COMPITO DEL 12/01/2017 FILA A

Es. 2 - Sia dato il sistema LTI a tempo discreto con risposta in frequenza $\bar{H}(f) = \left[2\sin^2(\pi f T) + \cos(4\pi f T)\right]e^{-j4\pi f T}$.

1) Scrivere la funzione di trasferimento (trasf. z) equivalente e la risposta imp. del sistema

2) Scrivere l'equaz. alle diff. e disegnare la forma canonica del sistema.

3) Calcolare la risposta $y(n) = u(n) \otimes h(n)$ del sistema al gradino unitario $u(n)$ e farne il grafico.

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1) $$\bar{H}(f) = \left[2\sin^2(\pi f T) + \cos(4\pi f T)\right]e^{-j4\pi f T}$$

$$= \left[\cancel{2}\,\frac{1-\cos(2\pi f T)}{\cancel{2}} + \cos(4\pi f T)\right]e^{-j4\pi f T}$$

$$= \left[1 - \frac{e^{j2\pi f T} + e^{-j2\pi f T}}{2} + \frac{e^{j4\pi f T} + e^{-j4\pi f T}}{2}\right]e^{-j4\pi f T}$$

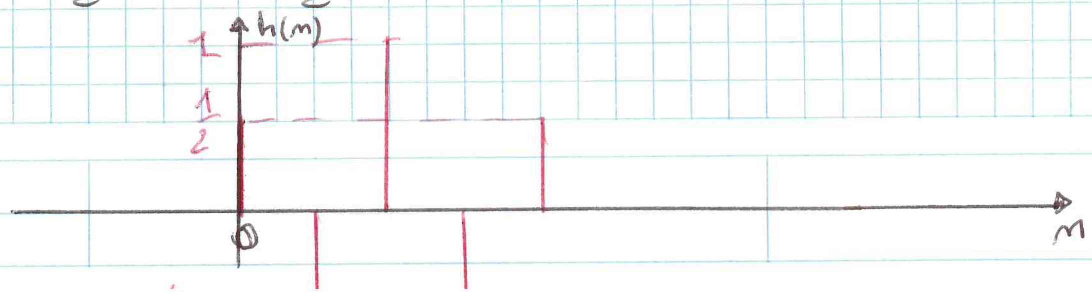
$$z = e^{j2\pi f T}$$

$$H(z) = z^{-2}\left[1 - \frac{z+z^{-1}}{2} + \frac{z^2+z^{-2}}{2}\right]$$

$$= \frac{1 - z^{-1} + 2z^{-2} - z^{-3} + z^{-4}}{2}$$

Filtro FIR

$$h(n) = \frac{1}{2}\delta(n) - \frac{1}{2}\delta(n-1) + \delta(n-2) - \frac{1}{2}\delta(n-3) + \frac{1}{2}\delta(n-4)$$

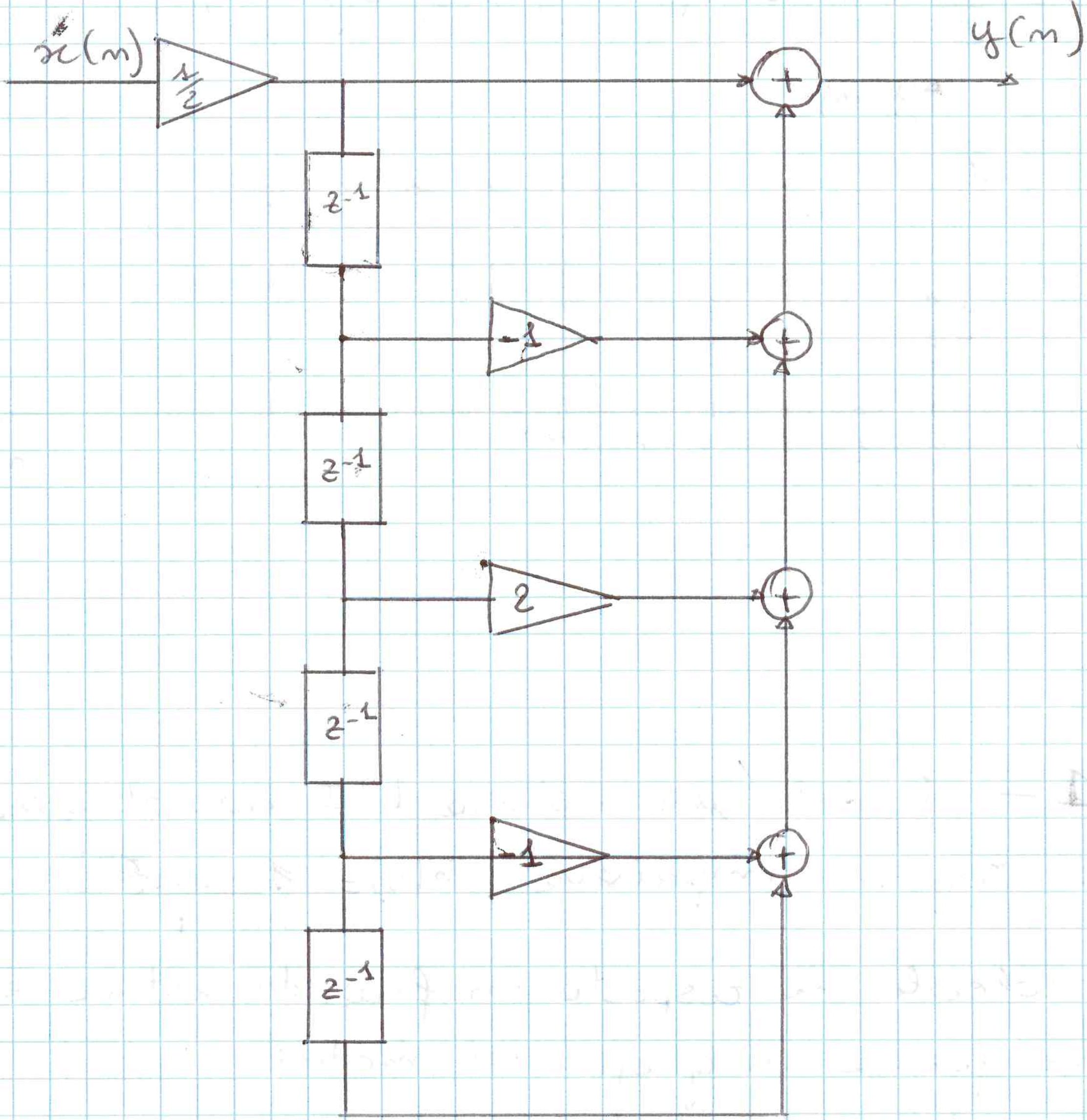


$$2Y(z) = X(z) - z^{-1}X(z) + 2z^{-2}X(z) - z^{-3}X(z) + z^{-4}X(z)$$

$$\Rightarrow y(n) = \frac{1}{2}x(n) - \frac{1}{2}x(n-1) + x(n-2) - \frac{1}{2}x(n-3)$$

