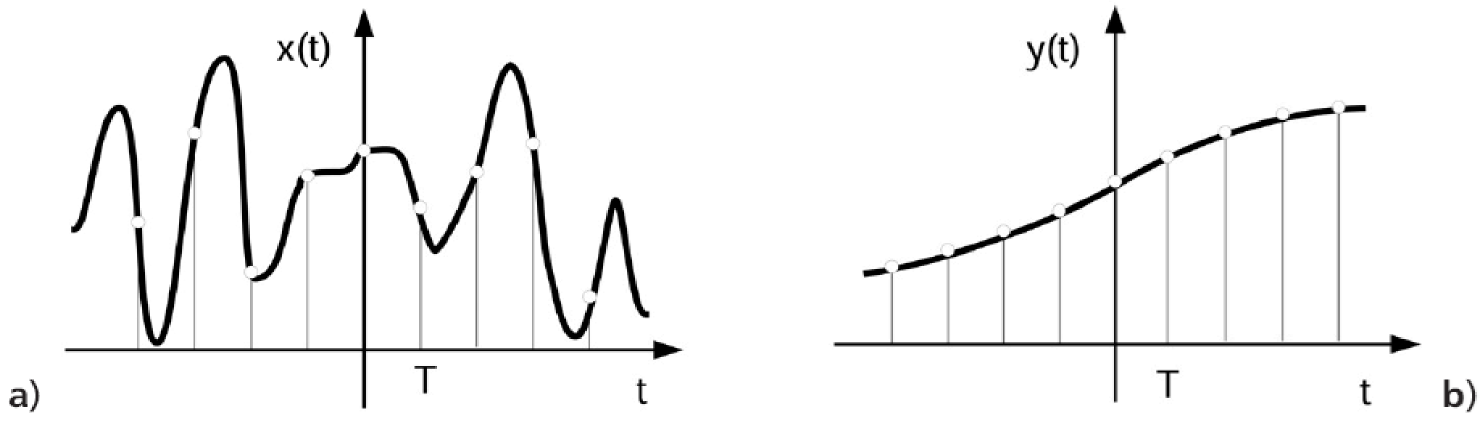
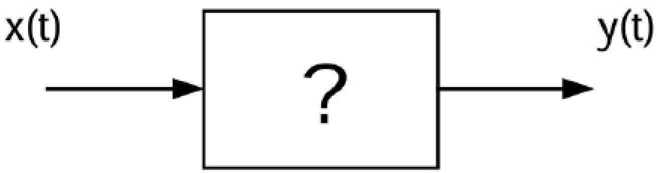
Nyquist rule in the time domain



The sampling period is adequate for y(t), but it is clearly too large for x(t). In this sense, the sampling frequency must be commensurate with the signal bandwidth (with the rate of change of the signal), as the Nyquist rule suggests

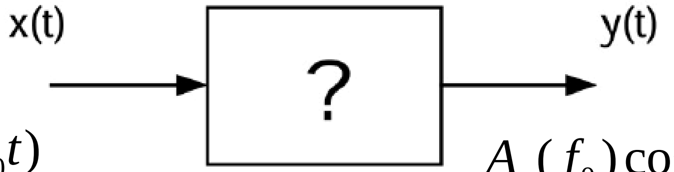
Filtering 1/2



- A **filter** is a «black box» (HW, SW in an ARM DSP processor) with a linear behavior.
- If $x(t)$ is sinusoidal, then $y(t)$ is sinusoidal at the same frequency: frequencies are not changed
- BUT what is changed is the amplitude/phase of the oscillation:

$$x(t) = \cos(2\pi f_0 t) \quad \Rightarrow \quad y(t) = A_y \cos(2\pi f_0 t + \vartheta_y)$$

Filtering 2/2



$$\cos(2\pi f_0 t) \qquad A_y(f_0)\cos(2\pi f_0 t + \vartheta_y(f_0))$$

- Not all frequencies are the same: according to the properties of the filter systems, some frequencies ar more altered then others (for instance, enhanced/attenuated)
- The filter *responds* differently according to the particular frequency of the input: the *frequency response* of the filter is

$$H(f_0) = A_y(f_0)e^{j\vartheta_y(f_0)}$$

What's the purpose of Filtering ?

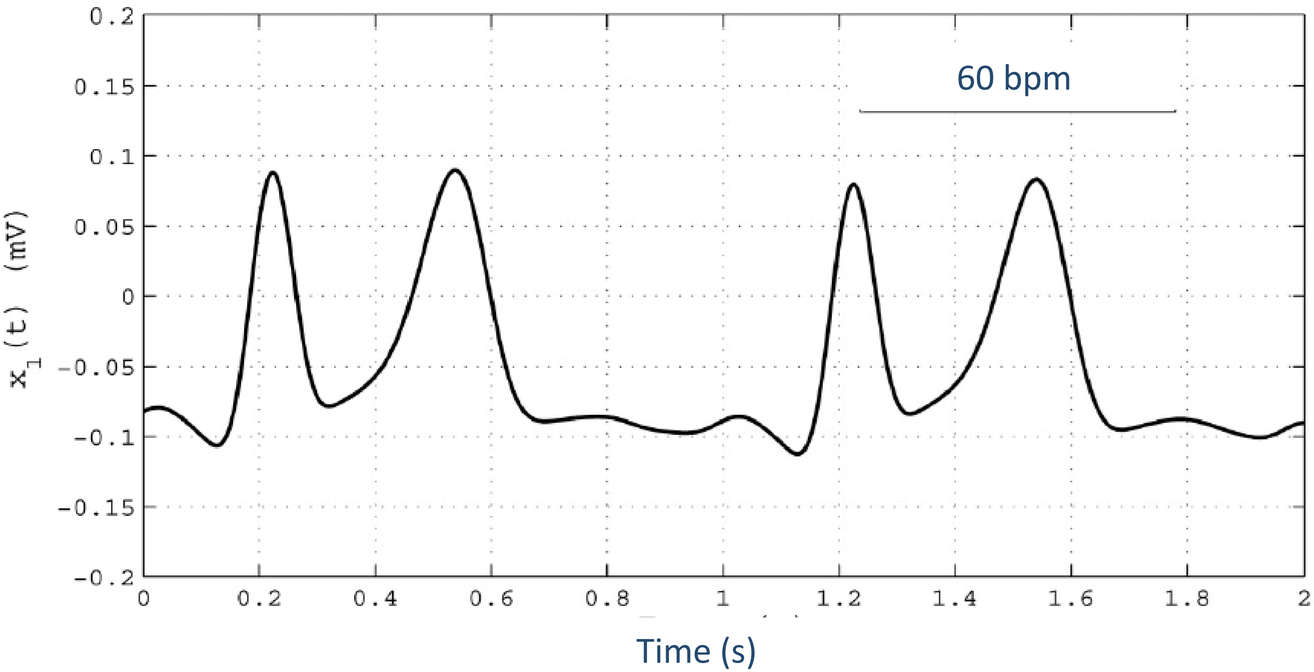
- A certain signal is made of the superposition of two components:

$$x(t) = x_1(t) + x_2(t)$$

The first component is the useful signal, the second is just *noise* (disturbance), an unwanted component we would like to rid of.

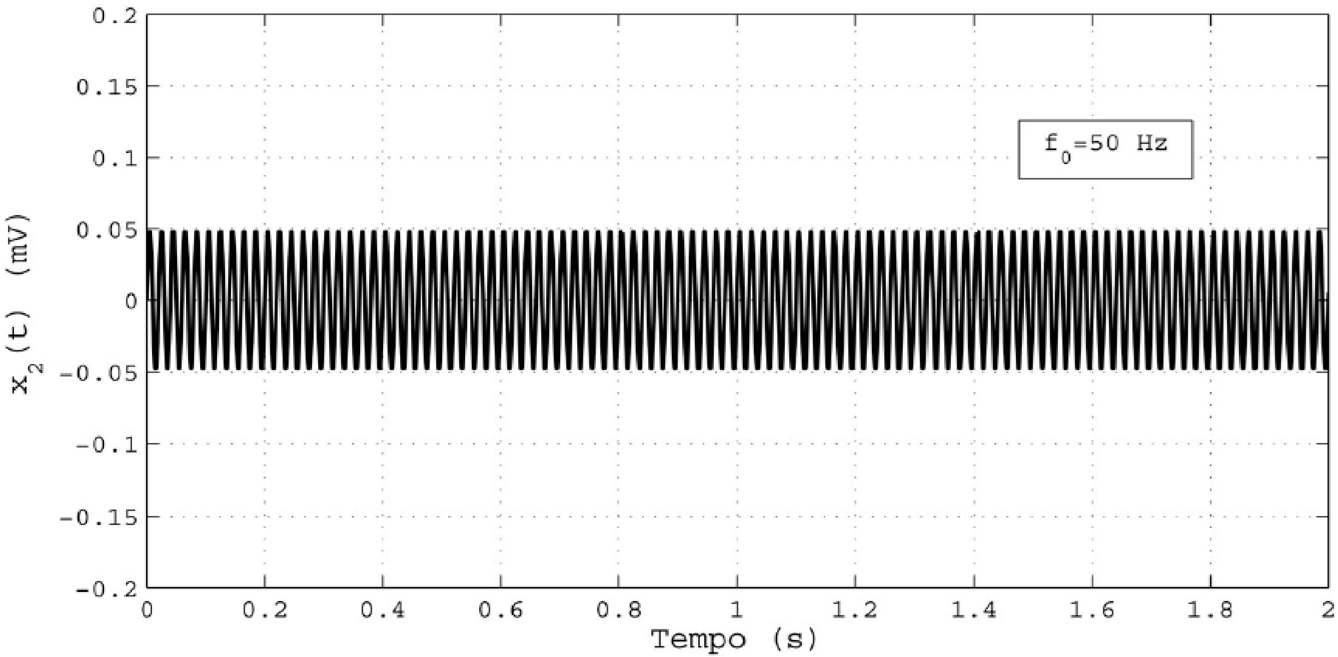
Example: (low-power, low amplitude) ECG signal plus an unwanted component (interference) coming from the 50 Hz AC power supply.

