STATICS AWARE VORONOI GRID-SHELLS

research seminar
Bath, 18 December 2014

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Host: Dr. C. J. K. Williams

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Statics Aware Voronoi Grid-Shells

Free-form grid-shells, designed with a novel polygonal pattern, that are at the same time:

- ***innovative;***
- ***aesthetically pleasing;***
- ***structurally sound.***

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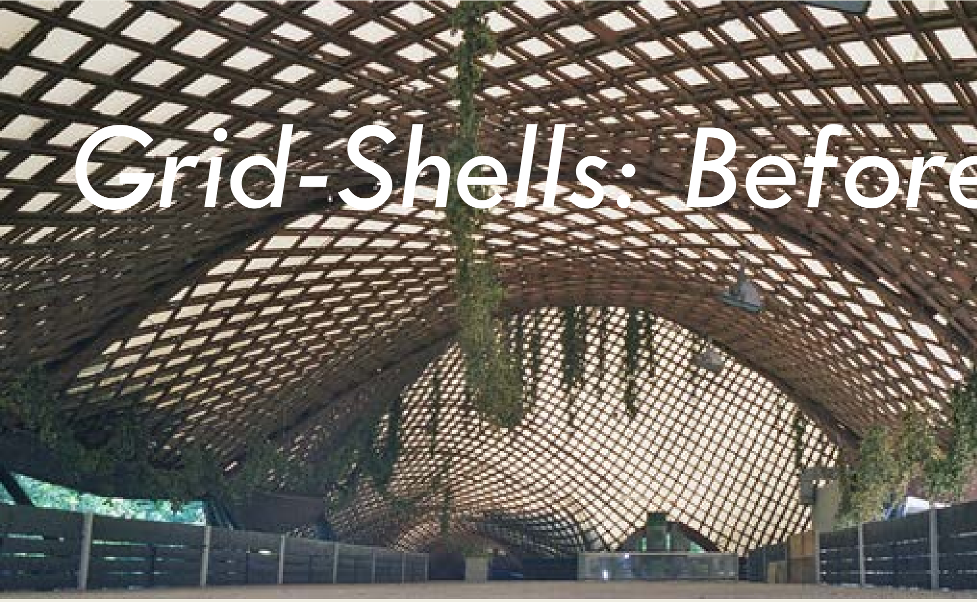
Grid-Shells: Before and Now



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Grid-Shells: Before and Now

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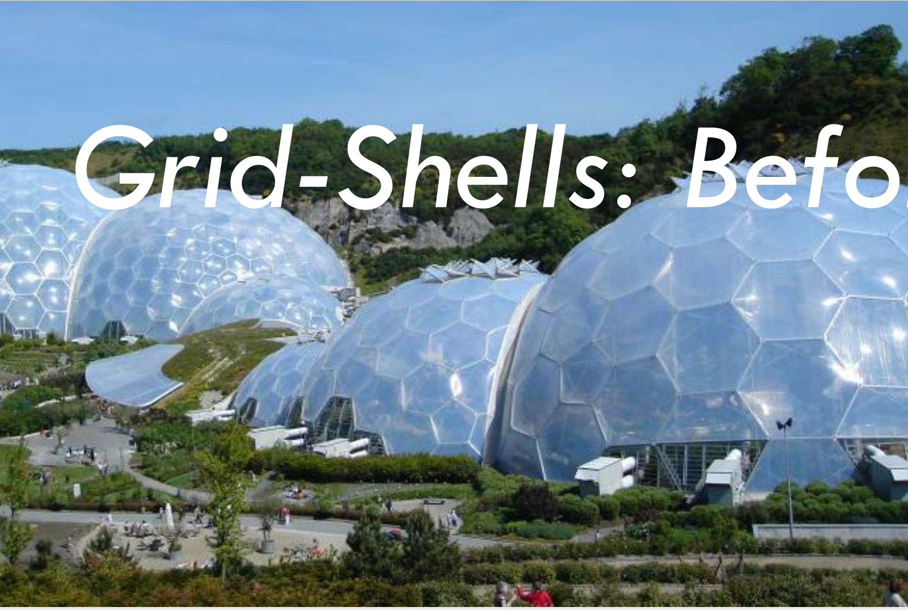
Grid-Shells: Before and Now



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What is a Grid-Shell

A ***colander*** is still a shell.

A ***sieve*** is a grid-shell.

