Network Security
Elements of Applied Cryptography

Random and Pseudorandom Bit Generators

- Random bit generators
- Pseudorandom bit generators
- Cryptographically Secure PRBG
- Statistical tests

Unpredictable quantities

- The security of many cryptographic systems depends on the generation of **unpredictable** quantities
- These quantities must be of **sufficient size** and **random** in the sense that
  - the probability of any particular value being selected must be sufficiently small to preclude an adversary from gaining advantage through optimizing a search strategy based on such probability

Random Bit Generator

- RBG requires a naturally occurring source of randomness

sequence of statistically **independent** and **unbiased** bits

Probability of emitting 1 does not depend on the previous bits

Probability of emitting 1 is equal to 0.5

Hardware-based RBG

- **HW-based RBGs exploit the randomness in some physical phenomena**
- elapsed time between emission of particles during radioactive decay
- thermal noise from a semiconductor diode or resistor
- the frequency instability of a free running oscillator
- the amount a metal-insulator semiconductor capacity is charged during a fixedperiod of time
- air turbolence within a sealed disk drive which causes random fluctuations in disk drive sector read latency times
- sound from a microphone or video from a camera

Software-based RBG

- **Random processes used by SW-based RBGs include**
- the system clock
- elapsed time between keystrokes or mouse movement
- content of input/output buffers
- user input
- operating system values such as system load and network statistics
- **A well-designed SW-based RBG uses as many sources as available**

Design and implementation problems

- RBG must not be subject to observation and manipulation by an adversary
- The natural source of randomness is subject to influence by external factors and to malfunction
- RBG must be tested periodically

De-skewing techniques

- A natural source of randomness may be defective and produce biased and correlated output bits
- De-skewing techniques make it possible to generate truly random bit sequences from the output bits of a defective generator
- De-skewing techniques
  - provable
  - practical

Pseudorandom bit generation

RBGs raise problems

- Generation of (truly) random bits is an inefficient procedure in most practical systems
- Storage and transmission of a large number of random bits may be impractical

- **These problems can be ameliorated by substituting a RBG with a Pseudorandom Bit Generator (PRBG)**

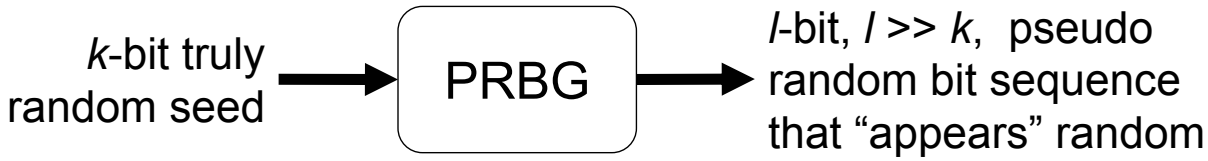
Famous quotes

"Random numbers should not be generated with a method chosen at random." —Donald E. Knuth

"The generation of random numbers is too important to be left to chance." —Robert R. Coveyou

"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin" —John von Neumann

Pseudorandom Bit Generator



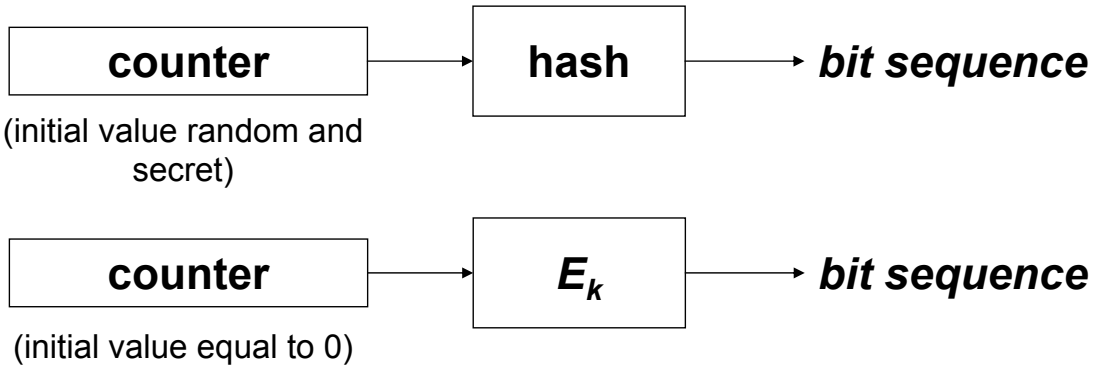
- PRBG is a **deterministic** algorithm
- An adversary must not **efficiently** distinguish between output sequences of PRBG and truly random bit sequences