The Notion of FILTER



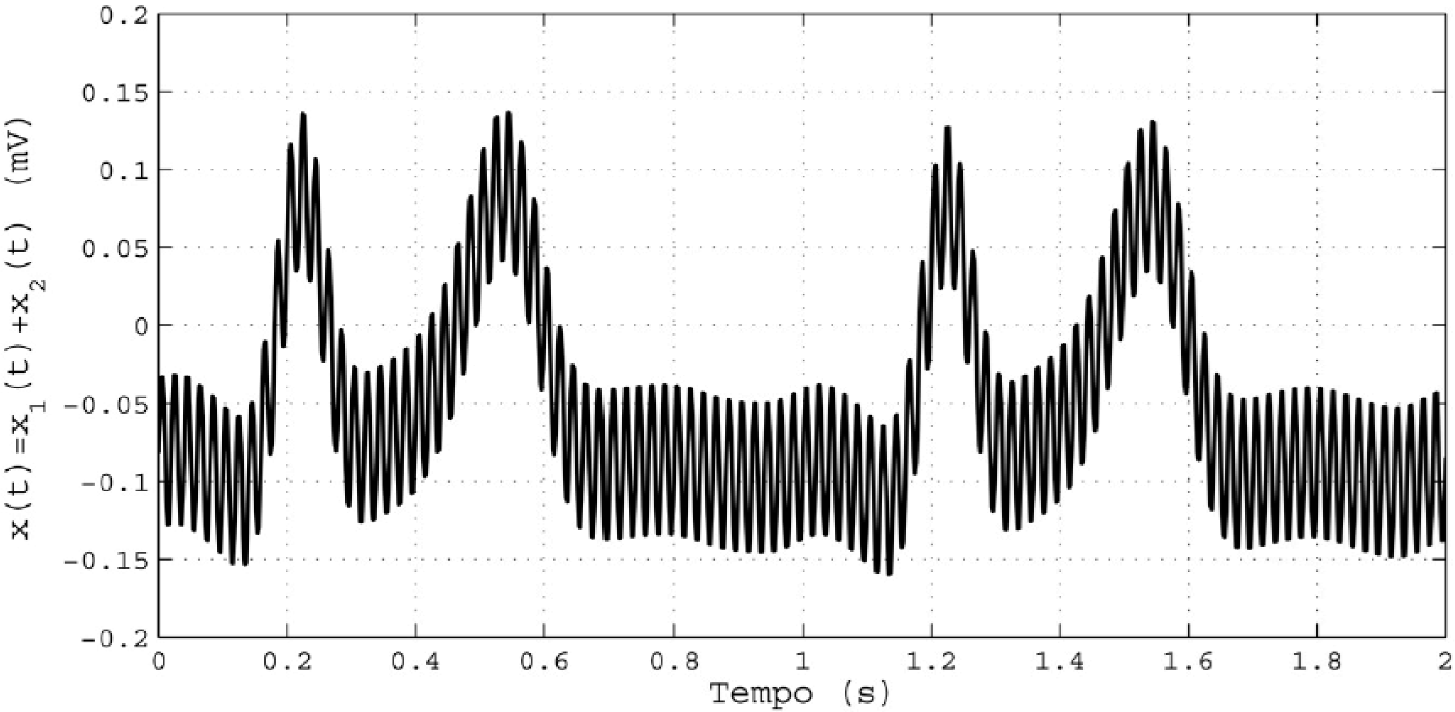
a)

The Notion of FILTER

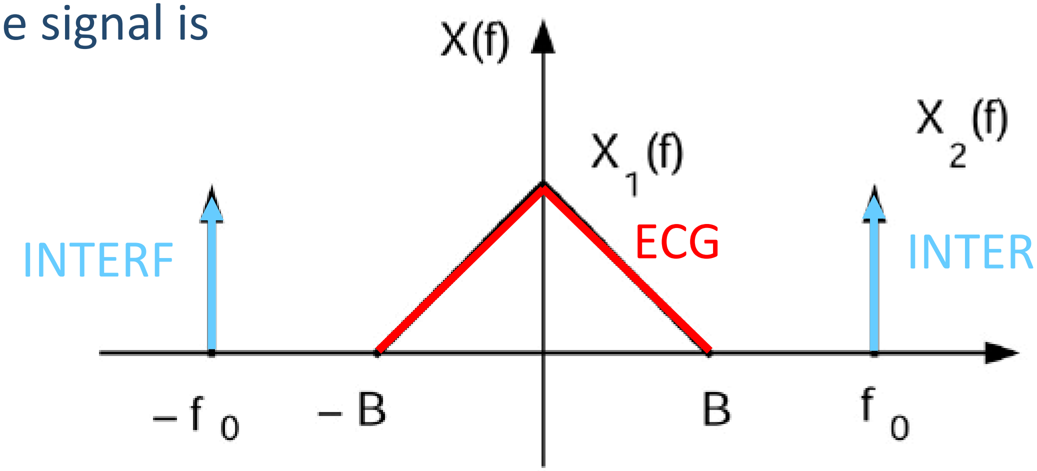


b)

The Notion of FILTER



Is there any hope to *reject* (eliminate) the unwanted component? Let us examine the problem in the *frequency domain*. The spectra of the signal is

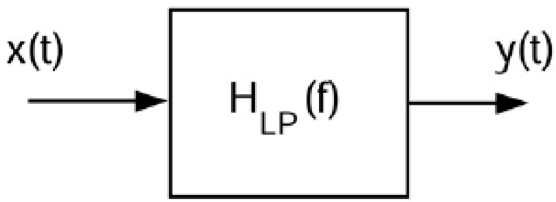


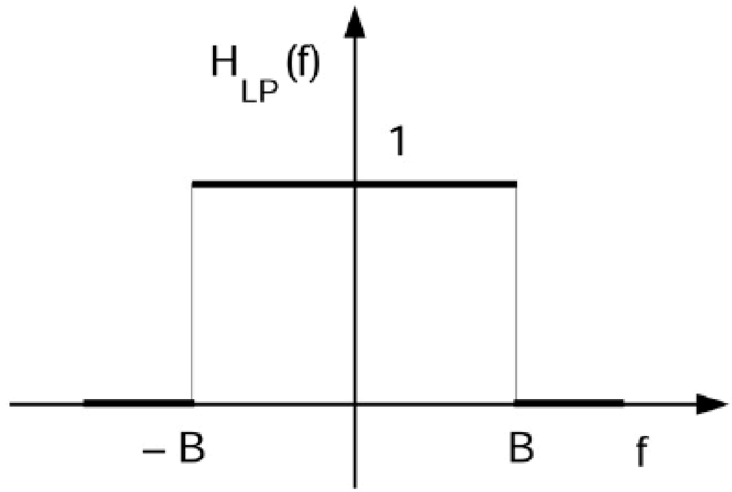
$$X(f) = X_1(f) + X_2(f)$$

Whilst the two signals fully overlap in time, their spectra lie in two separate frequency intervals (*bands*) ! We can separate the two (rejecting the interference) with a filter having a appropriate *frequency response* (with unequal treatment of frequency bands)

Low-pass FIlter

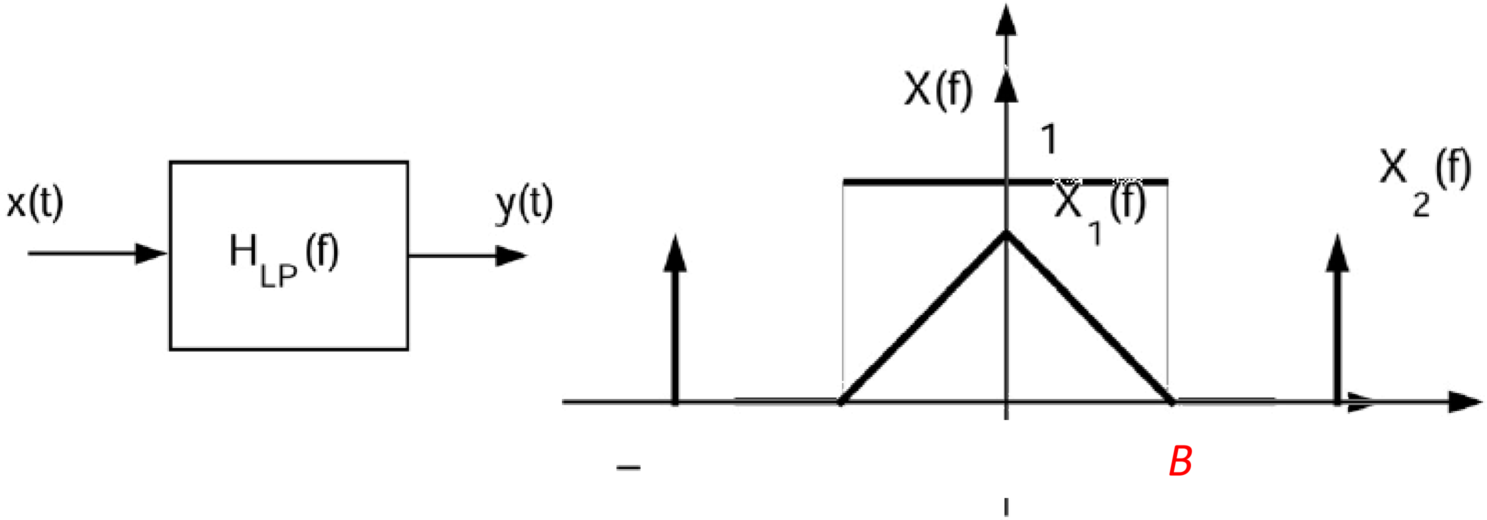
We can *filter out* (reject) the higher-frequency, unwanted component $x_2(t)$ applying to the whole signal $x(t)$ an LTI system that *passes* the low-frequency components and *blocks* the high-frequency ones: a low-pass filter with frequency response





$$Y(f) = X(f)\,H(f)$$

Low-pass Filter



B is the value of the filter bandwidth (to be considered on positive frequencies only)

$$Y(f) = X(f)\,H(f)$$

Filtering Example

a)

Input signal x (t_1,t_2)

Filtering Example

Low-pass filtered signal $y(t_1,t_2)$

Filtering Example

a)

High-pass filtered signal $y(t_1,t_2)$

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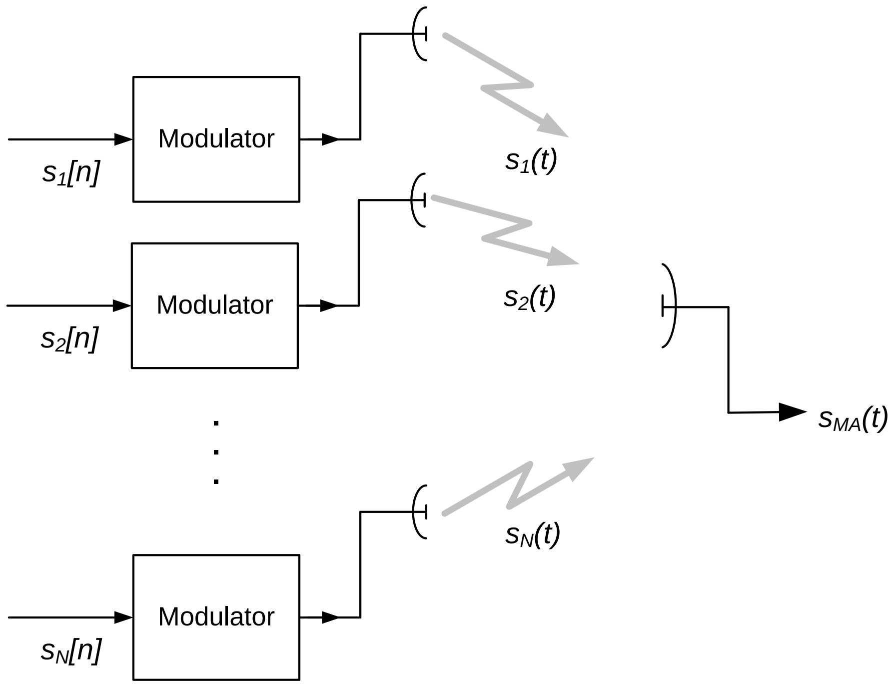
Filtering Example

b)

