Public Key Encryption

A case study

THE RSA CRYPTOSYSTEM

Rivest Shamir Adleman (1978)

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Key generation

1. Generate two large, distinct primes p, q (100÷200 decimal digits)
2. Compute $n = p \times q$ and $\varphi(n) = (p-1)\times(q-1)$
3. Select a random number $1 < e < \varphi(n)$ such that $\gcd(e, \varphi(n)) = 1$
4. Compute the unique integer $1 < d < \varphi$ such that $ed \equiv 1 \bmod \varphi$
5. (d, n) is the *private* key
6. (e, n) is the *public* key

At the end of key generation, ***p*** and ***q*** must be destroyed

RSA encryption and decryption

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Encryption. To generate c from m, Bob should do the following

1. Obtain *A*'s *authentic* public key (n, e)
2. Represent the message as an integer m in the interval $[0, n-1]$
3. Compute $c = m^e \bmod n$
4. Send c to A

Decryption. To recover m from c, Alice should do the following

1. Use the private key d to recover $m = c^d \bmod n$

RSA consistency

We have to prove that $D(d(E(e, m)) = m$, i.e.,

$c^d \equiv m^{de} \equiv m^{t \bullet \varphi(n)+1} \bmod n$, where t is some integer $\Rightarrow$

$m^{t \cdot \varphi(n)} \cdot m^1 \equiv (m^{\varphi(n)})^t \cdot m^1 \equiv m \bmod n$

The proof exploits the **Eulero's theorem**

$\forall$ integer $n > 1$, $\forall a \in \mathbb{Z}_n^*$, $a^{\varphi(n)} \equiv 1 \bmod n$ *where*

$\mathbb{Z}_n^* = \{ x \mid 1 < x < n, \gcd(x, n) = 1\}$

Example with artificially small numbers

Key generation

- Let $p = 47$ e $q = 71$
  $n = p \times q = 3337$
  $\varphi = (p-1) \times (q-1) = 46 \times 70 = 3220$
- Let $e = 79$
  $ed = 1 \bmod \varphi$
  $79 \times d = 1 \bmod 3220$
  $d = 1019$

Encryption

Let $m = 9666683$

Divide m into blocks $m_i < n$

$m_1 = 966$; $m_2 = 668$; $m_3 = 3$

Compute

$c_1 = 966^{79} \bmod 3337 = 2276$

$c_2 = 668^{79} \bmod 3337 = 2423$

$c_3 = 3^{79} \bmod 3337 = 158$

$c = c_1 c_2 c_3 = 2276\ 2423\ 158$

Decryption

$m_1 = 2276^{1019} \bmod 3337 = 966$

$m_2 = 2423^{1019} \bmod 3337 = 668$

$m_3 = 158^{1019} \bmod 3337 = 3$

$m = 966\ 668\ 3$

RSA

- RSA algorithms for key generation, encryption and decryption are easy
- They involve the following operations
  - Discrete exponentiation
  - Generation of large primes
  - Solving diophantine equations

Modular ops - complexity

Bit complexity of basic operations in Z_n

- Let **n** be on **k** bits (**$n < 2^k$**)
- Let **a** and **b** be two integers in **Z_n** (on k-bits)
  - **Addition $a + b$** can be done in time **$O(k)$**
  - **Subtraction $a – b$** can be can be done in time **$O(k)$**
  - **Multiplication $a \times b$** can be done in **$O(k^2)$**
  - **Division $a = q \times b + r$** can be done in time **$O(k^2)$**
  - **Inverse a^{-1}** can be done in **$O(k^2)$**
  - **Modular exponentiation a^k** can be done in **$O(k^3)$**

How to encrypt/decrypt efficiently

- RSA requires *modular exponentiation* c^d **mod** ***n***
  - Let ***n*** have ***k*** bits in its binary representation, ***k = log n + 1***
- ***Grade-school*** algorithm requires ***(d-1)*** modular multiplications
  - ***d*** is as large as **n** which is exponentially large with respect to ***k***
  - The grade-school algorithm is inefficient
- ***Square-and-multiply*** algorithm requires up to **2k** multiplications thus the algorithm can be done in $O(k^3)$

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How to encrypt and decrypt efficiently

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Exponentiation by repeated squaring and multiplication: m^e **mod** n requires **at most** $\log_2(e)$ multiplications and $\log_2(e)$ squares