$$+ \frac{1}{2}x(n-4)$$

FORMA CANONICA



3) $Y(z) = H(z)\,X(z)$

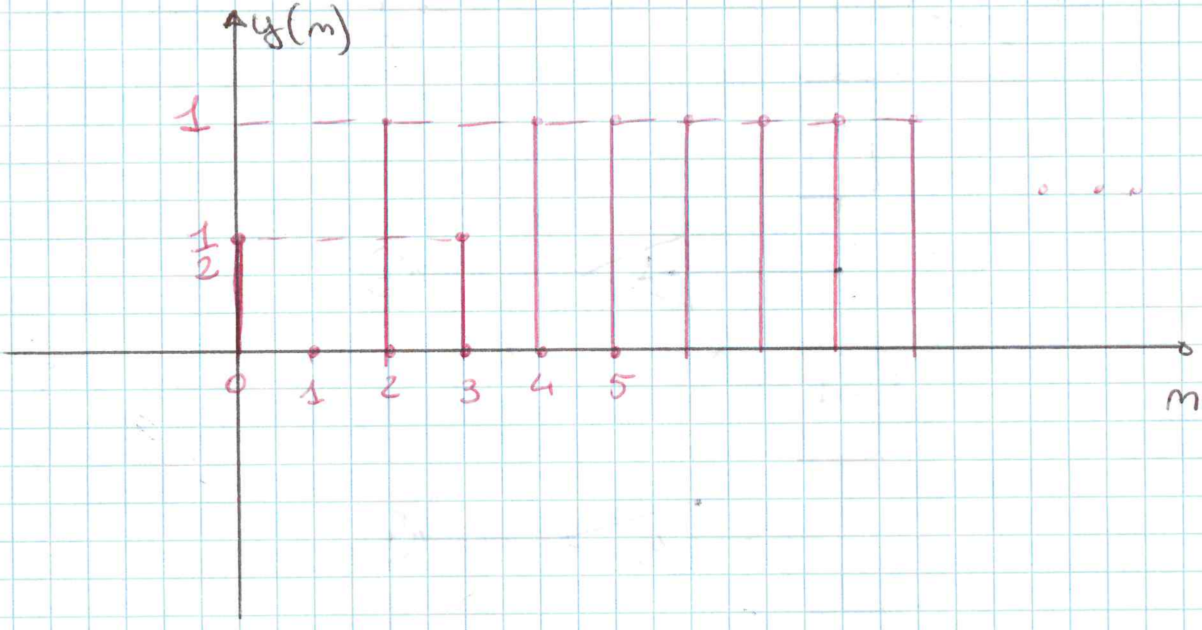
$$X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{1 - z^{-1} + 2z^{-2} - z^{-3} + z^{-4}}{2(1-z^{-1})}$$

$$= \frac{(1-z^{-1}) + 2z^{-2} - z^{-3}(1-z^{-1})}{2(1-z^{-1})}$$

$$= \frac{1}{2} + \frac{z^{-2}}{1-z^{-1}} - \frac{1}{2} z^{-3}$$

$$y(n) = \frac{1}{2}\delta(n) + u(n-2) - \frac{1}{2}\delta(n-3)$$



Es. 1 – È dato un sistema LTI caratterizzato dalla risposta impulsiva $h(t) = \frac{1-\cos(t)}{t}$

1) Si calcoli la risposta in freq. del sistema e se ne faccia il grafico del modulo

2) Il sistema del pt 1 viene messo in serie ad un filtro passa-basso ideale $H_2(f)$ di banda $B = 3/2\pi$. Si faccia il grafico del modulo della risposta equivalente $H_{eq}(f)$ del sistema costituito da $H(f)$ e $H_2(f)$ in cascata.

3) All'ingresso del sistema totale $H_{eq}(f)$ viene posto il segnale $x(t) = e^{-t} u(t)$. Si calcoli la trasformata di Fourier del segnale di uscita $y(t)$ e la sua energia

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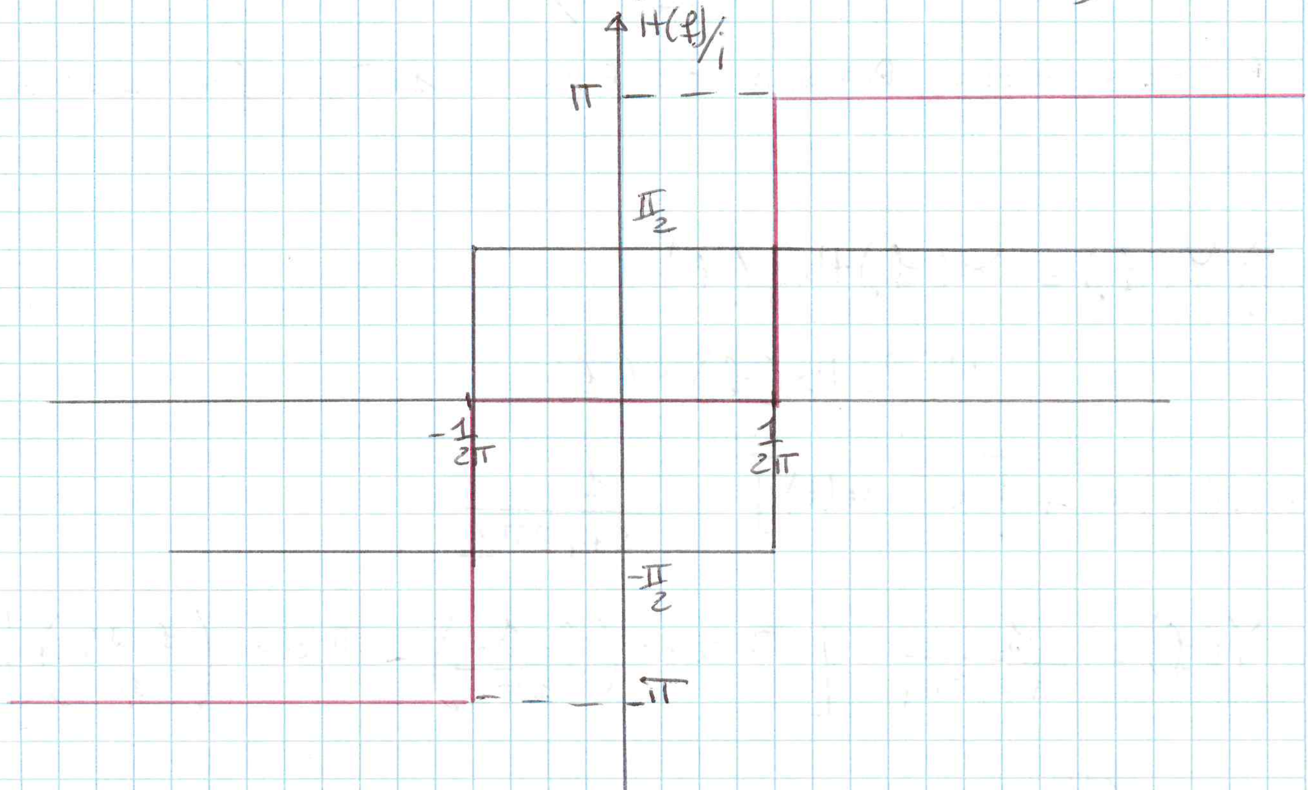
1) $h(t) = -z(t)\cos t$ con $z(t) = 1/t$

$$H(f) = -\frac{1}{2} Z\left(f - \frac{1}{2\pi}\right) - \frac{1}{2} Z\left(f + \frac{1}{2\pi}\right)$$

$$Z(f) = \mathcal{F}[1/t] = -j\pi \, \mathrm{sgn}(f)$$

$$\Rightarrow H(f) = +\frac{1}{2}\pi j \, \mathrm{sgn}\left(f - \frac{1}{2\pi}\right) + \frac{\pi}{2} j \, \mathrm{sgn}\left(f + \frac{1}{2\pi}\right)$$



$$|H(f)| = \pi\, u\left(-f - \frac{1}{2\pi}\right) + \pi\, u\left(f - \frac{1}{2\pi}\right)$$