Same, with inverted brightness scale

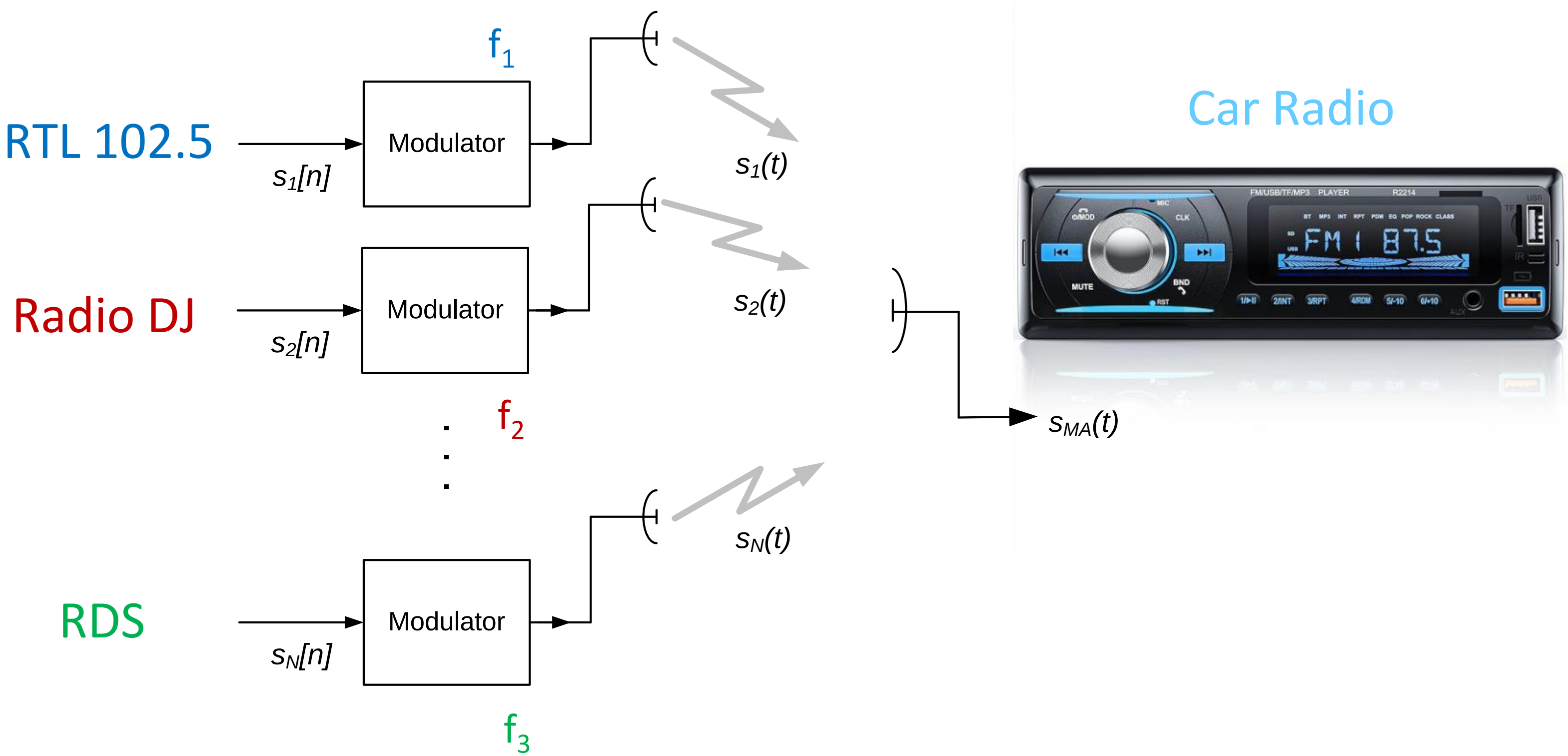
More on Filtering: Frequency-Division Multiple Access

- **Many users share the same physical medium (for example, the same band of frequency). How can they co-exist ?**

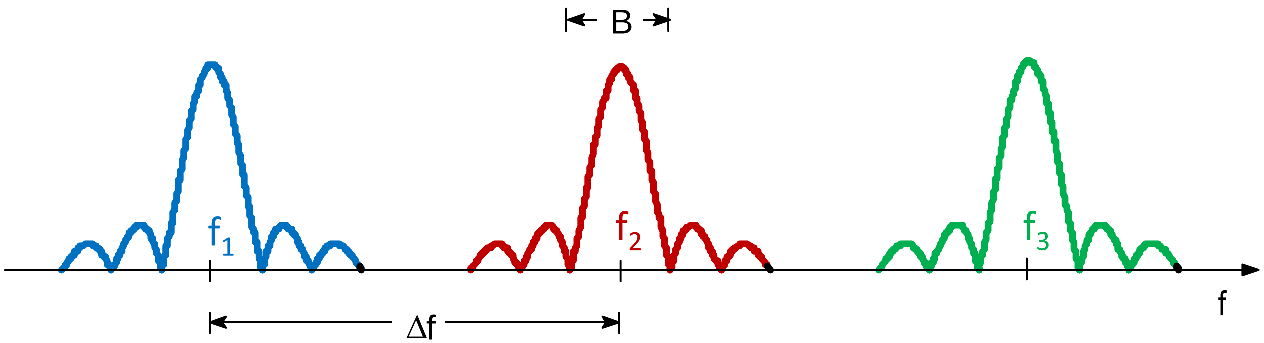


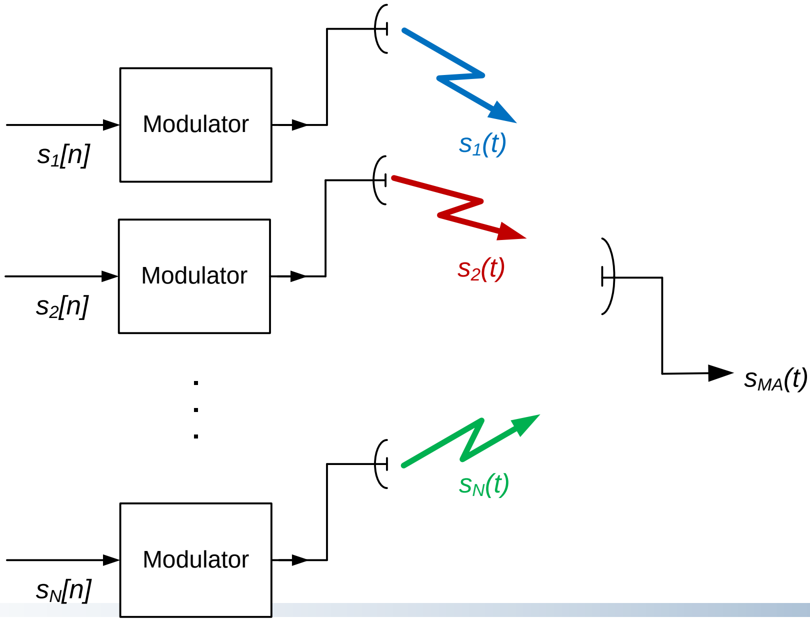
Example: FM Radio

- **Many users share the same physical medium (for example, the same band of frequency). How can they co-exist ?**



Frequency Division Multiple Access



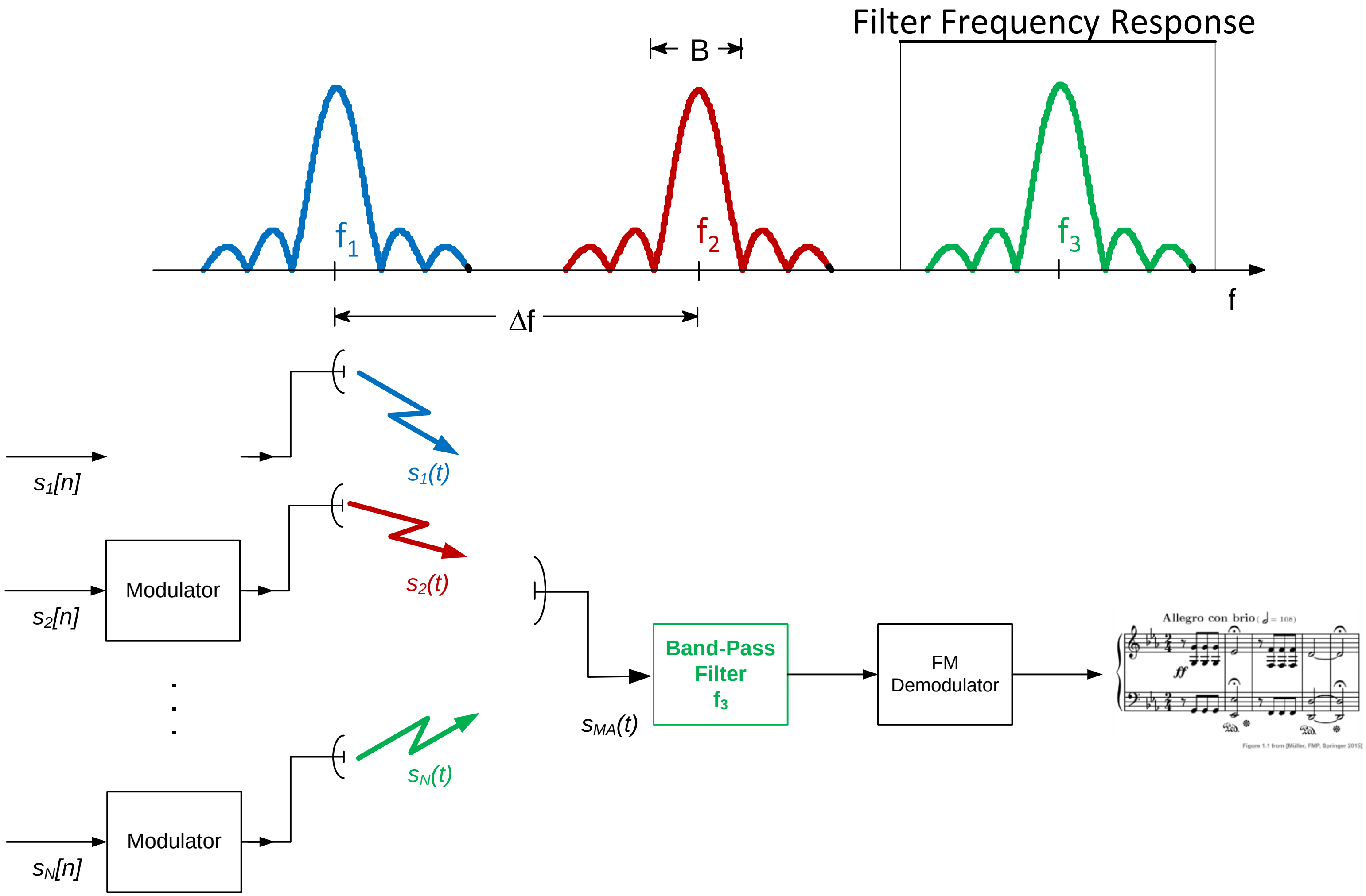


Different radio stations co-exist on the same FM frequency band 88-108 MHz by using separate bandwdiths and different carrier frequencies