Let $e_{k-1}, e_{k-2}, \dots, e_2, e_1, e_0$, where $k = \log_2 e$, the binary representation of **e**

$$m^e \bmod n = m^{\left(e_{k-1}2^{k-1}+e_{k-2}2^{k-2}+\dots+e_2 2^2+e_1 2+e_0\right)} \bmod n \equiv$$
$$m^{e_{k-1}2^{k-1}} m^{e_{k-2}2^{k-2}} \cdots m^{e_2 2^2} m^{e_1 2} m^{e_0} \bmod n \equiv$$
$$\left(m^{e_{k-1}2^{k-2}} m^{e_{k-2}2^{k-3}} \cdots m^{e_2 2} m^{e_1}\right)^2 m^{e_0} \bmod n \equiv$$
$$\left(\left(m^{e_{k-1}2^{k-3}} m^{e_{k-2}2^{k-4}} \cdots m^{e_2}\right)^2 m^{e_1}\right)^2 m^{e_0} \bmod n \equiv$$
$$\left(\left(\left(\left(m^{e_{k-1}}\right)^2 m^{e_{k-2}}\right)^2 \cdots m^{e_2}\right)^2 m^{e_1}\right)^2 m^{e_0} \bmod n$$

```
c ← 1
for (i = k-1; i >= 0; i --) {
    c ← c² mod n;
    if (e_i == 1)
        c ← c × m mod n;
}
```

- always k square operations
- **at most** k modular multiplications (***equal to the number of 1 in the binary representation of e***)

Square and multiply

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Exponentiation by repeated squaring and multiplication: a^x **mod** n requires **at most** $\log_2(x)$ multiplications and $\log_2(x)$ squares

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$$\left(\left(a^{x_{k-1}2^{k-3}} a^{x_{k-2}2^{k-4}} \cdots a^{x_2}\right)^2 a^{x_1}\right)^2 a^{x_0} \bmod n \equiv$$

...

$$\left(\left(\left(\left(a^{x_{k-1}}\right)^2 a^{x_{k-2}}\right)^2 \cdots a^{x_2}\right)^2 a^{x_1}\right)^2 a^{x_0} \bmod n$$

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- always k square operations
- **at most** k modular multiplications (***equal to the number of 1 in the binary representation of e***)

Fast encryption with short public exponent

- RSA ops with public key exponent ***e*** can be speeded-up
  - Encryption
  - Digital signature verification
- The public key ***e*** can be chosen to be a very small value
  - e = 3 #MUL + #SQ = 2
  - e = 17 #MUL + #SQ = 5
  - e = 2^{16}+1 #MUL + #SQ = 17
  - **RSA is still secure**
- There is no easy way to accelerate RSA when the private exponent ***d*** is involved

How to find a large prime

repeat *p* ← randomOdd(x); **until** isPrime(*p*);	▪ **FACT**. On average (ln *x*)/2 odd numbers must be tested before a prime *p* < *x* can be found

- Primality tests **do not** try to factor the number under test
  - *probabilistic primality test* (Solovay-Strassen, Miller-Rabin) polynomial in **log n**
  - *true primality test* (O(n^{12}) in 2002))

On computing the public exponent

- Solution of **$d \cdot e \equiv 1 \bmod \varphi(n)$** with **$\gcd(e, \varphi(n)) \equiv 1$** can be done by means of the **Extended Euclidean Algorithm** (EEA)
  - Exponent ***d*** can be generated efficiently (polytime)
  - Condition $\gcd(e, \varphi(n)) \equiv 1$

RSA one-way function

- One-way function $y = f(x)$
  - $y = f(x)$ is easy
  - $x = f^{-1}(y)$ is hard
- RSA one-way function
  - Multiplication is easy
  - Factoring is hard

Security of RSA

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The RSA Problem (RSAP)

- **DEFINITION. The RSA Problem** (**RSAP**): recovering plaintext *m* from ciphertext *c*, given the public key (*n*, *e*)

RSA VS FACTORING

- **FACT. RSAP $\leq_P$ FACTORING**
  - FACTORING is at least as difficult as RSAP or, equivalently, RSAP is not harder than FACTORING
  - ***It is widely believed that RSAP and Factoring are computationally equivalent, although no proof of this is known.***

RSA vs Factoring

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- **THM**. Computing the decryption exponent ***d*** from the public key (***n, e***) is computationally equivalent to factoring ***n***
  - If the adversary could somehow factor ***n***, then he could subsequently compute the private key ***d*** efficiently
  - If the adversary could somehow compute ***d***, then it could subsequently factor ***n*** efficiently