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What is a *Grid-Shell*

A ***well shaped SHELL*** (i.e. that minimizes some domain integral based-function, such as the strain energy U)

with ***enough boundary support*** (i.e. restrained in such a way that only deformations involving length changes over the surface are allowed)

is a ***very effective structure.***

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What is a Grid-Shell

A ***well shaped GRID-SHELL***

with ***enough boundary support***

might not be a very effective structure

because many ***other variables*** affect its behaviour.

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What is a *Grid-Shell*

Among these, the joint rigidity and the

grid TOPOLOGY

are the most crucial.

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What is a *Voronoi* diagram

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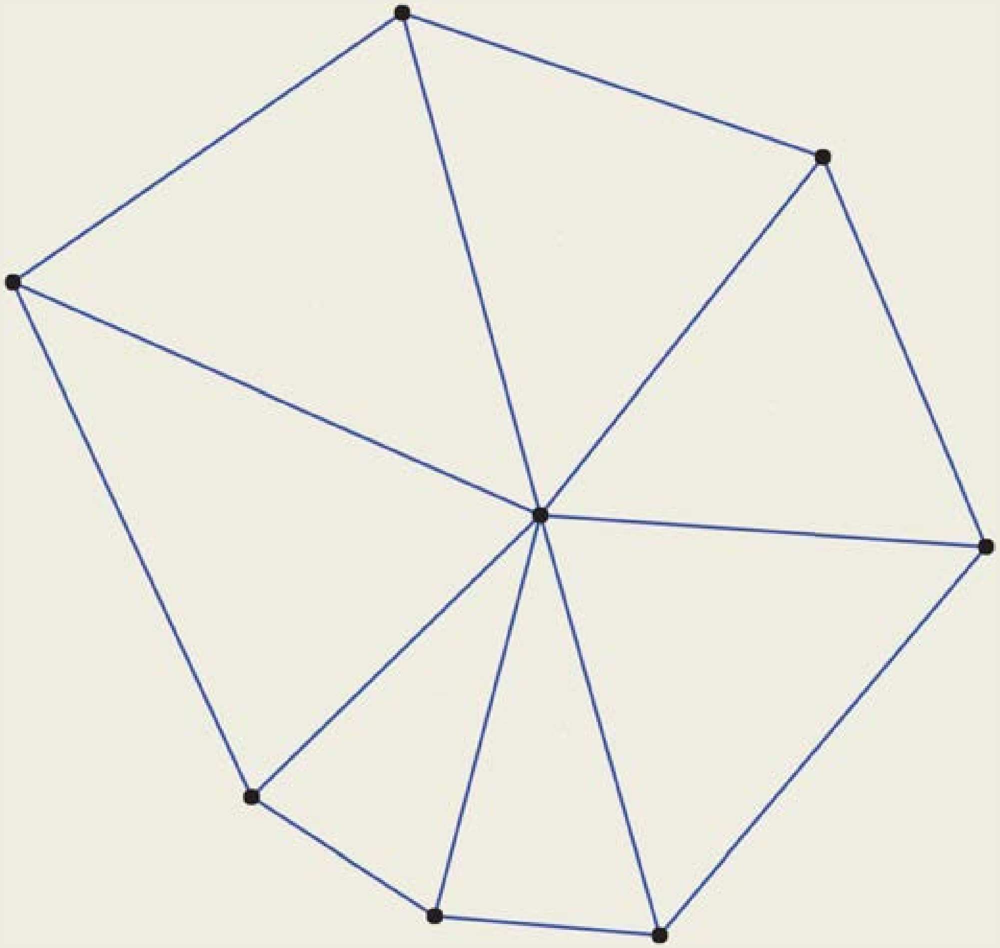
What is a Voronoi diagram