B=150 kHz

Δf=250 kHz

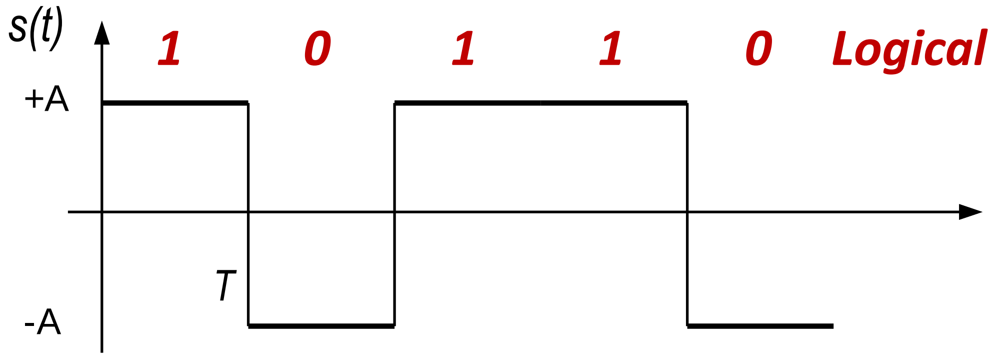
Demultiplexing and.. listening



Baseband Physical Data Signal

$\{a_k\} = \pm A$

Physical Values



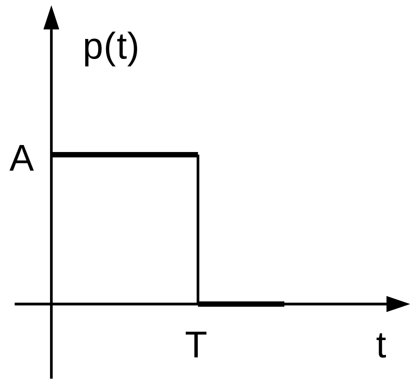
Logical Values

Binary Baseband Digital Signal

Pulse/Symbol Interval: T = Bit Interval: T_b

Pulse/Symbol Rate: $R=1/T$ = Bit Rate: $R_b=1/T_b$

Basic Pulse p(t):

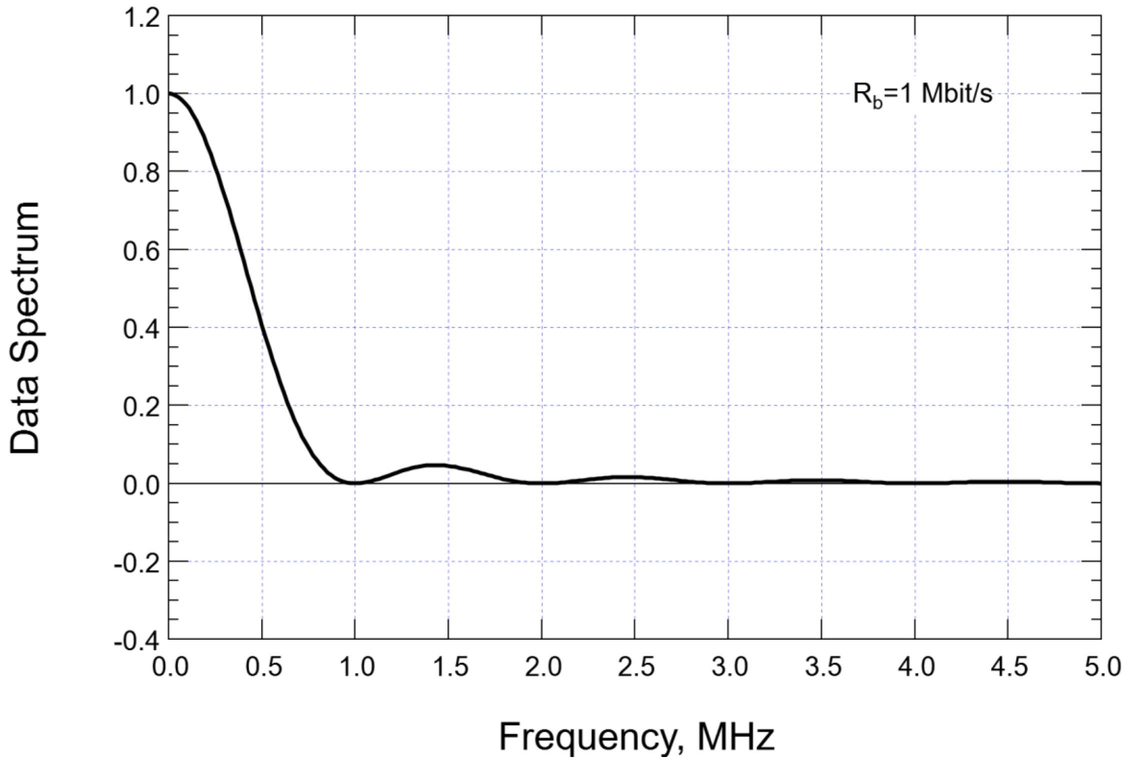


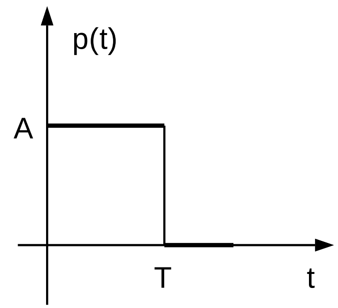
Bipolar Format

$$\boxed{s(t) = \sum_{k=-\infty}^{\infty} a_k \cdot p(t-kT)}$$

Power Spectral Density (PSD) of the Baseband Data Signal

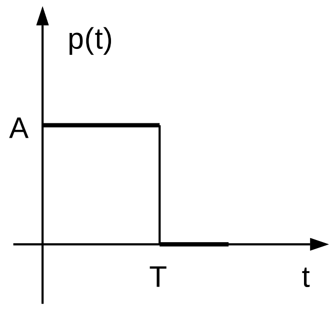
$$S(f) = A^2 T^2 \left(\frac{\sin(\pi f T)}{\pi f T} \right)^2$$