Factoring

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- **FACTORING**.
  - Given ***n* > 0**, find its prime factorization; that is, write $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ where $\boldsymbol{p_i}$ are pairwise distinct primes and each $\mathbf{e_i} \geq \mathbf{1}$**,**
- **Primality testing vs. factoring**
  - Deciding whether an integer is composite or prime seems to be, in general, much easier than the factoring problem
- **Factoring algorithms**
  - Brute force
  - Special purpose
  - General purpose
  - Elliptic Curve
  - Factoring on Quantum Computer (for the moment only theorethical)

Factoring algorithms

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- **Brute Force**
  - Unfeasible if *n* large and *p* len = *q* len
- **General purpose**
  - The running time depends solely on the size of n
    - Quadratic sieve
    - General number field sieve
- **Special purpose**
  - The running time depens on certain properties
    - Trial division
    - Pollard's rho algorithm
    - Pollard's *p* -1 algorithm
- **Elliptic curve algorithm**

Running times

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Trial division: $O\left(\sqrt{n}\right)$

Quadratic sieve: $O\left(e^{\left(\sqrt{\ln(n)\bullet\ln\ln(n)}\right)}\right)$

General number field sieve: $O\left(e^{\left(1.923\times\sqrt[3]{\ln(n)\bullet(\ln\ln(n))^2}\right)}\right)$

Security of RSA

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RSAP and e-th root

- A possible way to decrypt $c = m^e \bmod n$ is to compute the e-th root of c
- **THM**. Computing the e-th root is a computationally easy problem iff n is prime
- **THM**. If n is composite the problem of computing the e-th root is *equivalent* to factoring

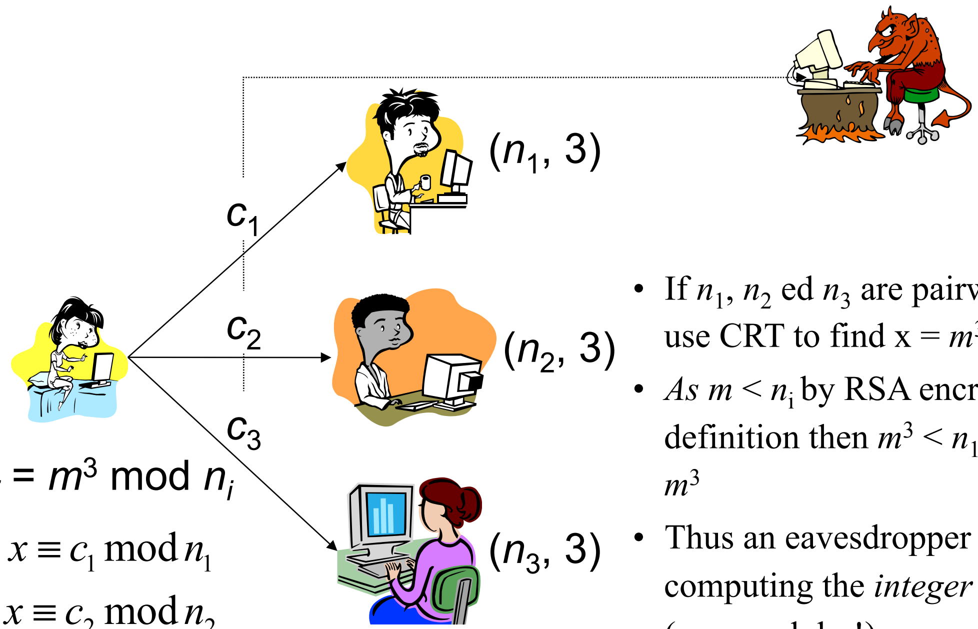
Security of RSA

- **Factoring vs totally breaking RSA**
  - A possible way to completely break RSA is to obtain φ
- **THM**. Knowing φ is computationally equivalent to factoring
  - **PROOF**.
    1. Given p and q, s.t. $n = pq$, computing φ is immediate.
    2. Let φ be given.
       a. From $\varphi(n) = (p-1)(q-1) = n - (p+q) + 1$, determine $x_1 = (p+q)$.
       b. From $(p - q)^2 = (p + q)^2 - 4n$, determine $x_2 = (p - q)$.
       c. Finally, $p = (x1 + x2)/2$ and $q = (x1 - x2)/2$.

Security of RSA

- A possible way to completely break RSA is an exhaustive attack to the private key *d*
  - This attack could be more difficult than factoring because (according to the choice for *e*) *d* can be much greater than *p* and *q*.

