Network Security
Elements of Applied Cryptography

Network Security
Digital Signatures

- Digital Signatures with appendix
- Digital signatures with message recovery
- Digital signatures based on RSA

Roadmap

- **Introduction**
- Classification
- Digital signatures based on RSA

Informal properties

- A digital signature is a number dependent on some secret known only to the signer and, additionally, on the content of the message being signed
- A digital signature must be verifiable, i.e., if a dispute arises an unbiased third party must be able to solve the dispute equitably, without requiring access to the signer's secret

Arbitrated digital signatures (symm. encrypt.)

Hypotheses

1. $A \overset{K_{AT}}{\leftrightarrow} T$
2. $B \overset{K_{BT}}{\leftrightarrow} T$
3. h is a OWHF

Protocol

1. $A \to B$: $A, B, m, E_{K_{AT}}\left(A \| h(m)\right)$
2. $B \to T$ $B, A, E_{K_{AT}}\left(A \| h(m)\right)$
3. $T \to B$ $A, E_{K_{BT}}\left(A \| h(m)\right)$

Bob decifra M3, ricavando $A||H$, e verifica se $H = h(m)$

- Bob let a third party G verify that Alice has originated message *m* by sending G the pair:

$$\boxed{\left(E_{K_{AT}}\left(A \| h(m)\right), E_{K_{BT}}\left(A \| h(m)\right)\right)}$$

- Then, G asks T to decrypt both ciphertexts so obtaining $A||H_1$ and $A||H_2$ and then
- G verifies that $H_1 = H_2 = h(m)$

Arbitrated digital signatures

- Security of arbitrated digital signatures systems depends on
  - security of the cipher
  - security of the hash function
  - security of the key distribution protocol
- The TTP must be unconditionally trusted
  - TTP becomes a single-point of failure
- Arbitrated digital signatures are more efficient than digital signatures based on public-key encryption
  - TTP becomes a bottleneck

Digital signatures based on asymmetric cryptography

Hypotheses

- (e_A, d_A): Alice's private-public key pair
- h: hash function

Signature generation

$A \to B$ A, m, s

where $s = E_{d_A}\left(h(m)\right)$

digital signature of m with key d_A

Signature verification

- By means of (m, s), a third party G verifies that Alice has originated messahe *m* as follows
- G obtains Alice's public key e_A;
- G obtains H by decrypting s with e_A;
- G verifies whether $H = h(m)$;

Roadmap

- Introduction
- **Classification**
- Digital signatures based on RSA

Classification

- **Digital signatures with appendix**
  - require the original message as input to the verification algorithm;
  - use hash functions
  - Examples: ElGamal, DSA, DSS, Schnorr
- **Digital signatures with message recovery**
  - do not require the original message as input to the verification algorithm;
  - the original message is recovered from the signature itself;
  - Examples: RSA, Rabin, Nyberg-Rueppel

Digital signatures with appendix

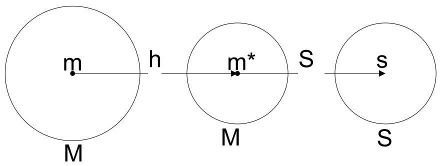
Definitions

- M is the message space
- h is a hash function with domain M
- M_h is the image of h
- S is the signature space

Key generation

- A selects a private key which defines a *signing algorithm* S_A which is a one-to-one mapping S_A: $M_h \rightarrow S$
- A defines the corresponding public key defining the *verification algorithm* V_A such that $V_A(m^*, s)$ = true if $S_A(m^*) = s$ and false otherwise, for all $m^* \in M_h$ and $s \in S$, where $m^* = h(m)$ for $m \in M$. M_A is constructed such that it may be computed without knowledge of the signer's private key
- S_A is the private key; V_A is the public key

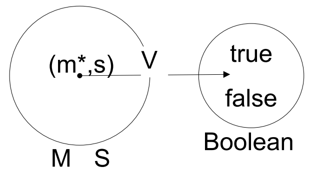
Digital signatures with appendix



Signature generation

- Compute $m^* = h(m)$ and $s = S_A(m^*)$
- A's digital signature for m is s*
- $\langle m, s^* \rangle$ are made available to anyone who may wish to verify the signature

Digital signatures with appendix



Signature verification

- Obtain A's public key V_A
- Compute $m^* = h(m)$, $u = V_A(m^*, s)$
- Accept the signature iff u = true

Digital signatures with appendix

Properties of S_A and V_A

- S_A should be efficient to compute
- V_A should be efficient to compute
- It should be computationally infeasible for an entity other than A to find an $m \in M$ and an $s \in S$ such that $V_A(m^*, s)$ = true, where $m^* = h(m)$