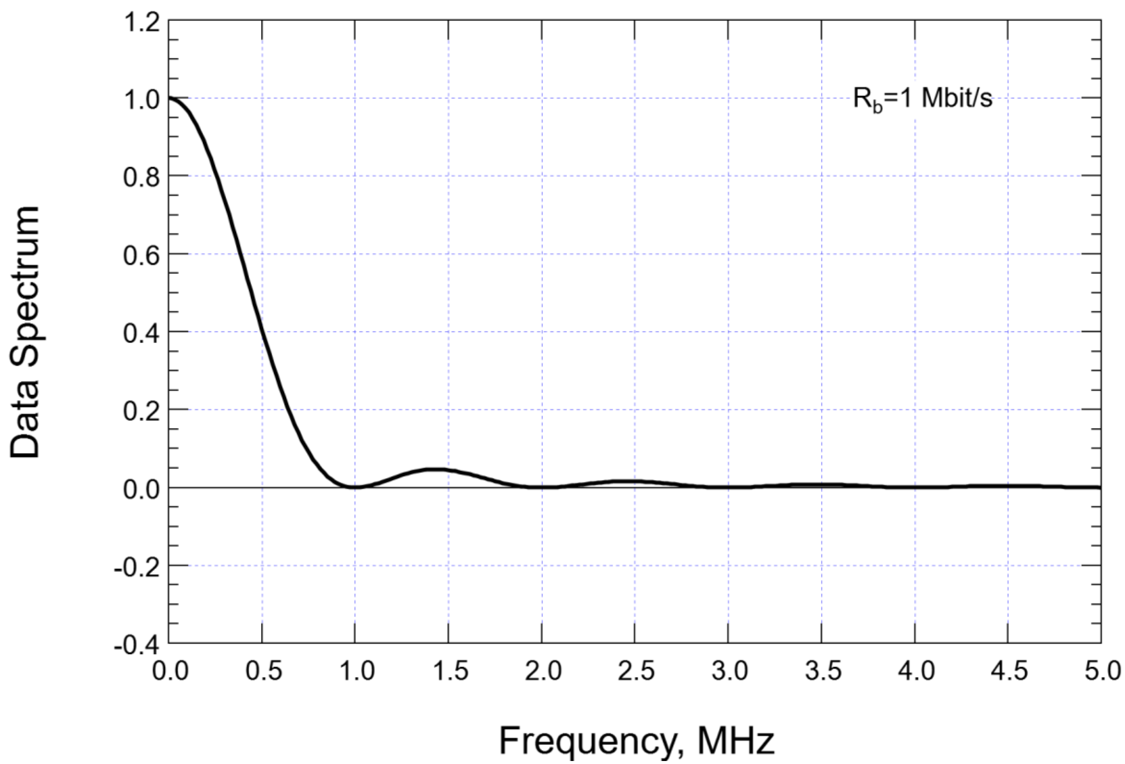




Power Spectral Density (PSD) of the Baseband Data Signal

Bandwdith=1/T=1/T$_b$=R$_b$





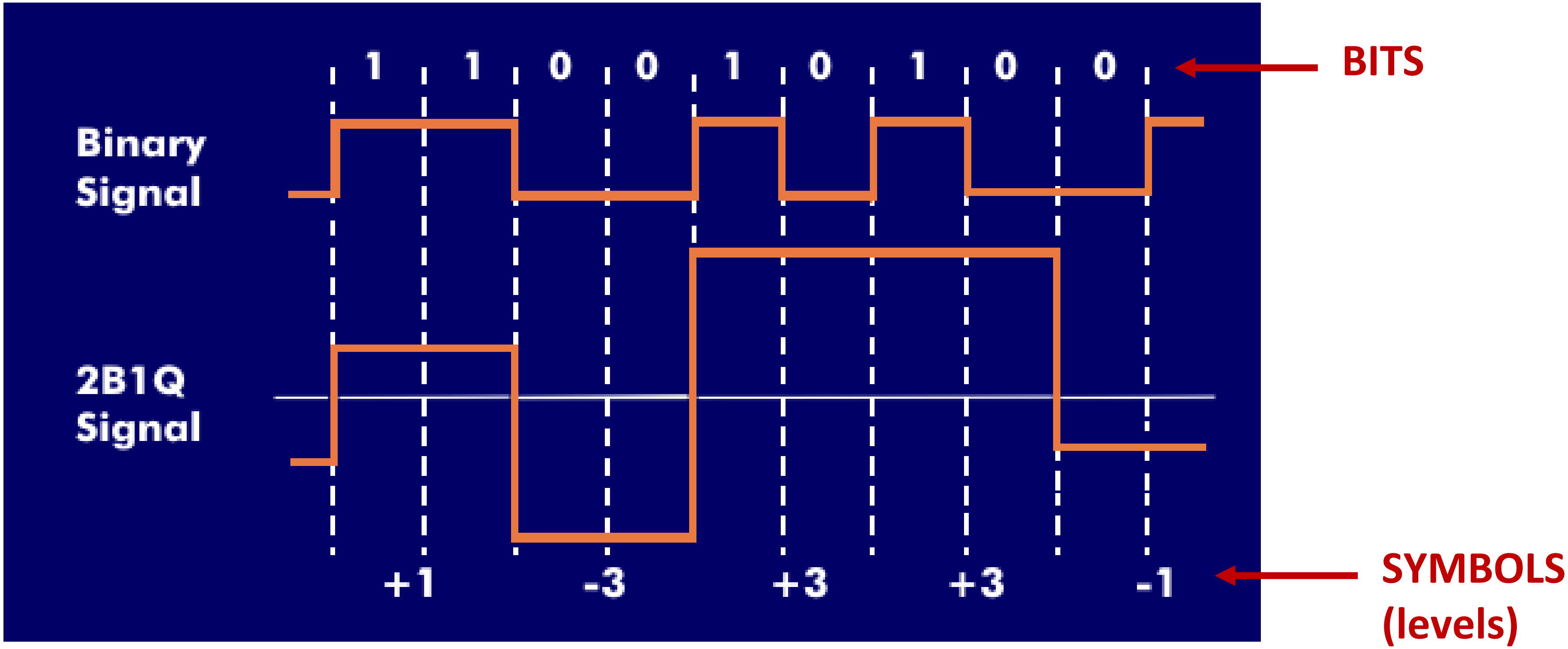
Bit Rate and Bandwidth

Is it true that the bandwdith (Hz) is the same thing as the bit rate (bit/s) of the link?

Is it true that the bandwdith (Hz) is the same thing as the bit rate (bit/s) of the link?

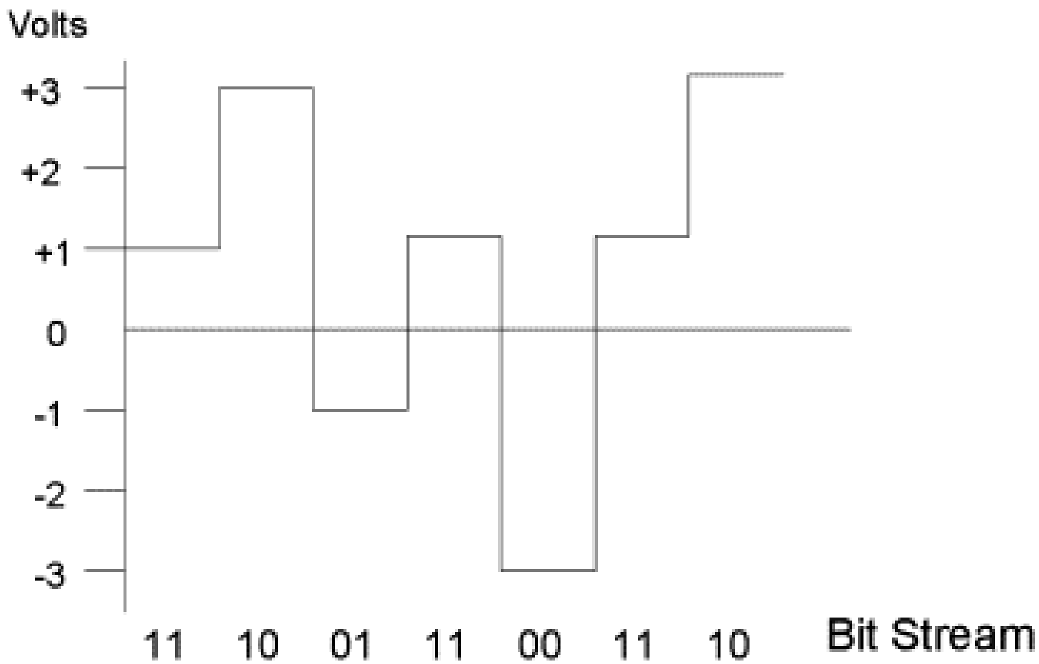
Pre-Internet Technology: ISDN

- 2B+D digital link on telephone lines @ R_b=2*64+16=144 kbit/s



Multilevel data signal…

- **There is a clear difference between *bits* and *symbols***
  - *They are both represented in the physical world by rectangular NRZ pulses*
  - *BUT a symbol pulse* is wider than a bit pulse
  - BECAUSE *a symbol carries more than 1 bit…*



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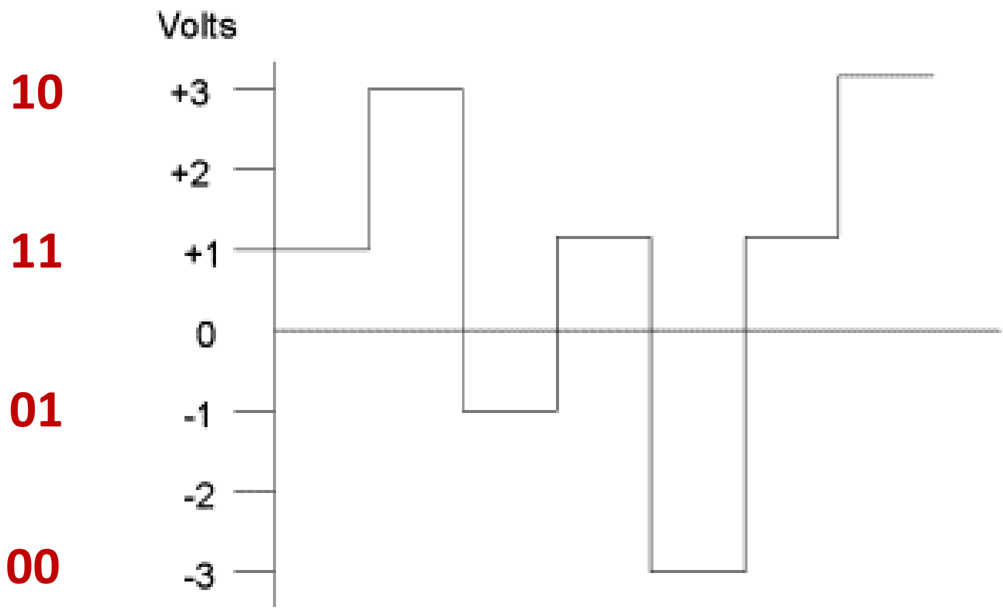
…and its *mapping*

- **Every symbol level is *mapped* to a (small) packet of bits**
  - *The number of bit in a packet is ust equal to the factor by which the symbol is wider in time than a bit*
  - *The receiver uses the reverse mapping to extract logical bits form physical symbol levels*