RSA: low exponent attack



$c_i = m^3 \bmod n_i$

$$\begin{cases} x \equiv c_1 \bmod n_1 \\ x \equiv c_2 \bmod n_2 \\ x \equiv c_3 \bmod n_3 \end{cases}$$

- If n_1, n_2 ed n_3 are pairwise coprime, use CRT to find x = $m^3 \bmod n_1 n_2 n_3$
- *As* $m < n_i$ by RSA encryption definition then $m^3 < n_1 n_2 n_3$, then $x = m^3$
- Thus an eavesdropper recovers m by computing the *integer cube root* of x (non modular!)

RSA in practice - padding

- We have described "schoolbook RSA"
- RSA implementation may be insecure
  - RSA is deterministic
  - PT values x = 0, x = 1 produce CT equal to 0 and 1
  - Small PT might be subject to attacks
  - RSA is malleable
- Padding is a possible solution
  - Optimal Asymmetric Encryption Padding (OAEP)
  - Public Key Cryptography Standard #1 (PKCS #1)

RSA is malleable

- RSA malleability is based on the homo-morphic property of RSA
- Attack
  - The attacker replaces CT = y mod n by CT' = $s^e \cdot y$ mod n, with s some integer
  - The receiver decrypts CT': $(s^e \cdot y)^d = s^{ed} \cdot x^{ed} = s \cdot x \bmod n$
  - By operating on the CT the adversary manages to multiply PT by s
  - **EX**. Let x be an amount of money. If $s = 2$ then the adversary doubles the amount
  - Possible solution: introduce redundancy: ex. $x \,||\, x$

RSA – Homomorphic property

- Let m_1 and m_2 two plaintext messages
- Let c_1 and c_2 their respective encryptions
- Observe that

$$(m_1 m_2)^e \equiv m_1^e m_2^e \equiv c_1 c_2 \pmod{n}$$

- In other words, the CT of the product $m_1 m_2$ is the product of CTs $c_1 c_2$ mod n

RSA in practice - PKCS #1

- Parameters
  - M = message
  - | M | = message len in bytes
  - k = | n | modulus len in bytes
  - | H | = hash function output len in bytes
  - L = optional label ("" by default)

RSA in practice - PKCS #1

- Padding
  1. Generate a string *PS* = 00…0; *PS* len = *k* – | *M* | - 2 |*H*| - 2 (*PS* len may be zero)
  2. *DB* = Hash(*L*) || *PS* || `0x01` || *M*
  3. *seed* = random(); *seed* len = | H |
  4. *dbMask* = MGF (*seed*, *k* - | *H* | - 1) (*)
  5. *maskedDB* = *DB* **xor** *dbMask*
  6. *seedMask* = MGF(*maskedDB*, | *H* |)
  7. *maskedSeed* = *seed* **xor** *seedMask*
  8. *EM* = `0x00` || *maskedSeed* || *maskedDB* (**)

(*) *MGF mask generation function (e.g., SHA-1)*
(**) *EM is the padded message*

RSA in practice

- ***RSA is substantially slower than symmetric encryption***
  – RSA is used for the transport of symmetric-keys and for the encryption of small quantities
- ***Recommended size of the modulus***
  – 512 bit: marginal security
  – 768 bit: recommended
  – 1024 bit: long-term security

RSA in practice

Selecting primes p and q

– ***p*** and ***q*** should be selected so that factoring ***n = pq*** is computationally infeasible, therefore