Digital signature with message recovery

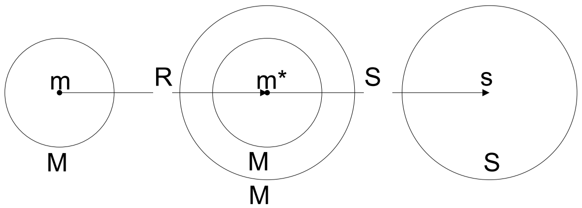
Definitions

- M is the message space
- M_S is the signing space
- S is the signature space

Key generation

- A selects a private key defining a *signing algorithm* S_A which is a one-to-one mapping $S_A: M_S \rightarrow S$
- A defines the corresponding public key defining the *verification algorithm* V_A such that $V_A \bullet S_A$ is identity map on M_S. V_A is constructed such that it may be computed without knowledge of the signer's private key
- S_A is A's private key; V_A is A's public key

Digital signatures with message recovery



Signature generation

1. Compute $m^* = R(m)$ and $s = S_A(m^*)$ (R is a **redundancy function**)
2. A's signature for m is s that is made available to entities which may wish to verify the signature and recover m from it

Signature verification

1. Obtain A's authentic public key S_A
2. Compute $m^* = V_A(s)$
3. Verify that $m^* \in M_R$, otherwise reject the signature
4. Recover $m = R^{-1}(m^*)$

Digital signatures with message recovery

Properties of S_A and V_A

- S_A should be efficient to compute
- V_A should be efficient to compute
- It should be computationally infeasible for an entity other than A to find an $s \in S$ such that $V_A(s) \in M_R$

Digital signatures with message recovery

The redundancy function

- R and R^{-1} are publicly known
- Selecting an appropriate R is critical to the security of the system

A bad redundancy function

- Let us suppose that $M_R \equiv M_S$
- R and S_A are bijections, therefore M and S have the same number of elements
- Therefore, for all $s \in S$, $V_A(s) \in M_R$, it is "easy" to find an m for which s is the signature, $m = R^{-1}(V_A(s))$
- s is a valid signature for m (existential forgery)

Digital signatures with message recovery

A good redundancy function

- $M = \{m : m \in \{0, 1\}^n\}$, $M_S = \{m : m \in \{0, 1\}^{2n}\}$
- $R: M \rightarrow M_S$, $R(m) = m \| m$
- $M_R \subseteq M_S$
- When n is large, $|M_R|/|M_S| = (1/2)^n$ is small. Therefore, for an adversary it is unlikely to choose an s that yields $V_A(s) \in M_R$
- ISO/IEC 9776 is an international standard that defines a redundancy function for RSA and Rabin

Scheme with appendix from message recovery

- Digital signature with appendix from scheme providing message recovery
- Signature generation
  - Compute $m^* = R(h(m))$, $s = S_A(m^*)$
  - A's digital signature for m is s*
  - $\langle m, s^* \rangle$ are made available to anyone who may wish to verify the signature
- Signature verification
  - Obtain A's public key V_A
  - Compute $m^* = R(h(m))$ and $u = V_A(m^*, s)$
  - Accept the signature iff u = true
- R is not security critical anymore and can be any one-to-one mapping

Types of attacks

BREAKING A SIGNATURE

1. **Total break** – adversary is able to compute the signer's private key
2. **Selective forgery** – adversary controls the messages whose signature is forged
3. **Existential forgery** – adversary has no control on the messages whose signature is forged

BASIC ATTACKS

1. **key-only attacks** – adversary knows only the signer's public key
2. **message attacks**
   a. **known-message attack** – adversary has signatures for a set of messages which are known by the adversary but not chosen by him
   b. **chosen-message attack** – in this case messages are chosen by the adversary
   c. **adaptive chosen-message attack** – in this case messages are adaptively chosen by the adversary

Attacks: considerations

- Adaptive chosen-message attack
  - It is the most difficult attack to prevent
  - Although an adaptive chosen-message attack may be infeasible to mount in practice, a well-designed signature scheme should nonetheless be designed to protect against the possibility
- The level of security may vary according to the application
  - Example 1. When an adversary is only capable of mounting a key-only attack, it may suffice to design the scheme to prevent the adversary from being successful at selective forgery.
  - Example 2. When the adversary is capable of a message attack, it is likely necessary to guard against the possibility of existential forgery.