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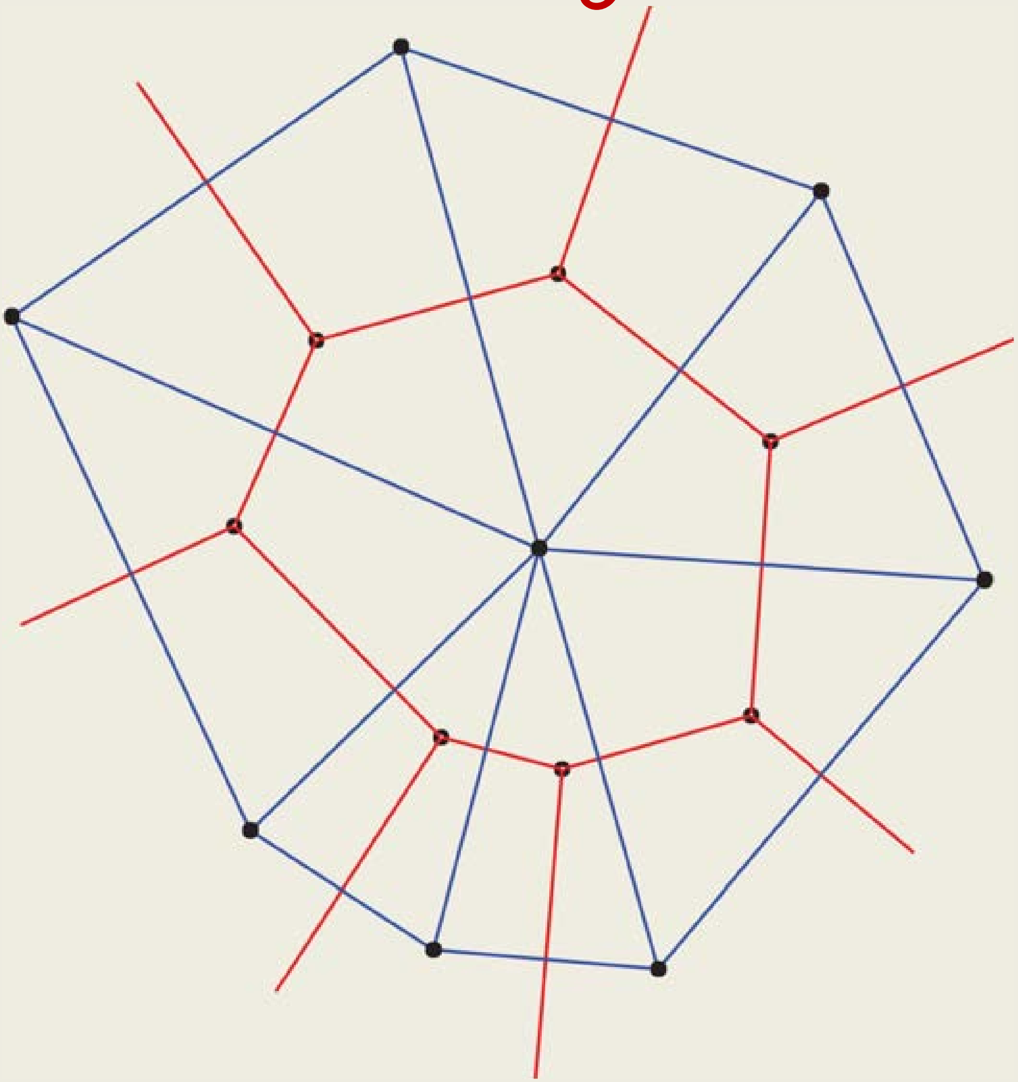
What is a Voronoi diagram