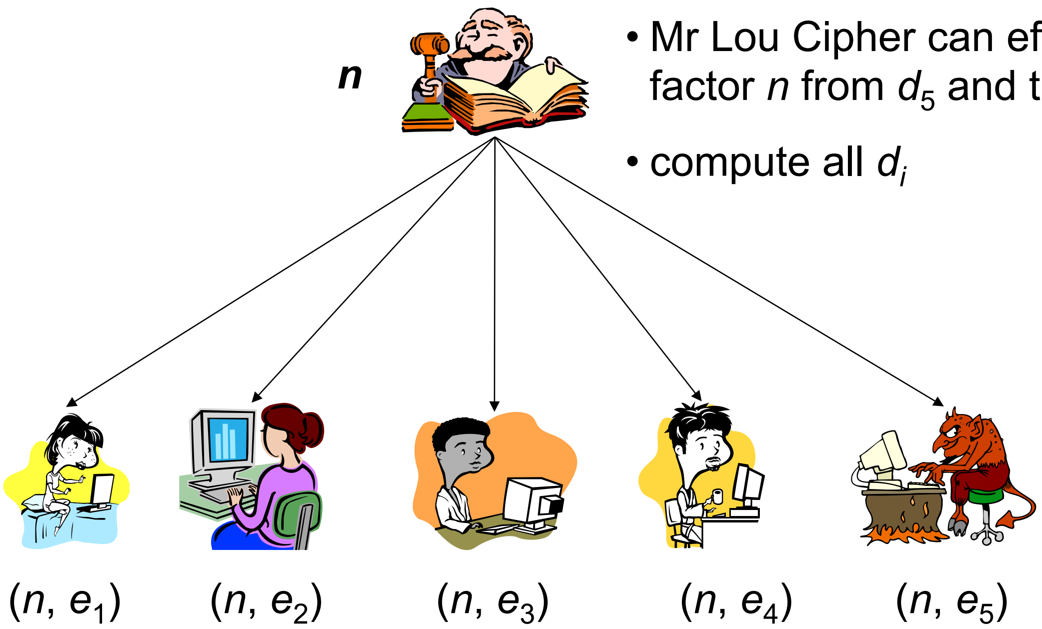
– ***p*** and ***q*** should be ***sufficiently large*** and about the ***same bitlenght*** (to avoid the elliptic curve factoring algorithm)

– ***p - q should be not too small***

RSA in practice

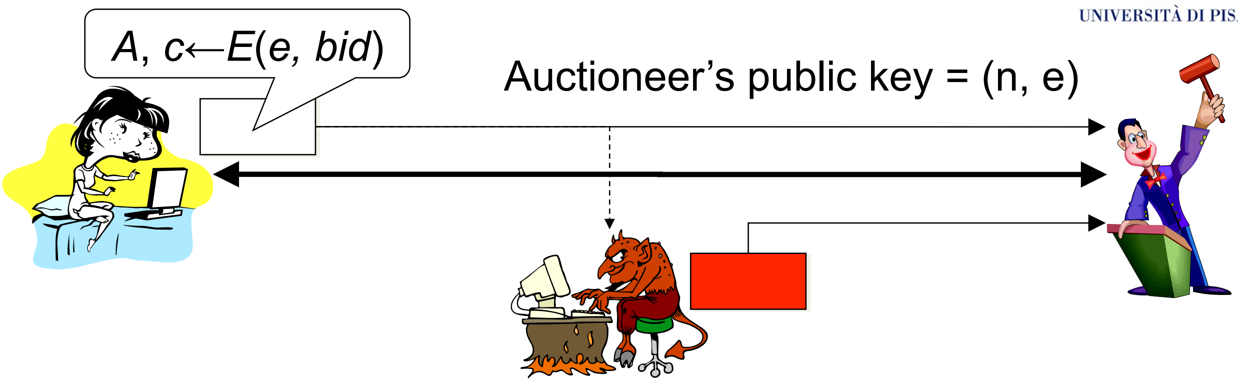
- Exponent *e* should be small or with a small number of 1's
  - **e = 3**
    [1 modular multiplication + 1 modular squaring]
    subject to small encryption exponent attack
  - **e = 2^{16} + 1 (Fermat's number)**
    [1 modular multiplication + 16 modular squarings]
    resistant to small encryption exponent attacks
- Decryption exponent *d* should be roughly the same size as *n*
  - Otherwise, if *d* is small, it could be possible to obtain *d* from the public information (*n*, *e*) or from a brute force attack

Common modulus attack



- Mr Lou Cipher can efficiently factor *n* from d_5 and then
- compute all d_i

Chosen-plaintext attack



The adversary encrypts all possible bids (e.g, 2^{32}) until he finds a **b** such that *E*(e, b) = *c*

Thus, the adversary sends a bid containing the minimal offer to win the auction: b' = b + 1

Salting is a solution: $r \leftarrow$ random(); $c \leftarrow E(e, r \,||\, bid)$

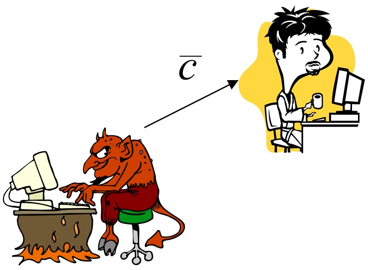
Homomorphic property of RSA

- Let m_1 and m_2 two plaintext messages
- Let c_1 and c_2 their respective encryptions
- Observe that

$$(m_1 m_2)^e \equiv m_1^e m_2^e \equiv c_1 c_2 \pmod{n}$$

  - In other words, the ciphertext of the product $m_1 m_2$ is the product of ciphertexts $c_1 c_2$ mod *n*