Bit Rate and Symbol Rate

In this case,

$T_s = 2T_b$

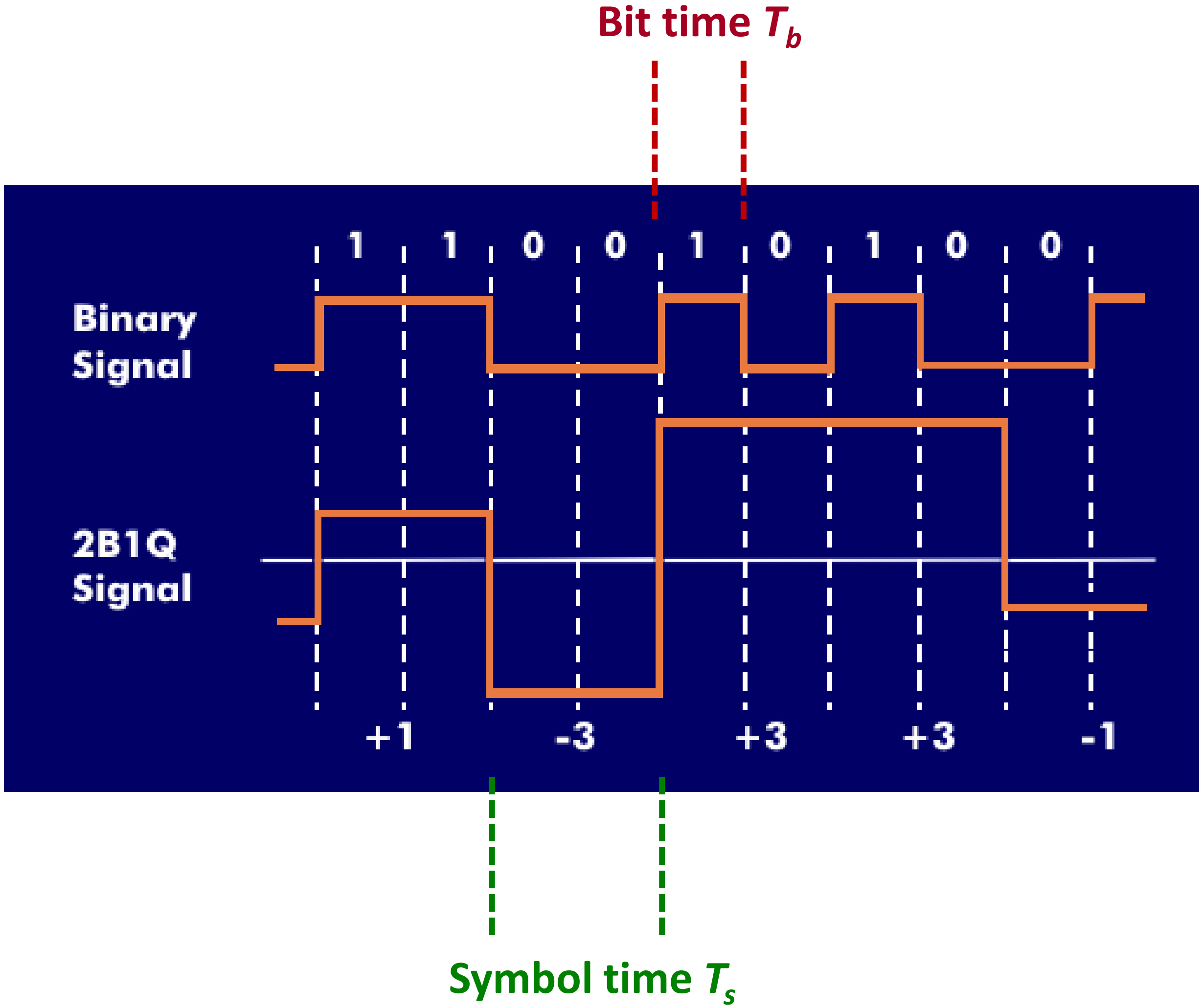
The symbol rate is

$R_s = 1/T_s$

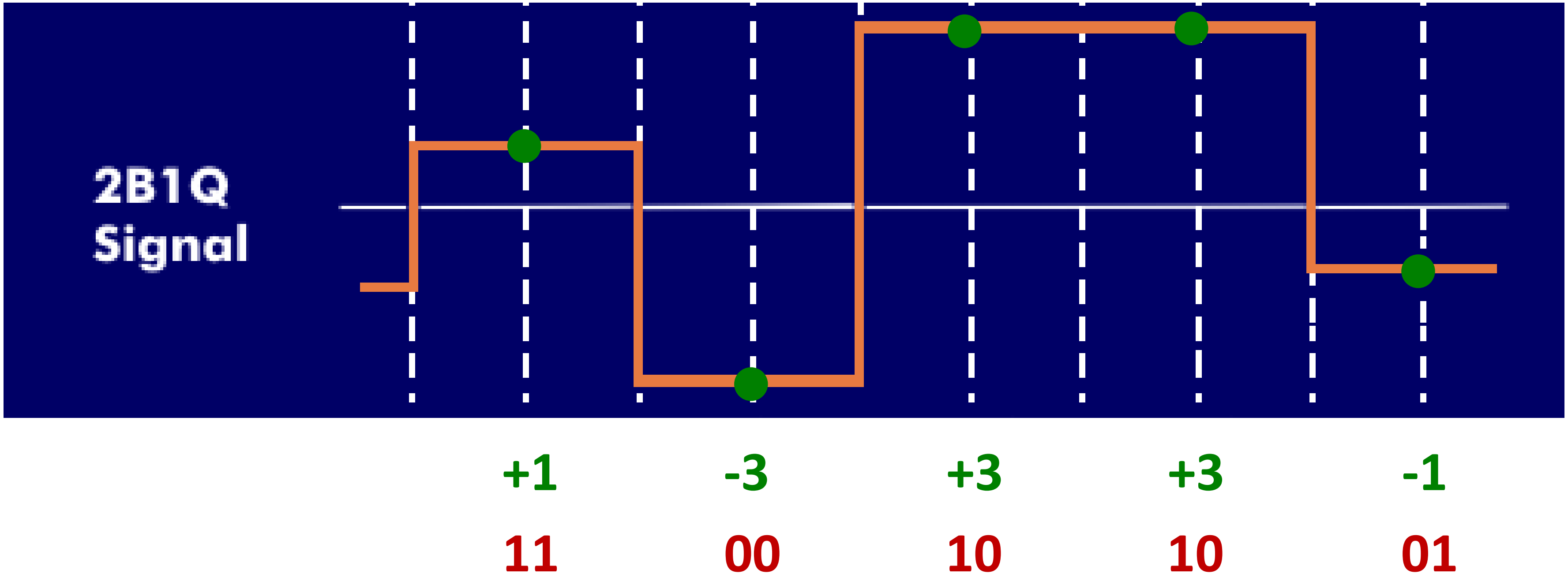
and the bit rate is

$R_b = 2R_s$

The bit rate is always *larger* than the symbol rate because one symbol "carries" more than one bit



Alice & Bob (encoding and decoding)



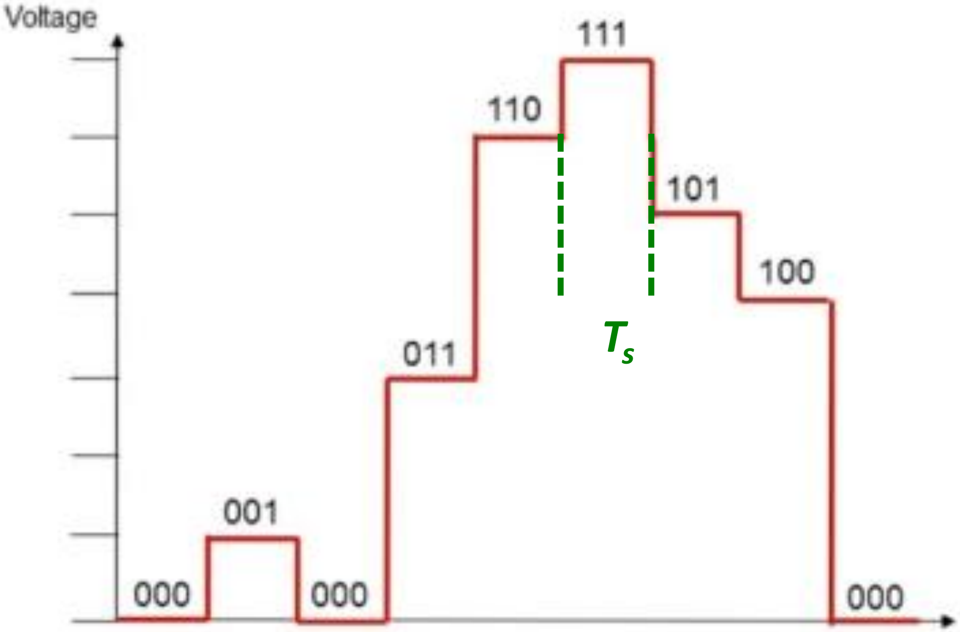
At the receiver, Bob re-clocks the signal at the center of each pulse (symbol) and form the observed values re-construct the stream of **symbols** and **bits**

Bit Rate, Symbol Rate, and Bandwidth 1/2

M=8 symbol levels → 3 bits/symbol

- The time width of any pulse carrying a symbol is T_s

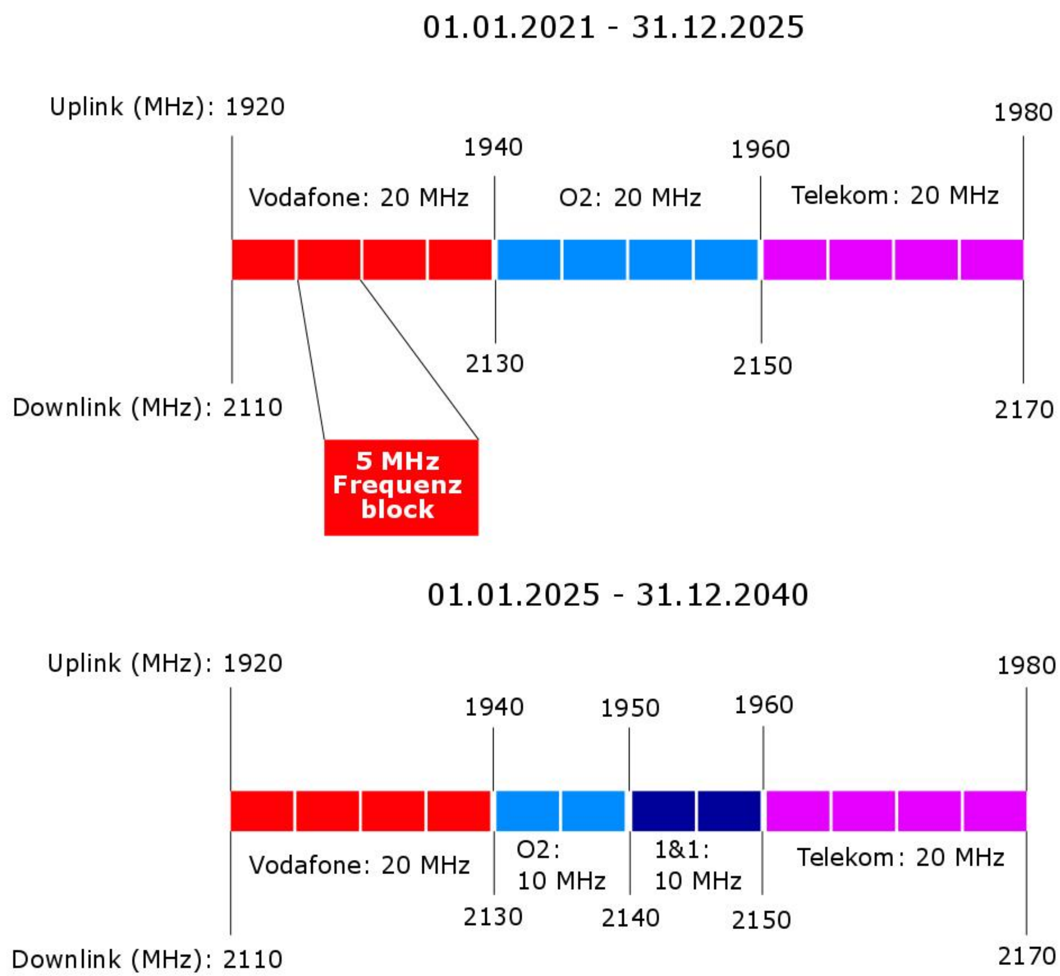
- Therefore, the *bandwidth* of the multilevel data signal is $B=1/T_s$
- The bandwidth is equal to SYMBOL RATE, not to the bit rate !!!



WHAT IS THE PURPOSE OF ALL THIS ?