Requirements

- **Minimum security requirement**
  - k should be sufficiently large to make an exhaustive search over 2^k seeds practically infeasible
- **General requirements**
  - A PRBG passes all **polynomial-time statistical tests** if no polynomial-time algorithm can correctly distinguish between an output sequence of the generator and a truly random sequence of the same length with probability significantly greater than 0.5
  - A PRBG passes the **next-bit test** if there is no polynomial-time algorithm which, on input of the first l-bits of an output sequence s can predict the $(l + 1)^{st}$ bit of s with probability significantly greater than 0.5
  - These two requirements are equivalent
- A PRBG that passes tests is said **cryptographically secure**

Ad-hoc PRBG

- **One-way functions** can be used to generate pseudo-random bit sequences



- Although **ad-hoc techniques have not proven** to be cryptographically secure, they **appear sufficient** for most applications

Ad-hoc PRBG: ANSI X9.17 generator

X9.17 generator is used to pseudorandomly generate keys and initialization vectors for use with DES

Let s be a 64-bit random seed, m be an integer, k be DES E-D-E encryption key, and D be a 64-bit representation of time/date

1. Let $I = E_k(D)$
2. For $i = 1$ to m do
    1. Let $x_i \leftarrow E_k(I \oplus s)$
    2. Let $s \leftarrow E_k(x_i \oplus s)$
3. Return$(x_1, x_2, \ldots, x_m)$

Ad-hoc PRBG: FIPS 186

- FIPS-approved methods for pseudo-randomly generating
  - DSA private key *a*
  - DSA per-message secret *k*
- Both algorithms use a **randomly generated secret seed** *s* and **one-way function** constructed by using either SHA-1 or DES

CSPRBG

The security of *Cryptographically Secure PRBG*s (*CSPRBG*) relies on the presumed intractability of an underlying number-theoretic problem

- **RSA pseudorandom bit generator** is a CSPRBG under the assumption that **RSA problem** is intractable
- Blum-Blum-Shub pseudorandom bit generator is a CSPRBG under the assumption that **integer factorization** is intractable
- These CSPRBGs make use of modular multiplication which makes them relatively slower than ad-hoc PRBG

RSA CSPRBG

1. Generate two primes p and q, and compute $n = pq$ and $\phi = (p - 1)(q - 1)$. Select a random integer e, $1 < e < \phi$, such that $\gcd(e, \phi) = 1$.
2. Select a random number x_0 (the *seed*) in the interval $[1, n - 1]$
3. For $i = 1$ to l do
   1. Let $x_i \leftarrow x^e_{i-1} \bmod n$
   2. Let $z_i \leftarrow \text{lsb}(x_i)$
4. Return$(z_1, z_2, \ldots, z_l)$

Statistical tests

- A set of statistical tests have been devised to measure the quality of a random bit generator
- While it is not possible to prove whether a generator is indeed a random bit generator, these tests detect certain kinds of weaknesses the generator may have
- Each test takes a sample output sequence and probabilistically determines whether it possesses a certain attribute that a truly random sequence would be likely to exhibit
  - A sequence should roughly have the same number of 1's as 0's
- A generator may be *rejected* or *accepted* (not rejected)

Statistical tests: basic tests

- **Frequency test** (**monobit test**). The purpose of this test is to determine whether the number of 0's and 1's are approximately the same
- **Serial test** (**two-bit test**). The purpose of this test is to determine whether the number of occurrences of 00, 01, 10, 11 are approximately the same
- **Poker test**. The purpose of this test is to determine whether the sequences of length *m* each appear approximately the same number of times
- **Runs test**. The purpose of this test is to determine whether the number of runs of various length is as expected for a random sequence
- **Autocorrelation test**. The purpose of this test is to check correlations between the sequence and shifted versions of it

Statistical tests

- **Statistical tests give only necessary conditions** for a periodic pseudorandom sequence to look random
  - *Linear congruential pseudorandom generator*

$$x_n = a\, x_{n-1} + b \bmod n,\ n \geq 1$$

passes statistical tests

However, it is **predictable** and hence entirely **insecure** for cryptographic purposes

- **FIPS 140-1** specifies statistical tests for randomness