An adaptive chosen-ciphertext attack



- Bob decrypts ciphertext except a given ciphertext *c*
- Mr Lou Cipher wants to determine the ciphertext corresponding to *c*

- Mr Lou Cipher selects *x* at random, s.t. gcd(*x*, *n*) =1, and sends Bob the quantity $\overline{c} = cx^e \bmod n$
- Bob decrypts it, producing $\overline{m} = (\overline{c})^d = c^d x^{ed} = mx \pmod{n}$
- Mr Lou Cipher determine *m* by computing $m = \overline{m}x^{-1} \bmod n$

The attack can be contrasted by imposing structural constraints on *m*

Hybrid systems

- An asymmetric cipher is subject to the chosen-plaintex attack
- An asymmetric cipher is three orders of magnitude slower than a symmetric cipher

therefore

- An asymmetric cipher is often used in conjunction with a symmetric one so producing an *hybrid system*

Hybrid systems

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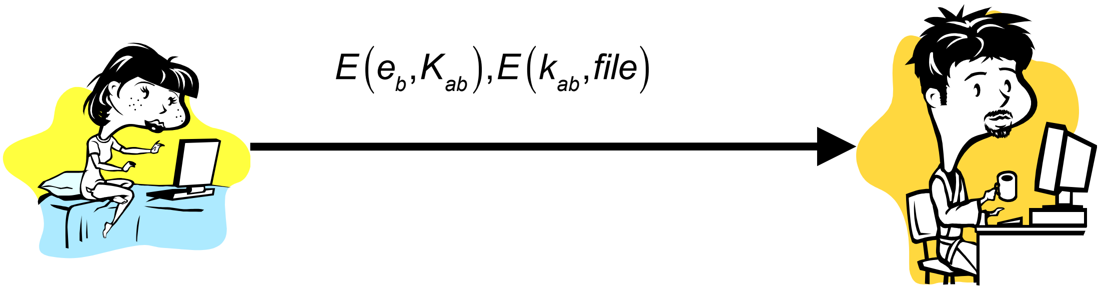
Alice confidentially sends Bob a file *file*



- File *file* is encrypted with a symmetric cipher
- Session key is encrypted with an asymmetric cipher
- Alice needs an *authentic* copy of Bob's public key

Public-key encryption

OTHER PUBLIC KEY CRYPTO-SYSTEMS

Other asymmetric cryptosystems

Discrete Logarithm Systems

- Let p be a prime, q a prime divisor of p–1 and $g \in [1, p-1]$ has order q
- Let x be the *private key* selected at random from $[1, q-1]$
- Let y be the corresponding *public key* $y = g^x \bmod p$
- **Discrete Logarithm Problem (DLP)**
- Given (p, q, g) and y, determine x

ElGamal encryption scheme

- **Encryption**
  - select ***k*** randomly
  - $c1 = g^k \bmod p$, $c_2 = m \times y^k \bmod p$
  - send (c_1, c_2) to recipient
- **Decryption**
  - $c_1^x = g^{kx} \bmod p = y^k \bmod p$
  - $m = c_2 \times y^{-k} \bmod p$
- **Security**
  - An adversary needs $y^k \bmod p$. The task of calculating $y^k \bmod p$ from **(g, p, q)** and **y** is equivalent to **DHP** and thus ***based*** on **DLP** in $\mathbb{Z}_p$

ElGamal in practice

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- Prime p and generator g can be common system-wide
- Prime p size
  - 512-bit: marginal
  - 768-bit: recommended
  - 1024-bit or larger: long-term
- Efficiency
  - Encryption requires two modular exponentiations
  - Message expansion by a factor of 2
- Security
  - Different random integers k must be used for different messages