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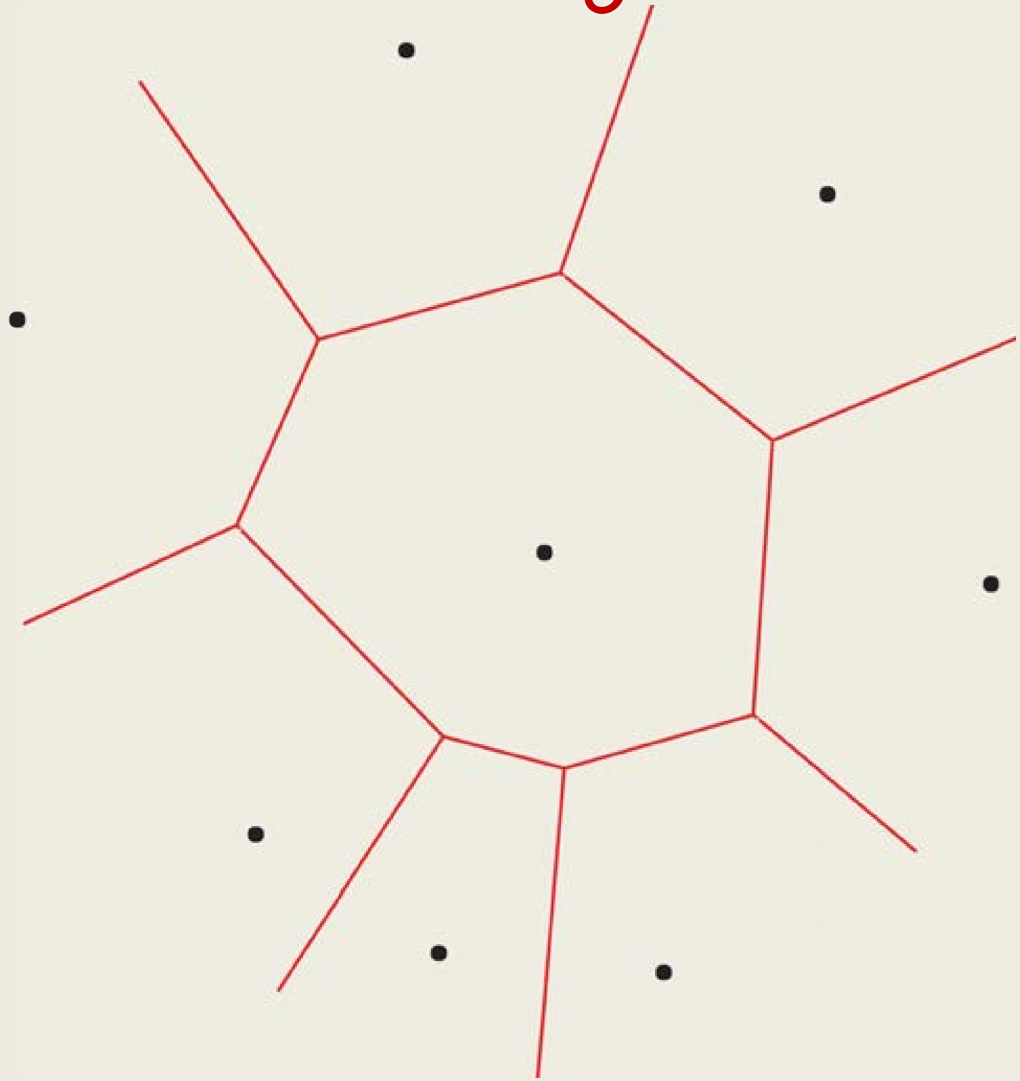
What is a Voronoi diagram



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What is a Voronoi-like Topology

It is a *hex-dominant* ***grid TOPOLOGY*** with *constant vertex valence* (3).

It is the only topology that leads to semi-regular meshes (i.e. not with the same polygon repeated) with all regular vertices (i.e. with the same valence 3).

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Voronoi-like Topology in Nature

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Voronoi-like Topology reproduced