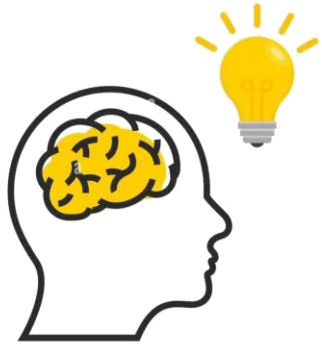
5G Bandwdith Allocation: a Sample from Germany

- A certain mobile operator is assigned very well defined chunks (blocks) of frequency under the regulation of public bodies (Ministry, Agency etc.)
    - Operators compete for frequencies in an AUCTION
- Total revenue to the Ministry from 5G frequencies auction in Italy (same in Germany): 6 G€
- You strive to squeeze as much bitrate as possible into such a bandwidth to serve as much users as possible with high-bit rate services



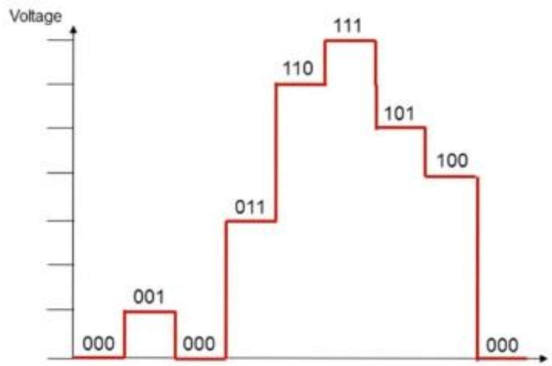
The idea

- The bandwidth *B* is assigned
- $R_s = B$ therefore, the (maximum) symbol rate R_s is assigned as well
- If I want to squeeze as much bitrate as possible into my bandwidth, I have to increase *M*



$$R_b = \log_2(M)\cdot R_s = \log_2(M)\cdot B$$

Number of bits carried by one symbol

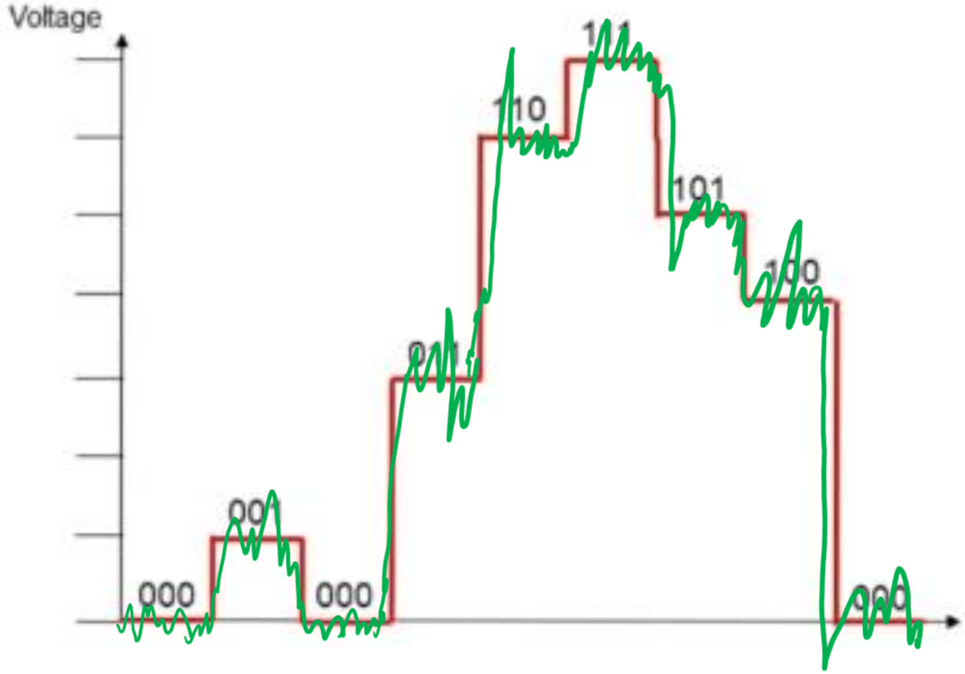


- The bit rate is indeed proportional to the bandwidth, but receives a "boost" from multilevel constellations – **up to *M*=256 levels in 5G** !

Why not increasing *M* as much as possible?

The actual physical signal received by Bob is corrupted by noise, distortion, interference especially when the distance between Alice & Bob is large and the received signals is weak – we have the **green** waveform instead of the ideal, theoretical **red**

M=8 symbol levels → 3 bits/symbol

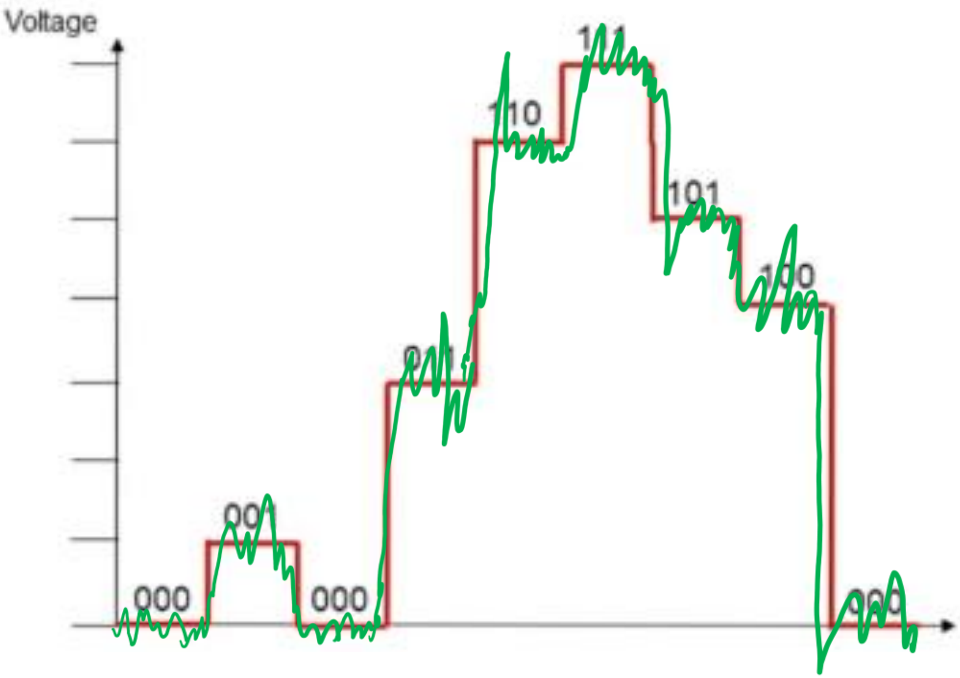


Then what?

Why not increasing *M* as much as possible?

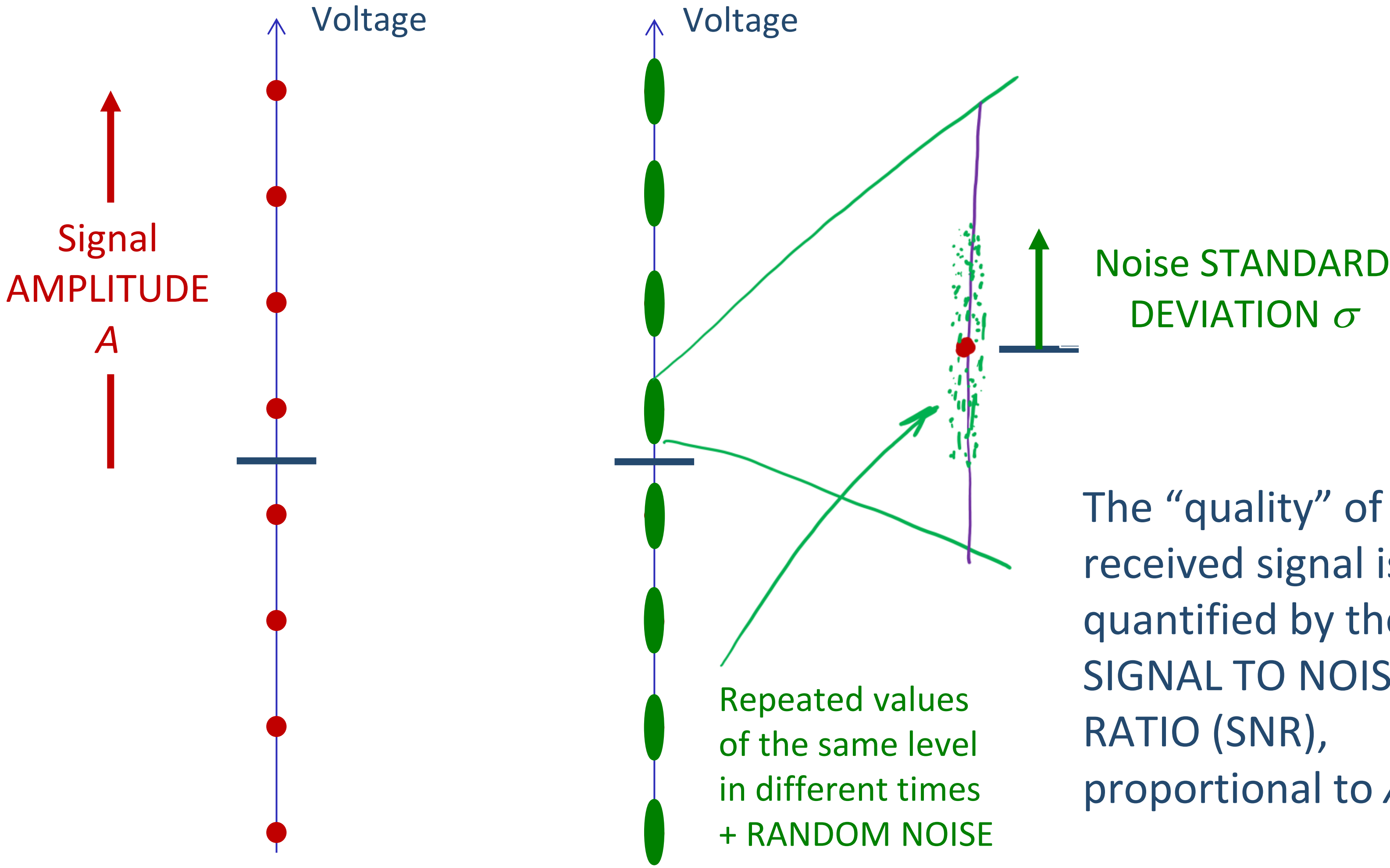
M=8 symbol levels → 3 bits/symbol

Look at the skyline from this “side” observation point



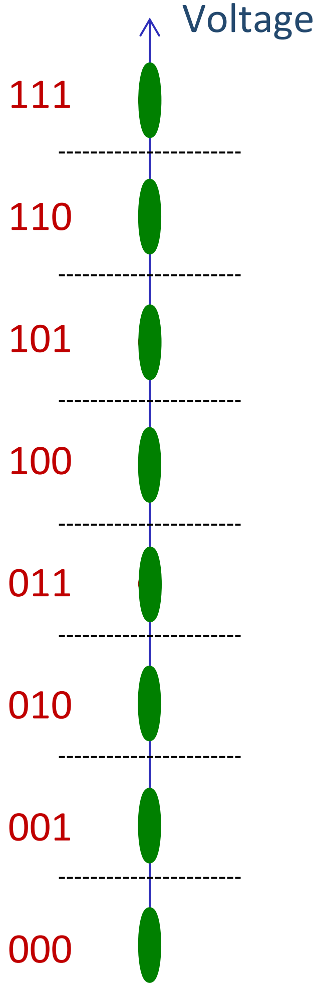
Can we compare the “theoretical” (red) view with the “actual” (green) one?