Ellyptic Curve Cryptography

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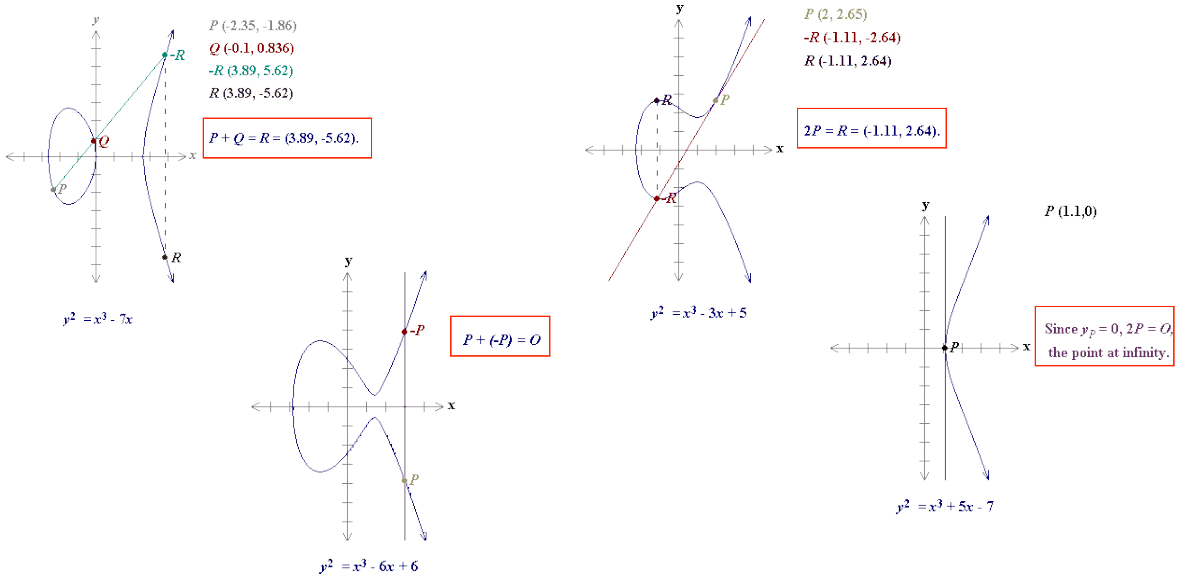
- Let p and $\in \mathbb{F}_p$
- Let E be an elliptic curve defined by
  $y^2 = x^3 + ax + b$ (mod p) where a, $b \in \mathbb{F}_p$ and $4a^3+27b^2=0$
- Example. E: $y^2 = x^3 + 2x + 4$ (mod p)
- The set of points $E(\mathbb{F}_p)$ with *point at infinity* ∞forms an additive Abelian group

Elliptic curves

Geometrical approach



Elliptic Cryptography (ECC)

- Algebric Approach
  - Elliptic curves defined on finite field define an Abelian finite field
- **Elliptic curve discrete logarithm problem**
  - **Given points G and Q such that Q=kG, find the integer *k***
  - **No sub-exponential algorithm to solve it is known**
- ECC keys are smaller than RSA ones

Ellyptic Curve Cryptography

- Let *P* have order ***n*** then the cyclic subgroup generated by ***P*** is ***G = <P, 2P,…, (n – 1)P>***
- ***p***, ***E***, ***P*** and n are the *public parameters*
- Private key ***d*** is selected at random in [1, *n*–1]
- Public key is ***Q =dP***

Ellyptic Curve Cryptography

- Encryption
  - A message ***m*** is represented as a point ***M***
  - $C_1 = kP$; $C_2 = M + kQ$
  - send $(C_1; C_2)$ to recipient
- Decryption
  - $dC_1 = d(kP) = kQ$
  - $M = C_2 - dC_1$
- Security
  - The task of computing *kQ* from the domain parameters, ***Q***, and $C_1=kP$, is the **ECDHP**

Comparison among crypto-systems

	Security level (bits)				
	80 (SKIPJACK)	112 (3DES)	128 (AES small)	192 (AES medium)	256 (AES large)
DL parameter q EC parameter n	160	224	256	384	512
RSA modulus n DL modulus p	1024	2048	3072	8192	15360

- Private key operations are more efficient in EC than in DL or RSA
- Public key operations are more efficient in RSA than EC or DL if small exponent *e* is selected for RSA

PHYSICAL ATTACKS

Physical attacks

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- Embedded systems change the threat model
  - The adversary may physically attack the system
    - E.g.: smart meter
  - The system is even given to the adversary
    - E.g.: a bank or telco smart card
  - The adversary physically interfere with the system
  - Main attacks
    - Fault injection
    - Time analysis
    - Power analysis

CRT and RSA optimization

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- **Chinese Remainder Theorem** allows us to compute RSA more efficiently
- **Problem**: Compute $m = c^d \pmod{n}$
  1. Compute $m_1 = c^d \pmod{p}$ and $m_2 = c^d \pmod{q}$
  2. Compute $\mathbf{m = a_1 m_1 q + a_2 m_2 p}$
     where $\mathbf{a_1}$ and $\mathbf{a_2}$ are properly computed coefficients
- **Advantage.**
  - $E_1 = c^d \pmod{p} = c^{(d \bmod p-1)} \pmod{p}$,
  - While d is on k bits, **p–1** is on **k/2** bits
  - Thus, multiplication takes $\mathbf{O(k^2/4)}$