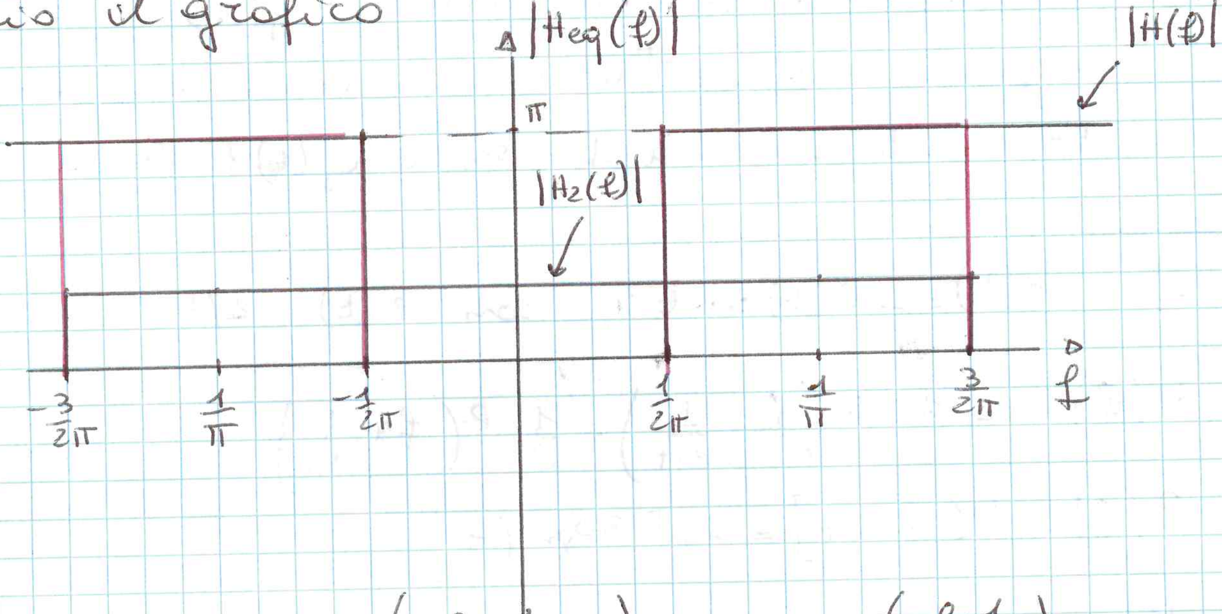
FILTRO PASSA-ALTO DI BANDA B

2) $H_2(f) = \mathrm{rect}\left(\frac{\pi f}{3}\right)$

$$H_{eq}(f) = H(f)\, H_2(f)$$

$$|H_{eq}(f)| = |H(f)|\,|H_2(f)|$$

Faccio il grafico



$$|H_{eq}(f)| = \pi \, rect\left(\frac{f-\frac{1}{\pi}}{\frac{1}{\pi}}\right) + \pi \, rect\left(\frac{f+\frac{1}{\pi}}{\frac{1}{\pi}}\right)$$

FILTRO PASSA-BANDA DI BANDA $1/\pi$ e freq. centrale $1/\pi$.

3) $Y(f) = X(f) H_{eq}(f)$

$$= H(f) H_2(f) X(f)$$

$$X(f) = \mathcal{F}\left[e^{-t} u(t)\right] = \frac{1}{1+j2\pi f}$$

$$Y(f) = \frac{1}{1+j2\pi f}\left[\pi \, rect\left(\frac{f-1/\pi}{1/\pi}\right) - \pi \, rect\left(\frac{f+1/\pi}{1/\pi}\right)\right]$$

$$E_Y = \int_{-\infty}^{+\infty} |Y(f)|^2 df$$ applicando Parseval

Data la simmetria della funzione

$$E_Y = 2\int_{0}^{+\infty} \frac{1}{1+4\pi^2 f^2} \pi^2 \, rect\left(\frac{f-1/\pi}{1/\pi}\right) df$$

$$= 2\pi^2 \int_{\frac{1}{2\pi}}^{\frac{3}{2\pi}} \frac{1}{1+4\pi^2 f^2}\, df$$

Facciamo la sostituzione $x = 2\pi f$, da cui

$$= \pi \int_{1}^{3} \frac{1}{1+x^2}\, dx = \pi \left[\operatorname{arctg} 3 - \operatorname{arctg} 1 \right] =$$

$$= \pi \left[\operatorname{arctg} 3 - \frac{\pi}{4} \right]$$

I poli sono complessi coniugati

$$z_{1/2} = \pm j\sqrt{\frac{1}{3}} = \pm j\,\frac{\sqrt{3}}{3}$$

La zona di convergenza causale è $|z| > \frac{\sqrt{3}}{3}$

2) Calcoliamo la risposta impulsiva del sistema

$$H(z) = \frac{2}{3}\left[\frac{z^2+1}{z^2+\frac{1}{3}}\right] = \frac{2}{3}\left[1 + \frac{2}{3}\,\frac{1}{z^2+\frac{1}{3}}\right]$$

$$= \frac{2}{3} + \frac{4}{9}\,\frac{1}{z^2+\frac{1}{3}}$$

$$\frac{z^2}{z^2+\frac{1}{3}} \Rightarrow \left(\frac{\sqrt{3}}{3}\right)^n \cos\left(n\frac{\pi}{2}\right)u(n)$$

$$\frac{1}{z^2+\frac{1}{2}} \Rightarrow \left(\frac{\sqrt{3}}{3}\right)^{n-2} \cos\left[(n-2)\frac{\pi}{2}\right]u(n-2)$$

Di conseguenza

$$h(n) = \frac{2}{3}\,\delta(n) + \frac{4}{9}\left(\frac{\sqrt{3}}{3}\right)^{n-2} \cos\left[(n-2)\frac{\pi}{2}\right]u(n-2)$$

$h(0) = \frac{2}{3}$ $\quad$ $h(n = \text{dispari}) = 0$

$h(2) = \frac{4}{9}$

3) $Y(z) = H(z)\,X(z)$

$$X(z) = \frac{z}{z-\frac{1}{4}}$$

$$\Rightarrow Y(z) = \frac{2}{3}\,\frac{z^2+1}{\left(z^2+\frac{1}{2}\right)}\cdot\frac{z}{\left(z-\frac{1}{4}\right)}$$

Non è una frazione propria poichè $N = M$.
Consideriamo dunque $Y(z)/z$

$$\frac{Y(z)}{z} = \frac{2}{3}\left[\frac{z^2+1}{\left(z^2+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right] = \frac{2}{3}\left[\frac{A_1}{z-j\frac{\sqrt{2}}{2}} + \frac{A_1^*}{z+j\frac{\sqrt{2}}{2}} + \frac{A_2}{z-\frac{1}{4}}\right]$$

$$A_1 = \left.\frac{z^2+1}{\left(z-\frac{1}{4}\right)\left(z+j\frac{\sqrt{2}}{2}\right)}\right|_{z=j\frac{\sqrt{2}}{2}} = \frac{-\frac{1}{2}+1}{\left(j\frac{\sqrt{2}}{2}-\frac{1}{4}\right)j\sqrt{2}} = -\frac{2}{4+j\sqrt{2}}$$

$$A_1^* = -\frac{2}{4-j\sqrt{2}}$$

$$A_2 = \left.\frac{z^2+1}{z^2+\frac{1}{2}}\right|_{z=\frac{1}{4}} = \frac{\frac{1}{16}+1}{\frac{1}{16}+\frac{1}{2}} = \frac{17}{9}$$

$$Y(z) = \frac{2}{3}\left[-\frac{2}{4+j\sqrt{2}}\,\frac{z}{z-j\frac{\sqrt{2}}{2}} - \frac{2}{4-j\sqrt{2}}\,\frac{z}{z+j\frac{\sqrt{2}}{2}} + \frac{17}{9}\,\frac{z}{z-\frac{1}{4}}\right]$$

$$y(n) = \frac{2}{3}\left[-\frac{2}{3\sqrt{2}}e^{-j\theta_0}\left(j\frac{\sqrt{2}}{2}\right)^n u(n) - \frac{2}{3\sqrt{2}}e^{j\theta_0}\left(-j\frac{\sqrt{2}}{2}\right)^n u(n) + \frac{17}{9}\left(\frac{1}{4}\right)^n u(n)\right]$$

dove $\theta_0 = \operatorname{arctg}\frac{\sqrt{2}}{4}$

$$= -\frac{2}{3}\,\frac{2}{3\sqrt{2}}\left(\frac{\sqrt{2}}{2}\right)^n u(n)\left[e^{-j\theta_0}e^{jn\frac{\pi}{2}} + e^{j\theta_0}e^{-jn\frac{\pi}{2}}\right]$$

$$+\frac{34}{27}\left(\frac{1}{4}\right)^n u(n)$$

correggere cont.

$$= -\frac{8}{9\sqrt{2}}\left(\frac{\sqrt{2}}{2}\right)^n \cos\left(n\frac{\pi}{2}-\theta_0\right)u(n)$$

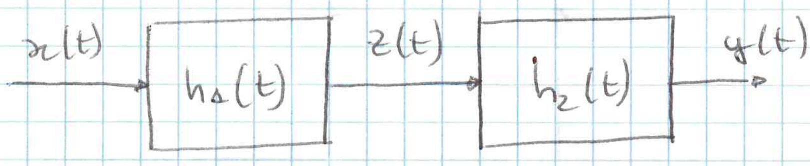
$$+\frac{34}{27}\left(\frac{1}{4}\right)^n u(n)$$

Es.2 - Sia dato il sistema LTI in cui $h_1(t) = e^{-|t|/T}$ e $h_2(t) = \frac{1}{T}\,\mathrm{sinc}\left(\frac{2t}{T}\right)\cos\left(\frac{6\pi t}{T}\right)$.