Attacks: considerations

- Hash functions and digital signature processes
  - When a hash function h is used in a digital signature scheme (as is often the case), h should be a fixed part of the signature process so that an adversary is unable to take a valid signature, replace h with a weak hash function, and then mount a selective forgery attack.
  - For example, let $\langle m, s \rangle$ where $s = S_A(h(m))$, the adversary may
    1. replace h with a weaker hash function g that is vulnerable to selective forgery. Thus, the adversary can
       1. determine m′ such that $g(m') = h(m)$;
       2. replace m with m′

Roadmap

- Introduction
- Classification
- **Digital signatures based on RSA**

Introductory comments

- Since the encryption transformation is a bijection, digital signatures can be created by reversing the roles of encryption and decryption
- Digital signature with message recovery
- $M_S \equiv S \equiv \mathbb{Z}_n$
- A redundancy function $R: M \rightarrow \mathbb{Z}_n$ is chosen and is public knowledge

Key generation

1. Generate two **large**, **distinct primes** p, q (100÷200 decimal digits)
2. Compute $n = p \times q$ and $\phi = (p-1) \times (q-1)$
3. Select a **random number** $1 < e < \phi$ such that $\gcd(e, \phi) = 1$
4. Compute the **unique** integer $1 < d < \phi$ such that $ed \equiv 1 \bmod \phi$
5. (d, n) is the private key
6. (e, n) is the public key

At the end of key generation, p and q must be destroyed

Signature generation and verification

Signature generation. In order to sign a message m, A does the following

1. Compute $m^* = R(m)$ an integer in $[0, n-1]$
2. Compute $s = m^{*d} \bmod n$
3. A's signature for m is s

Signature verification. In order to verify A's signature s and recover message m, B does the following

1. Obtain A's authentic public key (e, n)
2. Compute $m^* = s^e \bmod n$
3. Verify that m^* is in M_R; if not reject the signature
4. Recover $m = R^{-1}(m^*)$

Proof that verification works

- If s is a signature for a message m, then $s = m^{*d} \bmod n$ where $m^* = R(m)$.
- Since $ed = 1 \pmod{\phi}$, $s^e = m^{*ed} = m^* \pmod{n}$. Finally, $R^{-1}(m^*) = R^{-1}(R(m)) = m$.

Possible attacks

- Integer factorization
  - Factorization of n lead to total break.
  - A should choose p and q so that factoring n is a computationally infeasible task
- Multiplicative property of RSA
  - Condizione necessaria ma non sufficiente per difendersi da existential forgery è che la funzione di ridondanza non deve essere moltiplicativa

RSA signature in practice

- Reblocking problem
  - If A wants to send a secret and signed message to B then it must be $n_A < n_B$
  - There are various ways to solve the problem
    - **reordering**: the operation with the smaller modulus is performed first; however the preferred order is always to sign first and encrypt later
    - **two moduli for entity**: each entity has two moduli; moduli for signing (e.g., t-bits) are always smaller of all possible moduli for encryption (e.g., t+1-bits)
    - **prescribing the form of the modulus**

RSA signature in practice

- Redundancy function
  - A suitable redundancy function is necessary in order to avoid existential forgery
  - IOS/IEC 9796 (1991) defines a mapping that takes a k-bit integer and maps it into a 2k-bits integer
- The RSA digital signature scheme with appendix
  - MD5 (128 bit)
  - PKCS#1 specifies a redundancy function mapping 128-bit integer to a k-bit integer, where k is the modulus size ($k \geq 512$, $k = 768$, 1024)

RSA signature in practice

- Performance characteristics
  - Let $|p| = |q| = k$ then
  - signature generation requires $O(k^3)$ bit operations
  - signature verification, in the case of small public exponent, requires $O(k^2)$ bit operations
  - Suggested value for e in practice are 3 and $2^{16}+1$. Of course, p and q must be chosen so that $\gcd(e, (p-1)(q-1)) = 1$.
  - The RSA signature scheme is ideally suited to situations where signature verification is the predominant operation being performed.
    - Example. A trusted third party creates a public-key certificate for an entity A. This requires only one signature generation, and this signature may be verified many times by various other entities

RSA signature in practice

- Parameter selection
  - bitsize of the modulus: miminum 768; at least 1024 for signatures of longer lifetime or critical for overall security of a large network (i.e., the private key of a certification authority)
  - No weaknesses have been reported when the public exponent e is chosen to be a small number such as 3 or $2^{16}+1$.
  - It is not recommended to restrict the size of the private exponent d in order to improve the efficiency of signature generation
- Bandwidth efficiency
  - By definition, $BWE = \log2(|M_S|) / \log2(|M_R|)$
  - For (RSA, ISO/IEC 9796), BWE = 0.5, that is, with a 1024-bits modulus can be signed 512-bits messages

RSA signature in practice

- System wide parameters
  - Each entity must have a distinct RSA modulus; it is insecure to use a system-wide modulus
  - The public exponent e can be a system-wide parameter, and is in many applications. In this case, the low exponent attack must be considered
- Short vs. long messages
  - Suppose n is a 2k-bit RSA modulus which is used to sign k-bit messages (i.e., BWE is 0.5)
  - Suppose entity A wishes to sign a kt-bit message m
  - For t = 1 RSA with message recovery is more efficient;
  - For t > 1, RSA with appendix is more efficient