CRT and RSA optimization

- **Chinese Remainder Theorem** allows us to compute RSA (decryption, signing) more efficiently
- **Problem**: Compute $y = x^d \pmod{n}$
  1. Compute $x_p = x \bmod p$ and $x_q = x \bmod q$
  2. Compute $y_p = x_p^{d \bmod (p-1)} \bmod p$ and $y_q = x_q^{d \bmod (q-1)} \bmod q$
  3. Compute $y = a_p y_p q + a_q y_q p$ where a_p and a_q are properly (pre-)computed coefficients
- **Advantage.**
  – Computation of y_p and y_q is the most demanding
  – It requires #MUL+#SQ = 1.5*t*, on average
  – Each squaring/multiplication involves *t*/2-bit operands ➔ multiplication/ squaring takes $O(k^2/4)$
  – Thus **the total speedup obtained through CRT is a factor of 4**.

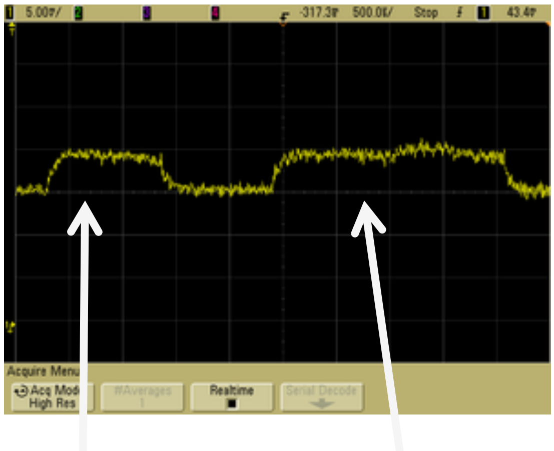
A fault-injection attack against CRT-based RSA

- **Attack intuition:** by injecting a fault the adversary is able to factorize **n**
- **The attack**
  – Cause an **hw fault** while computing y_p which produces y'_p and thus $y' = a_p y'_p q + a_q y_q p$
  – It follows that $y - y' = a_p(m'_p - m_p)q$
  – Thus, $\gcd(y - y', n) = q$ *which can be efficiently computed with the Euclide's algorithm*
- **Practical considerations**
  – **causing hw fault**: tamper with computing circuitry
  – **countermeasures**: checking results (10% *slow down*)

Power Analysis

- Power analysis is a **side channel** attack in which the attacker studies the power consumption of a cryptographic hardware device
  - smart card, tamper-resistant "black box", or integrated circuit
- The attack is non-invasive
- **Simple power analysis (SPA)** involves *visual examination* of graphs of the current used by a device over time.
  - Variations in power consumption occur as the device performs different operations.

Power Analysis of RSA

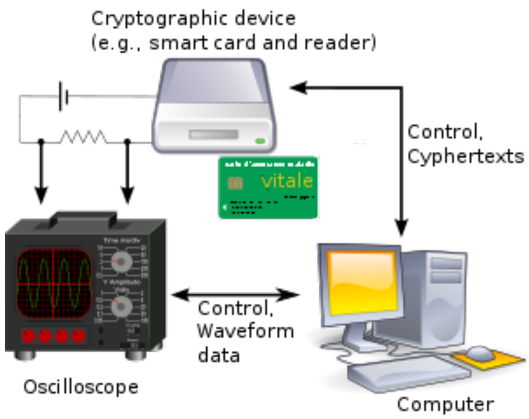


Key bit = 0
No multiplication

Key bit = 1
multiplication

Power Analysis

- **Differential power analysis (DPA)** involves statistically analyzing power consumption measurements from a cryptosystem.
  - DPA attacks have signal processing and error correction properties which can extract secrets from measurements which contain too much noise to be analyzed using simple power analysis.



Timing attack

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- A **timing attack** is a **side channel** attack in which the attacker attempts to compromise a cryptosystem by analyzing the time taken to execute cryptographic algorithms
  - Execution time depends on inputs (e.g., key!)
  - Precise measurement of time
  - Attack is application dependent
  - E.g., square-and-multiply for exp mod n
    - time depends on number of “1” in the key
    - Statistical analysis of timings with same key and different inputs