1) Calcolare la risposta in frequenza del sistema e fare il grafico del modulo.

Si supponga ora che il segnale di ingresso sia dato da $x(t) = \mathrm{sinc}\left(\frac{t}{T}\right)c(t)$, con $c(t) = \sum_{n=-\infty}^{+\infty}(-1)^n\delta(t-nT)$.

2) Calcolare ~~l'energia~~ lo spettro del segnale di uscita $y(t)$.



1) $h_1(t) = e^{-|t|/T} \Leftrightarrow H_1(f) = \dfrac{\frac{2}{T}}{\frac{1}{T^2}+4\pi^2 f^2}$

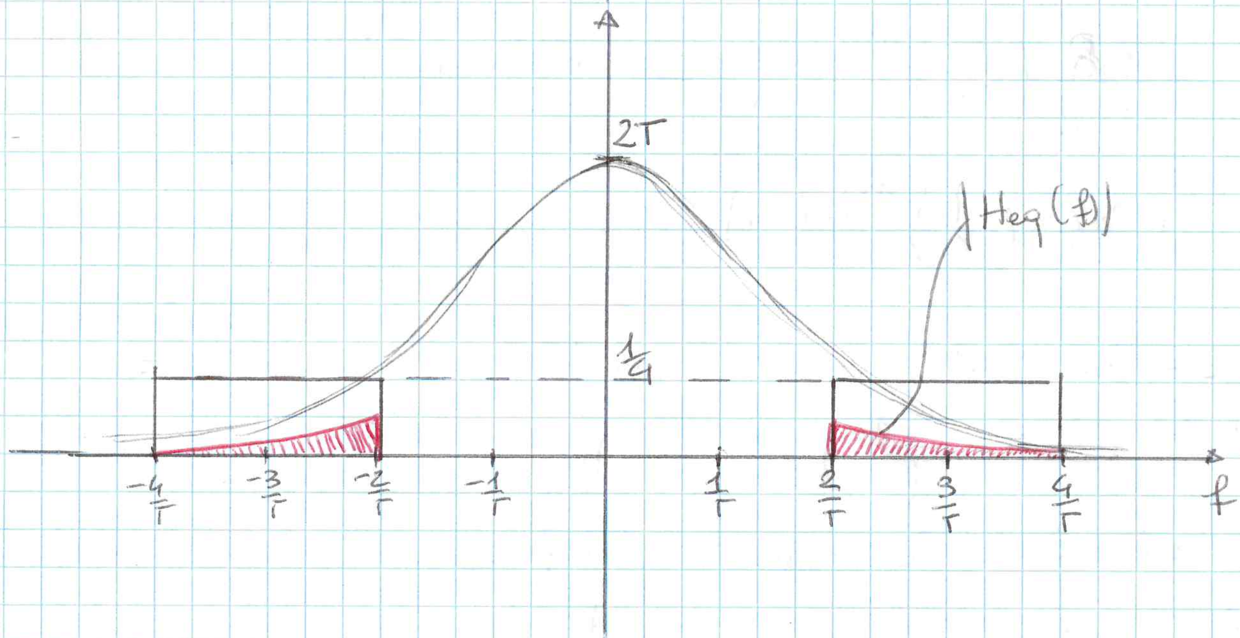
$h_2(t) = r(t)\cos\left(\frac{6\pi t}{T}\right)$ dove $r(t) = \frac{1}{T}\,\mathrm{sinc}\left(\frac{2t}{T}\right)$

$$H_2(f) = \frac{1}{2}R\left(f-\frac{3}{T}\right) + \frac{1}{2}R\left(f+\frac{3}{T}\right)$$

$$R(f) = \frac{1}{T}\,\frac{T}{2}\,\mathrm{rect}\left(\frac{f}{2/T}\right) = \frac{1}{2}\,\mathrm{rect}\left(\frac{Tf}{2}\right)$$

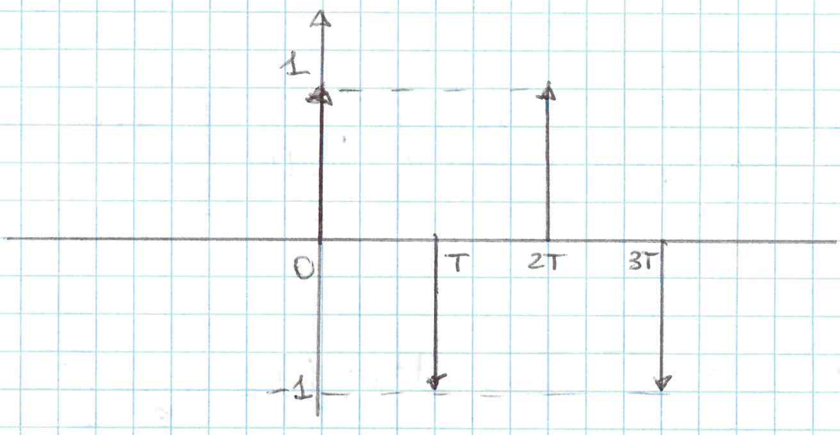
$$H_2(f) = \frac{1}{4}\,\mathrm{rect}\left(\frac{f-\frac{3}{T}}{2/T}\right) + \frac{1}{4}\,\mathrm{rect}\left(\frac{f+\frac{3}{T}}{2/T}\right)$$

$$|H_{eq}(f)| = |H_1(f)|\,|H_2(f)|$$



2) $x(t) = \mathrm{sinc}\left(\frac{t}{T}\right) c(t) \qquad c(t) = \sum_{n=-\infty}^{+\infty} (-1)^n \delta(t-nT)$

$$X(f) = T\,\mathrm{rect}(fT) \otimes C(f)$$



$$c_0(t) = \delta(t) - \delta(t-T) \qquad c(t) = \sum_{n=-\infty}^{+\infty} c_0(t-2nT)$$

$$C(f) = \sum_{k=-\infty}^{+\infty} C_k\,\delta\left(f-\frac{k}{2T}\right) \qquad C_k = \frac{1}{2T}\,C_0\left(\frac{k}{2T}\right)$$

$$c_0(f) = 1 - e^{-j2\pi f T} = e^{-j\pi f T}\, 2j \sin(\pi f T)$$

$$c_k = \frac{1}{\cancel{2}T}\, e^{-j\pi \frac{k}{2\cancel{T}}\cancel{T}}\, \cancel{2} j \sin\left(\pi \frac{k}{2\cancel{T}} \cancel{T}\right) =$$

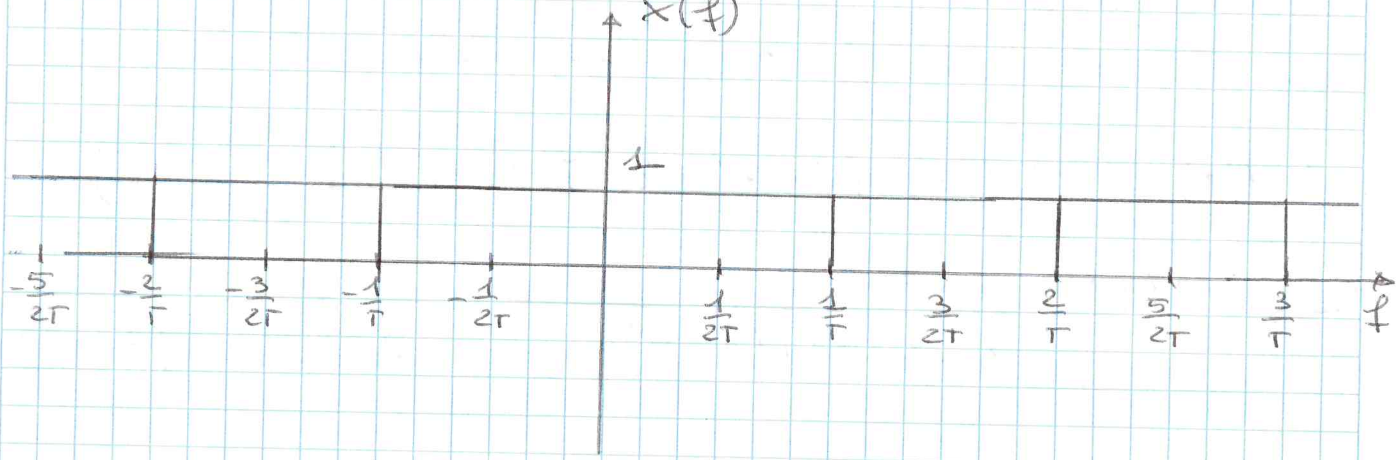
$$= \frac{1}{T}\, e^{-j\frac{k\pi}{2}}\, j \sin\left(\frac{k\pi}{2}\right)$$