SIGNAL and NOISE (The Skyline)

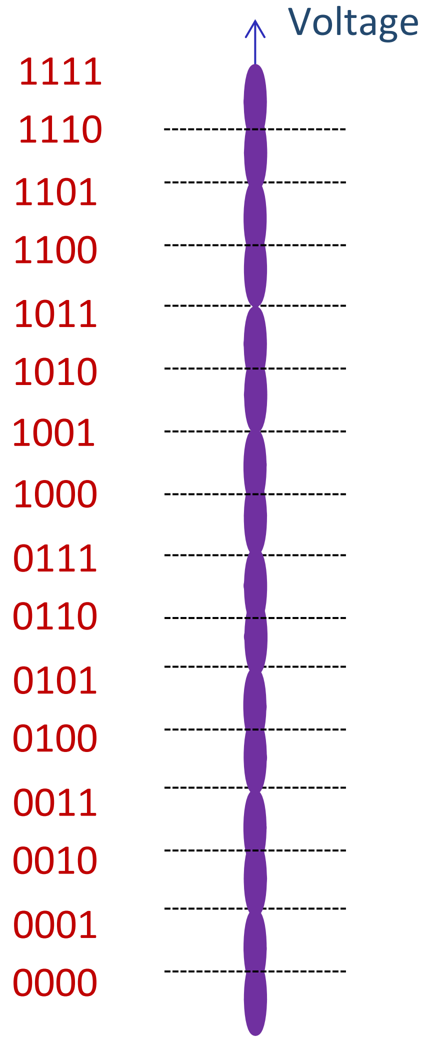


The "quality" of the received signal is quantified by the SIGNAL TO NOISE RATIO (SNR), proportional to A^2/σ^2

Doubling *M*



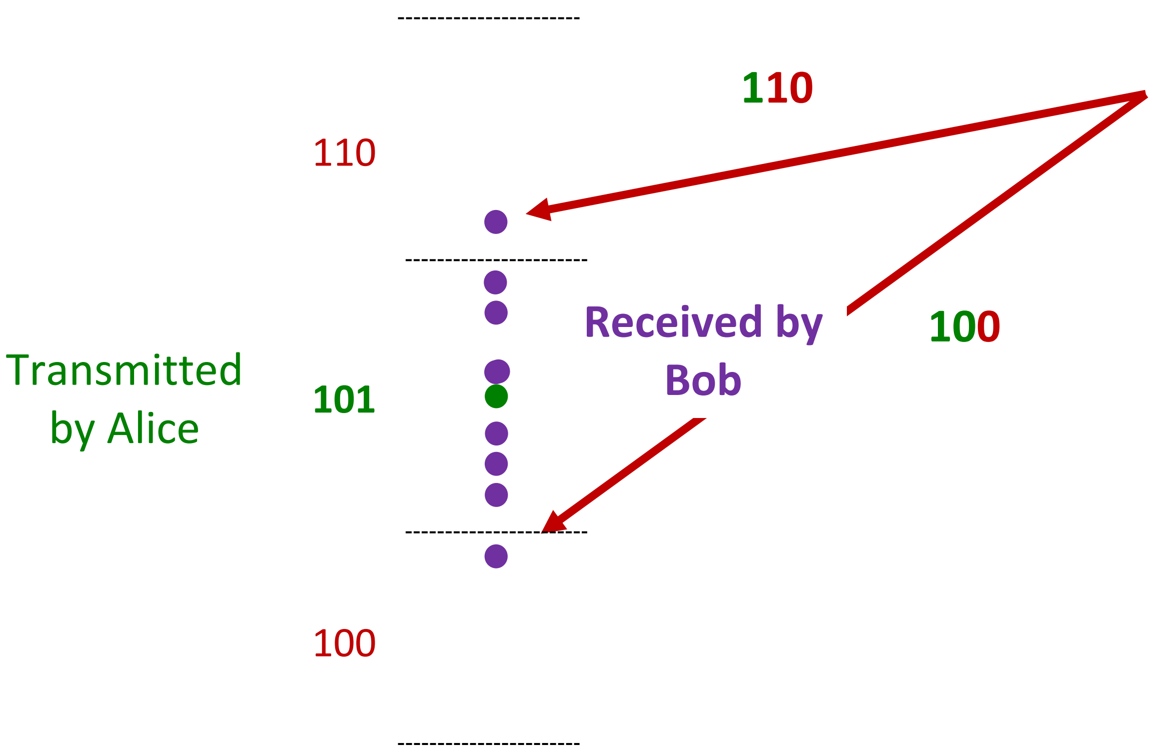
With this SNR, the "clouds" are still inside their own respective "decision regions" separated by the dashed boundaries



For the *same* SNR, the "clouds" invade the neighboring decision regions (the levels are *too many*).

Then what ?

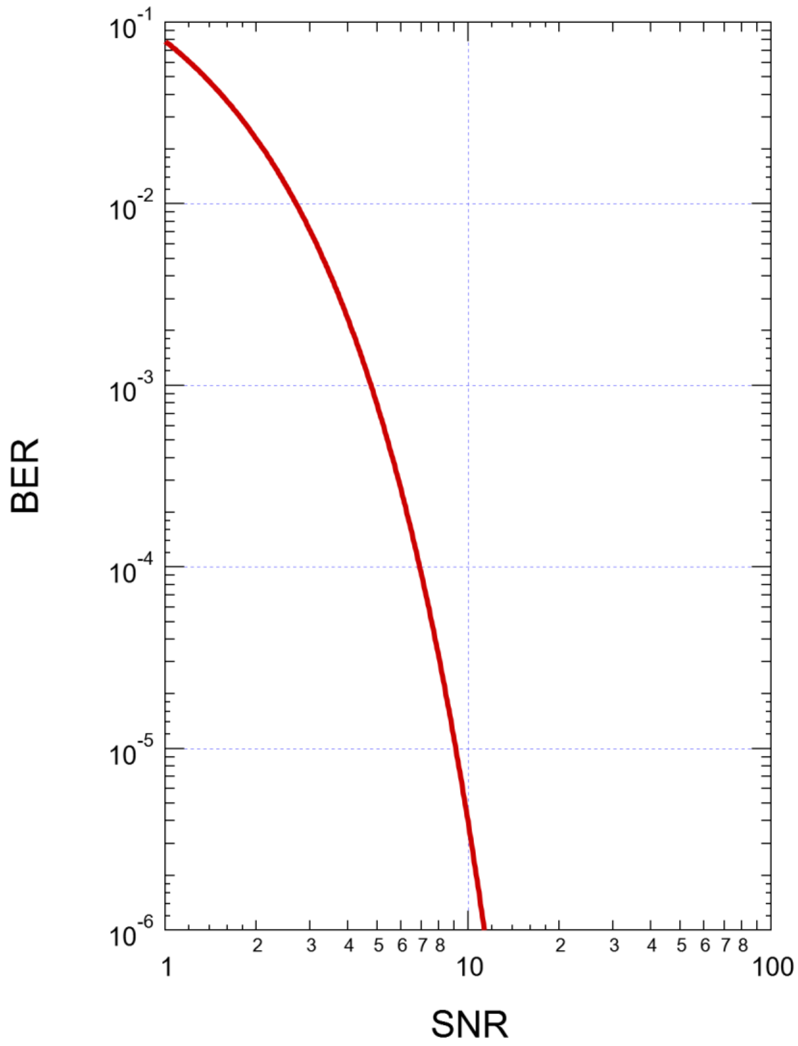
The SER / BER



SYMBOL ERRORS lead to BIT ERRORS….

With a certain frequency, the digital link is affected by bit errors. The BIT ERROR RATE (BER) depends of the SNR on the link

BER vs SNR

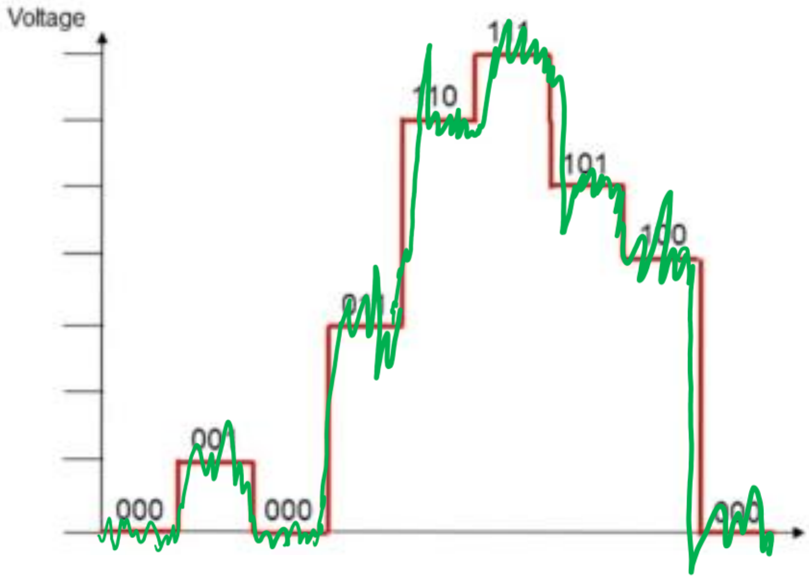


- **The dependence of the BER on the SNR is exponential**
- **The SNR depends on the application: 100 ÷ 1000 for cellular, 1 ÷ 10 for satellite, up to $10^4 \div 10^5$ for ADSL**
- **The effect of the BER strongly depends on the application:**
  - 1 error in 1000 bits (10^{-3}) is good for voice, terrible for image/video
  - Commercial TV needs 10^{-10}
  - Bank transfers $<10^{-12}$

BER Sample – 5 wrong pixels in 10^4

Shannon Capacity

Q



The higher the number of levels *M*, the less robust is the signal versus noise. The upper limit of the bit-rate (i.e., of the constellation size) for a certain SNR to attain RELIABLE COMMUNICATIONS is given by Shannon's capacity formula

$$R_b \leq B \log_2(1+\mathrm{SNR}) \quad (bit/s)$$