Random and Pseudorandom Bit Generators

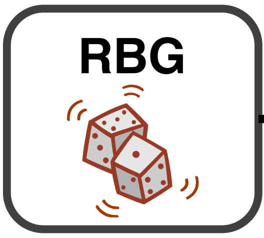
- **Random bit generators**
- **Pseudorandom bit generators**
- **Cryptographically Secure PRBG**
- **Statistical tests**

Unpredictable quantities

- The security of many cryptographic systems depends on the generation of **unpredictable** quantities
- These quantities must be of **sufficient size** and **random** in the sense that
  - *the probability of any particular value being selected must be sufficiently small to preclude an adversary from gaining advantage through optimizing a search strategy based on such probability*

Random bit generator

- RBG requires a naturally occurring source of randomness



Sequence of **statistically independent** and **unbiased** bits

- Probability of emitting a bit (1 or 0) value does not depend on the previous bits
- Probability of emitting a bit value (1 or 0) is equal to 0.5

Hardware-based RBG

- **HW-based RBGs exploit the randomness in some physical phenomena**
- elapsed time between emission of particles during radioactive decay
- thermal noise from a semiconductor diode or resistor
- the frequency instability of a free running oscillator
- the amount a metal-insulator semiconductor capacity is charged during a fixed period of time
- air turbulence within a sealed disk drive which causes random fluctuations in disk drive sector read latency times
- sound from a microphone or video from a camera

Software-based RBG

- **Random processes used by SW-based RBGs include**
- the system clock
- elapsed time between keystrokes or mouse movement
- content of input/output buffers
- user input
- operating system values such as system load and network statistics
- **A well-designed SW-based RBG uses as many sources as available**

Design and implementation problems

- RBG must not be subject to observation and manipulation by an adversary
- The natural source of randomness is subject to influence by external factors and to malfunction
- RBG must be tested periodically

De-skewing techniques

- A natural source of randomness may be defective and produce biased and correlated output bits
- **De-skewing techniques** make it possible to generate truly random bit sequences from the output bits of a defective generator
- De-skewing techniques
  - provable
  - practical

Pseudorandom bit generation

RBGs raise problems

- Generation of (truly) random bits is an inefficient procedure in most practical systems
- Storage and transmission of a large number of random bits may be impractical

- **These problems can be ameliorated by substituting a RBG with a Pseudorandom Bit Generator (PRBG)**

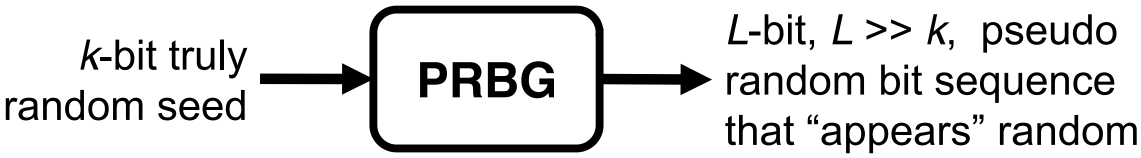
Famous quotes

"Random numbers should not be generated with a method chosen at random." —***Donald E. Knuth***

"The generation of random numbers is too important to be left to chance." —***Robert R. Coveyou***

"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin" —***John von Neumann***

Pseudorandom Bit Generator



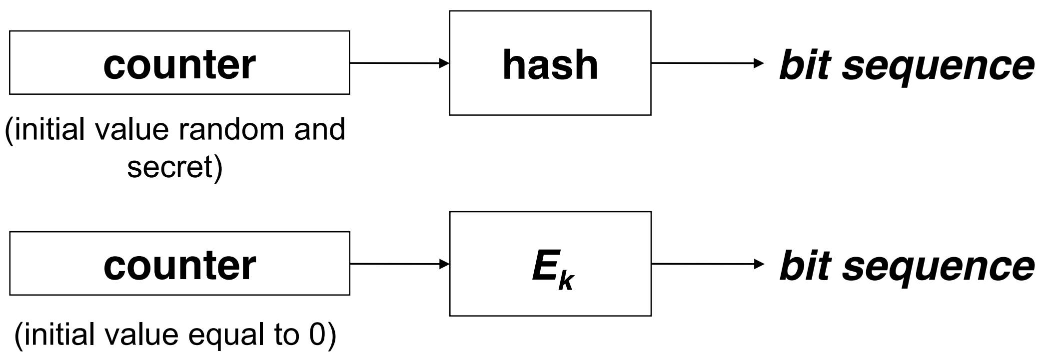
- PRBG is a **deterministic** algorithm
- An adversary must not **efficiently** distinguish between output sequences of PRBG and truly random bit sequences