$$X(f) = \sum_{k=-\infty}^{+\infty} j\, e^{-j\frac{k\pi}{2}} \sin\left(\frac{k\pi}{2}\right) \mathrm{rect}\left(\frac{f - \frac{k}{2T}}{1/T}\right)$$

$$\left.\begin{array}{ll} k = 0 & c_k = 0 \\ k = 1 & c_k = j(-j) = 1 \\ k = 2 & c_k = 0 \\ k = -1 & c_k = j\, j(-1) = 1 \end{array}\right\} \quad \begin{array}{l} c_k = 0 \text{ per } k \text{ pari} \\ c_k = 1 \text{ per } k \text{ dispari} \end{array}$$

$$\Rightarrow X(f) = \sum_{k=-\infty}^{+\infty} \mathrm{rect}\left(\frac{f - \frac{(2k+1)}{2T}}{1/T}\right)$$

Facciamo il grafico



$$X(f) = 1$$

Si poteva vedere banalmente che $x(t) = \mathrm{sinc}\left(\frac{t}{T}\right) c(t)$ ha un unico campione $\neq 0$ in 0

$\Rightarrow x(t) = \delta(t) \quad \Rightarrow X(f) = 1$

$Y(f) = H_{eq}(f)$

COMPITO 12/01/17 FILA C

Es. 1 - Sia dato un sistema lineare tempo-inv. discreto caratterizzato dall'equaz. alle differenze

$4y(n) - 4y(n-1) + (1-\alpha^2)\, y(n-2) = x(n) - x(n-1)$,

dove α è un parametro reale positivo.

1) Si trovino i valori di α per cui il sistema causale risulta stabile.

2) Si fissi $\alpha = 1/2$; si disegni la forma canonica del sistema e si calcoli la risposta impulsiva $h(n)$.

3) Si calcoli la risposta del sistema al segnale $x(n) = \left(\frac{1}{4}\right)^n u(n)$

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1) Perché il sistema causale risulti stabile tutti i poli devono avere $|\,| < 1$. Calcoliamo $H(z)$.

$$4\,Y(z) - 4z^{-1}Y(z) + (1-\alpha^2)\, z^{-2}Y(z) = X(z) - z^{-1}X(z)$$

$$\Rightarrow \quad H(z) = \frac{1 - z^{-1}}{4 - 4z^{-1} + (1-\alpha^2)z^{-2}} = \frac{z(z-1)}{4z^2 - 4z + (1-\alpha^2)}$$

Calcoliamo i poli

$$4z^2 - 4z + (1-\alpha^2) = 0 \qquad z_{1/2} = \frac{4 \pm \sqrt{16 - 16(1-\alpha^2)}}{8}$$

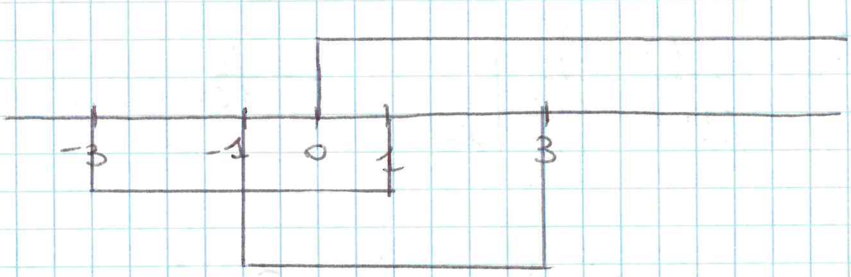
$$z_1 = \frac{1-\alpha}{2} \; ; \quad z_2 = \frac{1+\alpha}{2}$$

I valori di α accettabili sono quelli che soddisfano le 3 relazioni

1) $\alpha > 0$
2) $\left|\frac{1-\alpha}{2}\right| < 1$
3) $\left|\frac{1+\alpha}{2}\right| < 1$

$$\Rightarrow \begin{cases} \alpha > 0 \\ -1 < \frac{1+\alpha}{2} < 1 \\ -1 < \frac{1-\alpha}{2} < 1 \end{cases}$$

$$\begin{cases} \alpha > 0 \\ -3 < \alpha < 1 \\ \alpha > -1, \; \alpha < 3 \end{cases}$$



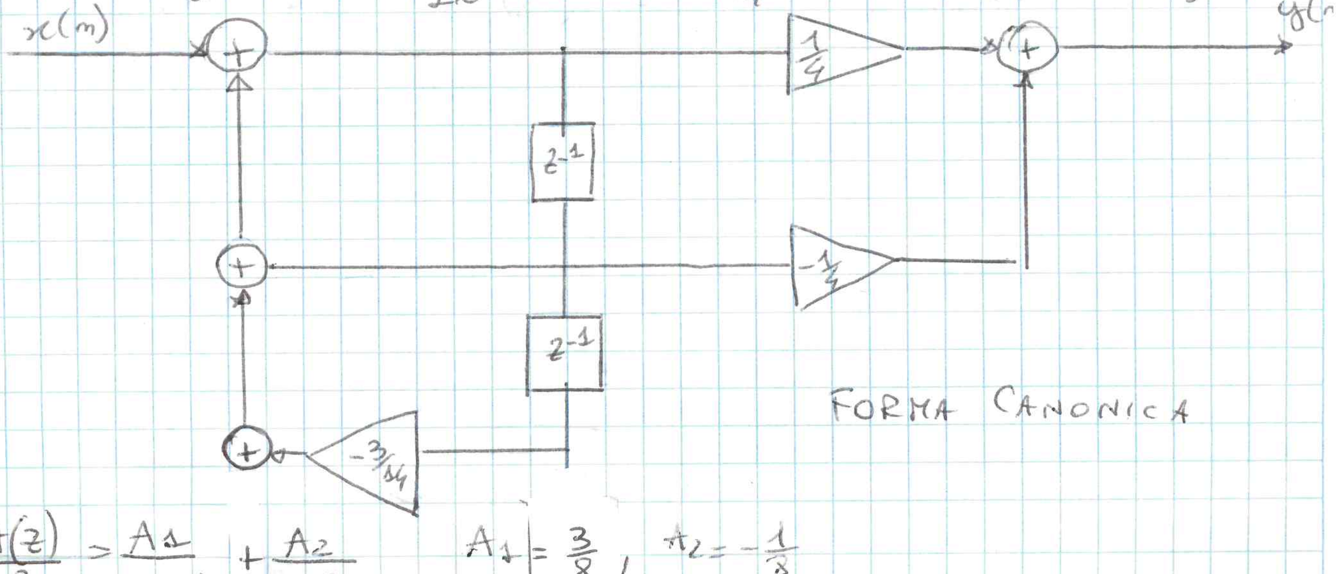
$$\Rightarrow \boxed{0 < \alpha < 1}$$

2) Con $\alpha = \frac{1}{2}$ il sistema è stabile e i 2 poli sono

$z_1 = \frac{1}{4}$ $\quad z_2 = \frac{3}{4}$

$$\Rightarrow \quad H(z) = \frac{z(z-1)}{4\left(z-\frac{1}{4}\right)\left(z-\frac{3}{4}\right)}$$

$$y(n) = y(n-1) - \frac{3}{16}\, y(n-2) + \frac{1}{4}\, x(n) - \frac{1}{4}\, x(n-1)$$



FORMA CANONICA

$$\frac{H(z)}{z} = \frac{A_1}{z-\frac{1}{4}} + \frac{A_2}{z-\frac{3}{4}} \qquad A_1 = \frac{3}{8}, \quad A_2 = -\frac{1}{8}$$

$$h(n) = \frac{3}{8}\left(\frac{1}{4}\right)^n u(n) - \frac{1}{8}\left(\frac{3}{4}\right)^n u(n)$$

3) Risposta a $x(n) = \left(\frac{1}{4}\right)^n u(n)$

$$Y(z) = \frac{z^2(z-1)}{4\left(z-\frac{3}{4}\right)\left(z-\frac{1}{4}\right)^2}$$

$$\frac{Y(z)}{z} = \frac{A_1}{z-\frac{3}{4}} + \frac{A_2}{z-\frac{1}{4}} + \frac{A_3}{\left(z-\frac{1}{4}\right)^2}$$

$$A_1 = \left.\frac{z(z-1)}{4\left(z-\frac{1}{4}\right)^2}\right|_{z=\frac{3}{4}} = -\frac{3}{16}$$

$$A_3 = \left.\frac{z(z-1)}{4\left(z-\frac{3}{4}\right)}\right|_{z=\frac{1}{4}} = \frac{3}{32}$$

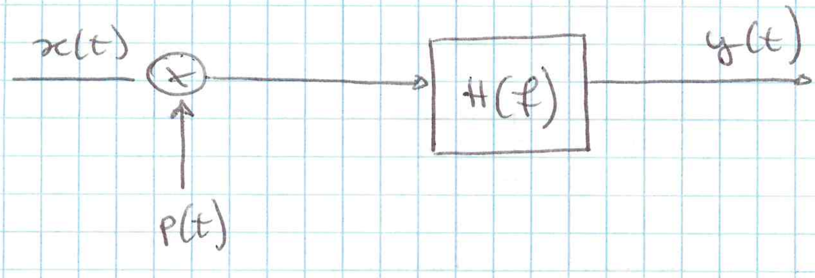
Per calcolare A_2 poniamo $z=1 \Rightarrow Y(z)=0$

$$-\frac{3}{16}\frac{1}{1-\frac{3}{4}} + \frac{A_2}{1-\frac{1}{4}} + \frac{3}{32}\frac{1}{\left(1-\frac{1}{4}\right)^2} = 0$$

$$-\frac{3}{4} + \frac{4}{3}A_2 + \frac{1}{6} = 0 \qquad A_2 = \frac{7}{16}$$

$$y(n) = -\frac{3}{16}\left(\frac{3}{4}\right)^n u(n) + \frac{7}{16}\left(\frac{1}{4}\right)^n u(n) + \frac{3}{8}\, n\left(\frac{1}{4}\right)^n u(n)$$

Es. 2 – Sia dato il sistema di fig. 1

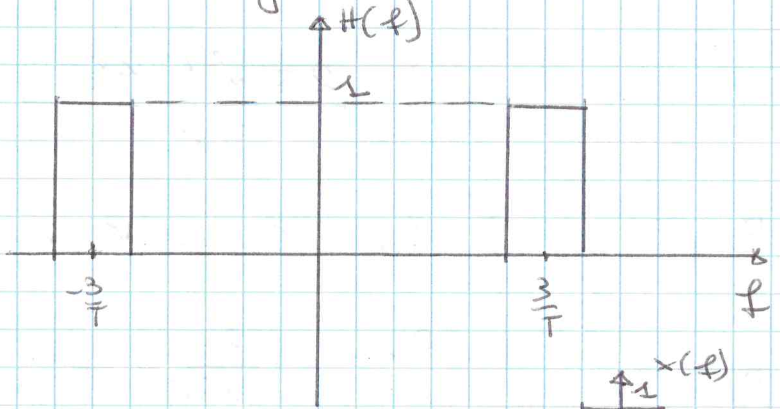


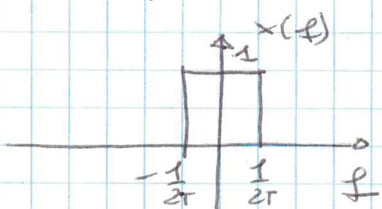
Il segnale di ingresso è $x(t) = \frac{1}{T}\,\mathrm{sinc}\left(\frac{t}{T}\right)$. Il segnale $p(t)$ è l'onda quadra così definita

$$p(t) = \sum_{m=-\infty}^{+\infty}\left(\mathrm{rect}\left(\frac{t-T/4-mT}{T/2}\right) - \mathrm{rect}\left(\frac{t+T/4-mT}{T/2}\right)\right).$$

1) Dopo aver disegnato $p(t)$, si calcoli lo spettro del segnale $z(t) = x(t)\,p(t)$ e se ne faccia il grafico del modulo.

2) Il filtro $H(f)$ è un passa-banda come in fig. 2. Si calcoli lo spettro e l'andamento temporale del segnale all'uscita $y(t)$.

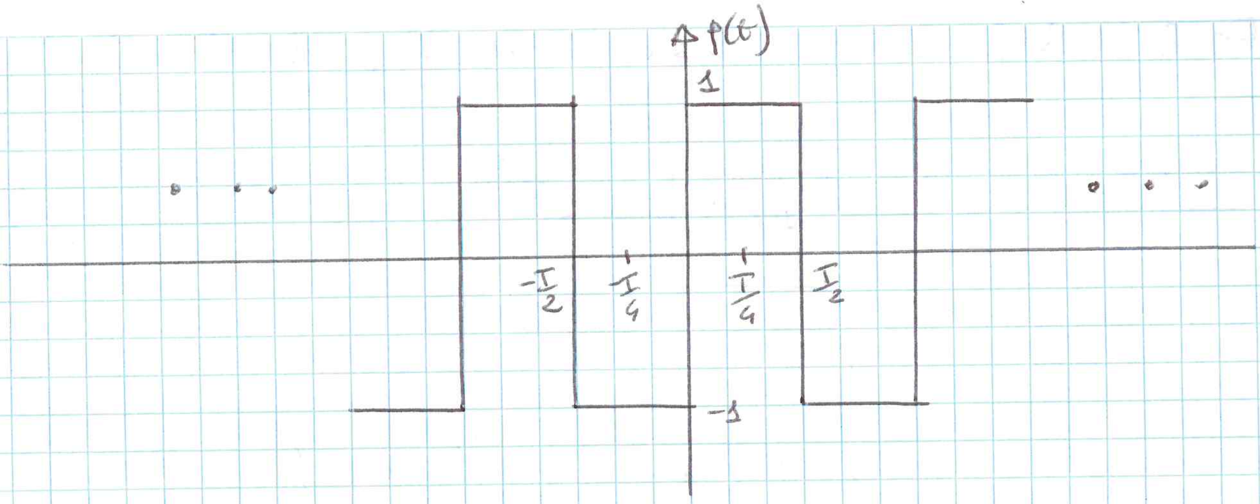




1) $X(f) = \mathrm{rect}(fT)$

$$p(t) = \sum_{k=-\infty}^{+\infty} p_0(t) \quad \text{con}$$

$$p_0(t) = \mathrm{rect}\left(\frac{t-T/4}{T/2}\right) - \mathrm{rect}\left(\frac{t+T/4}{T/2}\right)$$



p(t) è periodico di T

$$P_0(f) = \frac{T}{2}\,\mathrm{sinc}\left(\frac{fT}{2}\right) e^{-j\pi f \frac{T}{2}} - \frac{T}{2}\,\mathrm{sinc}\left(\frac{fT}{2}\right) e^{j\pi f \frac{T}{2}}$$

$$= T\,\mathrm{sinc}\left(\frac{fT}{2}\right)\left(-j\,\sin\left(\frac{\pi f T}{2}\right)\right)$$

$$P(f) = \sum_{k=-\infty}^{+\infty} P_k\,\delta\left(f - \frac{k}{T}\right)$$

con $P_k = \frac{1}{T} P_0\left(\frac{k}{T}\right) = -j\,\mathrm{sinc}\left(\frac{k}{2}\right)\sin\left(\frac{\pi k}{2}\right)$