Requirements

- **Minimum security requirement**
  - *k* should be sufficiently large to make an exhaustive search over 2^k seeds practically infeasible
- **General requirements**
  - A PRBG passes all **polynomial-time statistical tests** if no polynomial-time algorithm can correctly distinguish between an output sequence of the generator and a truly random sequence of the same length with probability significantly greater than 0.5
  - A PRBG passes the **next-bit test** if there is no polynomial-time algorithm which, on input of the first *l*-bits of an output sequence *s* can predict the $(l + 1)^{st}$ bit of *s* with probability significantly greater than 0.5
  - ***These two requirements are equivalent***
- A PRBG that passes tests is said **cryptographically secure**

Ad-hoc PRBG

- **One-way functions** can be used to generate pseudo-random bit sequences



- Although **ad-hoc techniques have not proven** to be cryptographically secure, they **appear sufficient** for most applications

Ad-hoc PRBG: ANSI X9.17 generator

X9.17 generator is used to pseudorandomly generate keys and initialization vectors for use with DES

Let s be a 64-bit random seed, m be an integer, k be DES E-D-E encryption key, and D be a 64-bit representation of time/date

1. Let $I = E_k(D)$
2. For $i = 1$ to m do
    1. Let $x_i \leftarrow E_k(I \oplus s)$
    2. Let $s \leftarrow E_k(x_i \oplus s)$
3. Return($x_1, x_2, \ldots, x_m$)

Ad-hoc PRBG: FIPS 186

- FIPS-approved methods for pseudo-randomly generating
    - DSA private key ***a***
    - DSA per-message secret ***k***
- Both algorithms use a **randomly generated secret seed *s*** and **one-way function** constructed by using either SHA-1 or DES

CSPRBG

The security of ***Cryptographically Secure PRBGs*** (***CSPRBG***) **relies on the presumed intractability of an underlying number-theoretic problem**

- **RSA pseudorandom bit generator** is a CSPRBG under the assumption that **RSA problem** is intractable
- Blum-Blum-Shub pseudorandom bit generator is a CSPRBG under the assumption that **integer factorization** is intractable
- These CSPRBGs make use of modular multiplication which makes them relatively slower than ad-hoc PRBG

RSA CSPRBG

1. Generate two primes p and q, and compute $n = pq$ and $\phi = (p-1)(q-1)$. Select a random integer e, $1 < e < \phi$, such that $\gcd(e, \phi) = 1$.
2. Select a random number x_0 (the *seed*) in the interval $[1, n-1]$
3. For $i = 1$ to l do
   1. Let $x_i \leftarrow x^e_{i-1} \bmod n$
   2. Let $z_i \leftarrow \mathrm{lsb}(x_i)$
4. Return$(z_1, z_2, \ldots, z_l)$

Statistical tests

- A set of statistical tests have been devised to measure the quality of a random bit generator
- While **it is not possible to prove whether a generator is indeed a random bit generator**, **these tests detect certain kinds of weaknesses** the generator may have (**necessary conditions**)
- Each test takes a sample output sequence and probabilistically determines whether it possesses a certain attribute that a truly random sequence would be likely to exhibit
  - Ex.: a sequence should roughly have the same number of 1's as 0's
- A generator may be *rejected* or *accepted* (not rejected)

Statistical tests: basic tests

- **Frequency test** (**monobit test**). The purpose of this test is to determine whether the number of 0's and 1's are approximately the same
- **Serial test** (**two-bit test**). The purpose of this test is to determine whether the number of occurrences of 00, 01, 10, 11 are approximately the same
- **Poker test**. The purpose of this test is to determine whether the sequences of length *m* each appear approximately the same number of times
- **Runs test**. The purpose of this test is to determine whether the number of runs of various length is as expected for a random sequence
- **Autocorrelation test**. The purpose of this test is to check correlations between the sequence and shifted versions of it

Statistical tests

- **Statistical tests give only necessary conditions** for a periodic pseudorandom sequence to look random
  - *Linear congruential pseudorandom generator*

$$x_n = a\, x_{n-1} + b \bmod n,\ n \geq 1$$

passes statistical tests

However, it is **predictable** and hence entirely **insecure** for cryptographic purposes

- **FIPS 140-1** specifies statistical tests for randomness