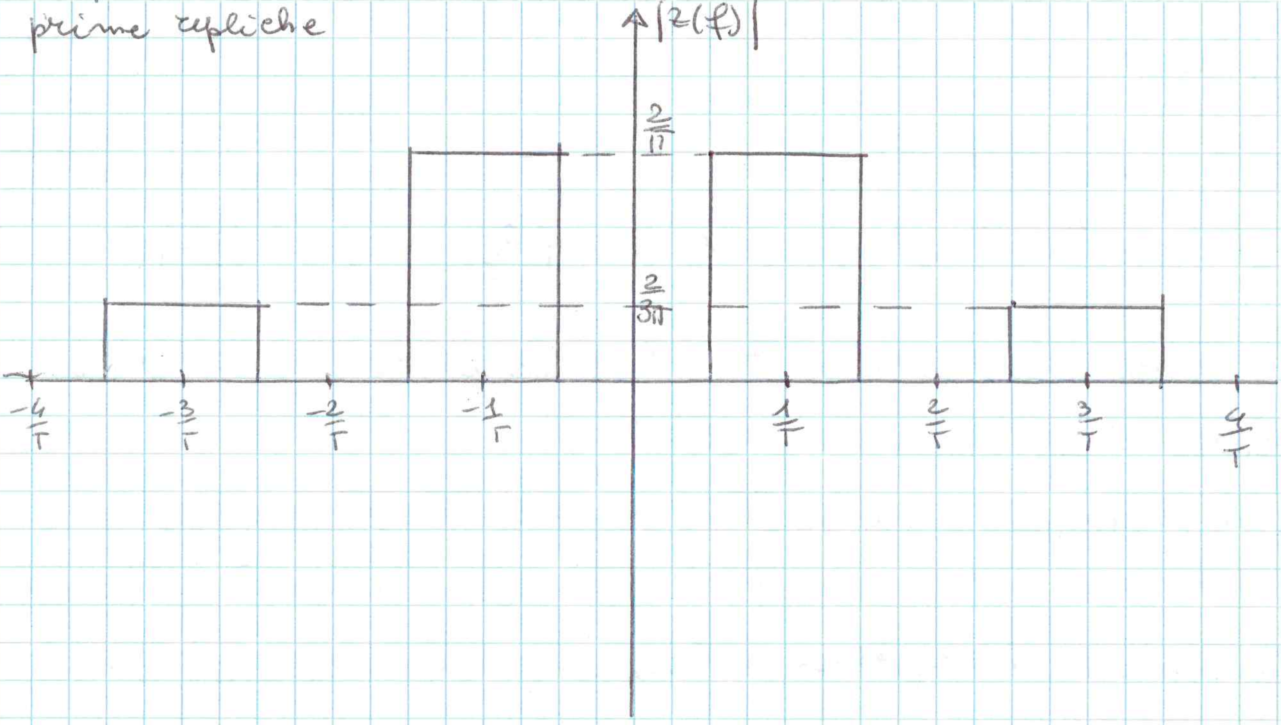
$$= -j\,\frac{2\sin^2(\pi k/2)}{\pi k}$$

$$\left.\begin{array}{ll} k=0 & P_k=0 \\ k=1 & P_k=-j\frac{2}{\pi} \\ k=-1 & P_k=j\frac{2}{\pi} \\ k=\pm 2 & P_k=0 \end{array}\right\} \quad \begin{array}{ll} P_k = 0 & \text{per } k \text{ pari e } 0 \\ P_k = -j\frac{2}{k\pi} & \text{per } k \text{ dispari} \end{array}$$

$$Z(f) = \sum_{k=-\infty}^{+\infty} P_k\,\mathrm{rect}\left(T\left(f - \frac{k}{T}\right)\right)$$

Faccio il grafico del modulo

Grafico delle prime repliche



2) Il filtro $H(f)$ seleziona le repliche centrate in $\pm\frac{3}{T}$, quindi

$$Y(f) = P_{-3}\,\mathrm{rect}\left(T\left(f+\frac{3}{T}\right)\right) + P_3\,\mathrm{rect}\left(T\left(f-\frac{3}{T}\right)\right)$$

$$= j\frac{2}{3\pi}\left[\mathrm{rect}\left(T\left(f+\frac{3}{T}\right)\right) - \mathrm{rect}\left(T\left(f-\frac{3}{T}\right)\right)\right]$$

Antitrasformiamo

$$y(t) = j\frac{2}{3\pi T}\left[\mathrm{sinc}\left(\frac{t}{T}\right)e^{-j2\pi\frac{3}{T}t} - \mathrm{sinc}\left(\frac{t}{T}\right)e^{+j2\pi\frac{3}{T}t}\right]$$

$$= j\frac{2}{3\pi T}\mathrm{sinc}\left(\frac{t}{T}\right)\left[e^{-j\frac{6\pi t}{T}} - e^{j\frac{6\pi t}{T}}\right]$$

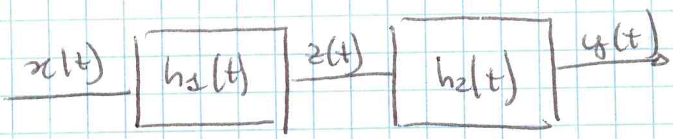
$$= \frac{4}{3\pi T}\,\mathrm{sinc}\left(\frac{t}{T}\right)\sin\left(\frac{6\pi t}{T}\right)$$

Fila D

Es. 1 Sia dato il sistema LTI in figura in cui $h_1(t) = e^{-t}u(t)$ e $h_2(t) = \frac{1}{T}\,\text{sinc}\left(\frac{2t}{T}\right)\cos\left(\frac{6\pi t}{T}\right)$.

1) Calcolare la risposta in frequenza del sistema e fare il grafico del modulo;

2) Si supponga ora che il segnale di ingresso sia dato da $x(t) = \text{sinc}\left(\frac{t}{T}\right)c(t)$, con $c(t) = 2\sum_{n=-\infty}^{+\infty}\delta(t-nT)$.
Calcolare lo spettro del segnale di uscita $y(t)$ e la sua energia.

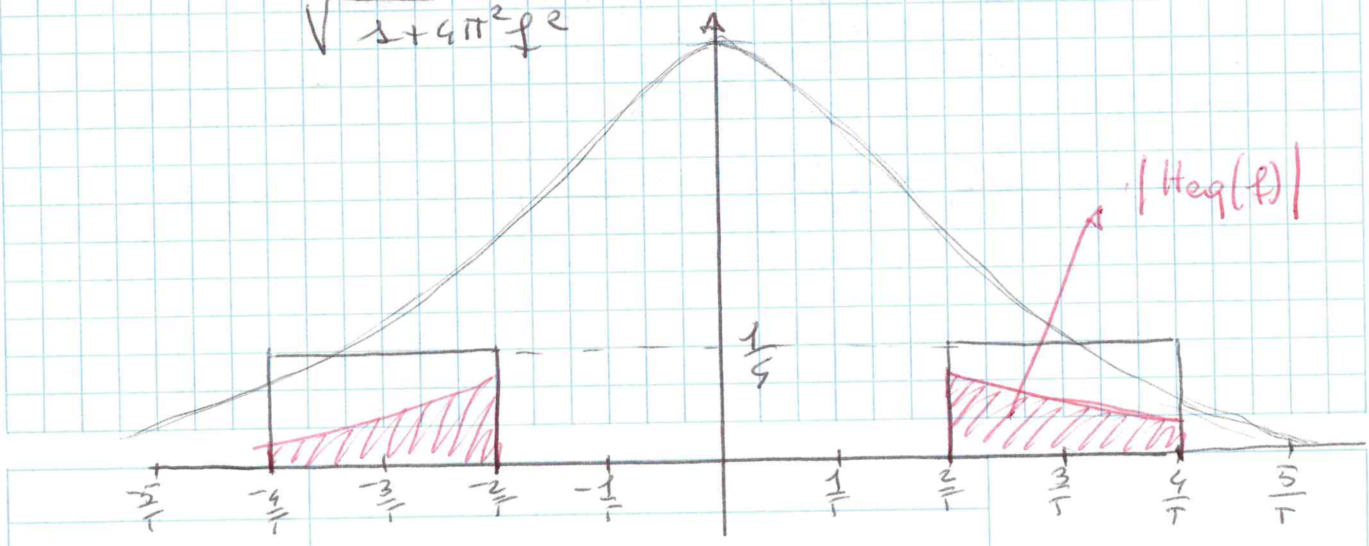


1) $$H_1(f) = \frac{1}{1+j2\pi f}$$

$$H_2(f) = \frac{1}{4}\,\text{rect}\left(\frac{f-\frac{3}{T}}{2/T}\right) + \frac{1}{4}\,\text{rect}\left(\frac{f+\frac{3}{T}}{2/T}\right)$$

$$|H_{eq}(f)| = |H_1(f)|\,|H_2(f)|$$

$$|H_1(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2}}$$



2) $x(t) = \text{sinc}\left(\frac{t}{T}\right) c(t)$

Data la forma di $\text{sinc}\left(\frac{t}{T}\right)$ $x(t) = 2\delta(t)$

$\Rightarrow X(f) = 2$

$$Y(f) = 2\, H_{eq}(f)$$

$$= \frac{1}{1+j2\pi f}\left[\frac{1}{2}\,\text{rect}\left(\frac{f-\frac{3}{T}}{2/T}\right) + \frac{1}{4}\,\text{rect}\left(\frac{f+\frac{3}{T}}{2/T}\right)\right]$$

Per calcolare l'energia applichiamo il teorema di Parseval

$$E_y = \int_{-\infty}^{+\infty} |Y(f)|^2 df$$

Data la simmetria del segnale

$$E_y = 2\int_0^{+\infty} \frac{1}{1+4\pi^2 f^2} \cdot \frac{1}{4}\,\text{rect}\left(\frac{f-\frac{3}{T}}{2/T}\right) df$$

$$= \frac{1}{2}\int_{\frac{2}{T}}^{\frac{4}{T}} \frac{1}{1+4\pi^2 f^2}\, df$$

$$= \frac{1}{2}\int_{\frac{4\pi}{T}}^{\frac{8\pi}{T}} \frac{1}{1+x^2}\, \frac{dx}{2\pi} =$$

$$= \frac{1}{4\pi}\left[\text{arctg}\left(\frac{8\pi}{T}\right) - \text{arctg}\left(\frac{4\pi}{T}\right)\right]$$

Es. 2 Sia dato il sistema LTI a tempo discreto caratterizzato dall'equaz. alle differenze

$$y(n) = x(n) - x(n-1) + 2x(n-2) - x(n-3) + x(n-4).$$

1) Scrivere la funzione di trasferimento e disegnare la forma canonica del sistema;
2) Calcolare la risposta $y(n)$ del sistema al gradino unitario $u(n)$ e farne il grafico;
3) Scrivere la risposta in frequenza del sistema e fare il grafico del modulo.

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1) $$Y(z) = X(z) - z^{-1}X(z) + 2z^{-2}X(z) - z^{-3}X(z) + z^{-4}X(z)$$

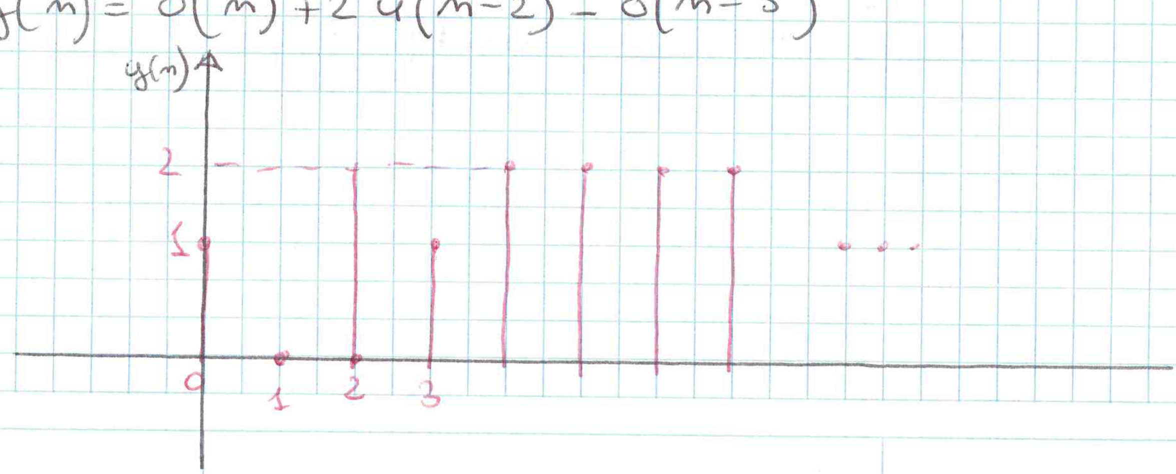
$$H(z) = 1 - z^{-1} + 2z^{-2} - z^{-3} + z^{-4}$$

2) $$U(z) = \frac{1}{1-z^{-1}}$$ da cui

$$Y(z) = \frac{(1-z^{-1}) + 2z^{-2} - z^{-3} + z^{-4}}{1-z^{-1}} =$$

$$= 1 + \frac{2z^{-2}}{1-z^{-1}} - z^{-3}$$

$$y(n) = \delta(n) + 2u(n-2) - \delta(n-3)$$



3) $\bar{H}(f) = 1 - e^{-j2\pi fT} + 2e^{-j4\pi fT} - e^{-j6\pi fT} + e^{-j8\pi fT}$

$$= 1 - e^{-j2\pi fT} + 2e^{-j4\pi fT} - e^{-j6\pi fT}\left[1 - e^{-j2\pi fT}\right]$$

$$= \left(1 - e^{-j2\pi fT}\right)\left(1 - e^{-j6\pi fT}\right) + 2e^{-j4\pi fT}$$

$$= \left[4\sin(\pi fT)\sin(3\pi fT) + 2\right] e^{-j4\pi fT}$$

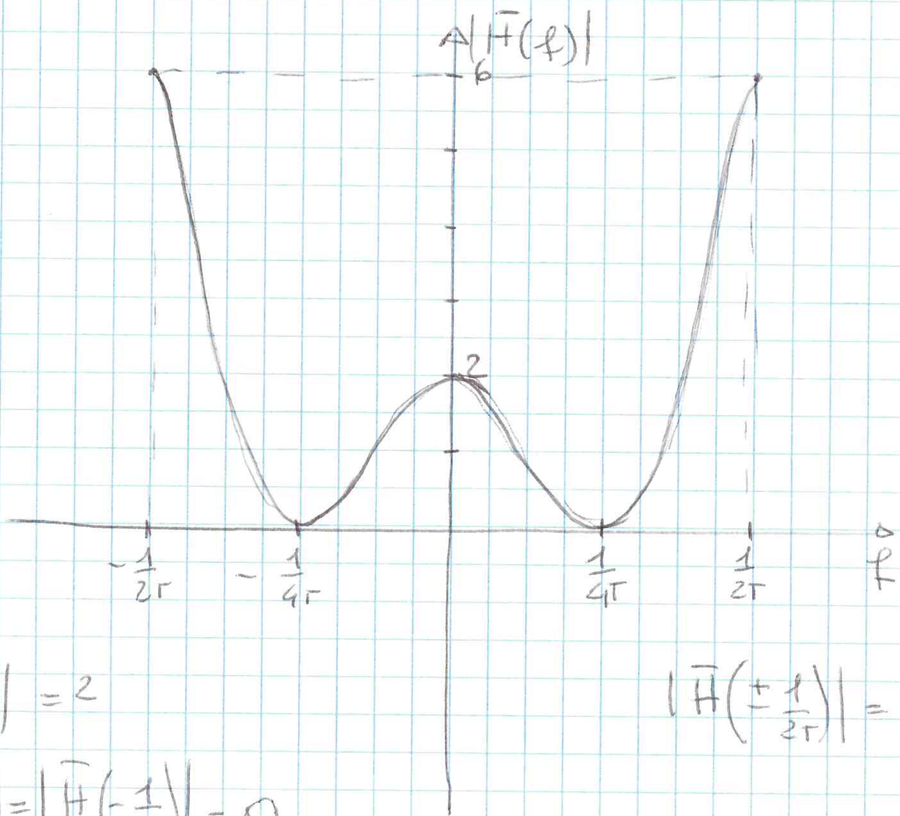
Ricordando che $-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$$\bar{H}(f) = 2\left[\cos 4\pi fT + 2\sin^2 \pi fT\right] e^{-j4\pi fT}$$

$$|\bar{H}(f)| = 2\left|\cos 4\pi fT + 2\sin^2 \pi fT\right|$$

$$= 2\left(\cos 4\pi fT + 2\sin^2 \pi fT\right)$$

Qualitativamente



$|\bar{H}(0)| = 2$

$|\bar{H}(\pm\frac{1}{2T})| = 6$

$|\bar{H}(\frac{1}{4T})| = |\bar{H}(-\frac{1}{4T})| = 0$