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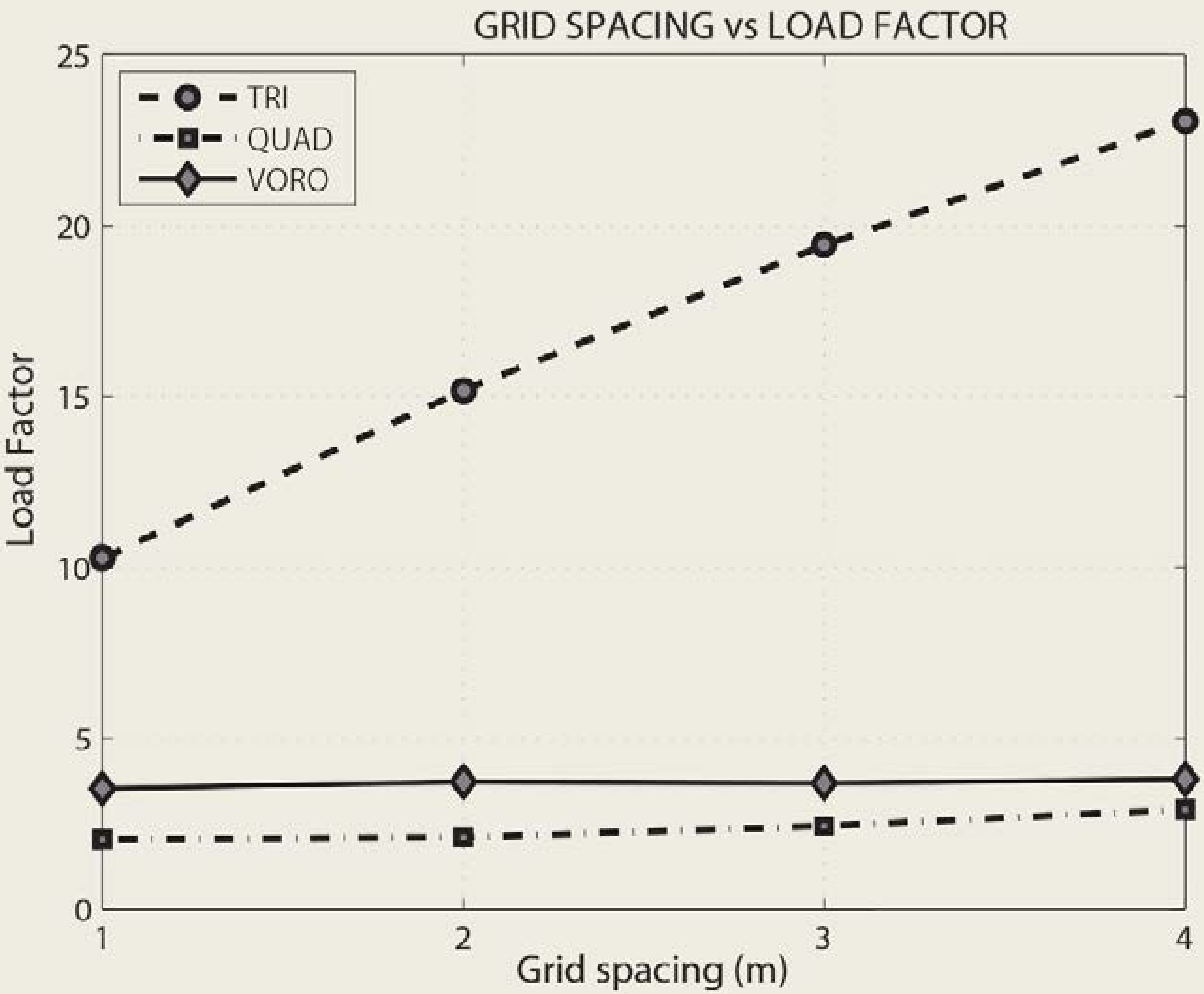
Topology and Mechanics



M_tot = Const does not guarantee ***equivalence*** in terms of statics.

M_tot = Const + L_tot = Const does.

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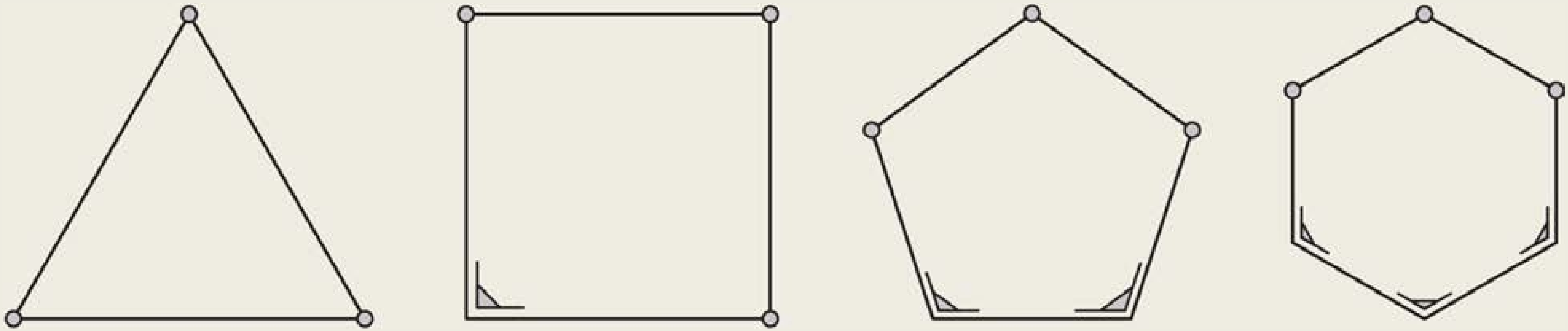
Topology and *Mechanics*



Every *n*-gonal polygon is a mechanism, unless *n*-3 vertices are rigid.

Whichever polygonal cell is less stiff than a triangle.

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Topology and Mechanics

Topology	Eq. Membrane Stiffness	Eq. Bending Stiffness	Eq. Thickness
Triangular	$\frac{2}{\sqrt{3}} \frac{EA}{L}$	$\frac{\sqrt{3}}{4} \frac{EI}{L} \left(3 + \frac{GJ}{EI}\right)$	4.1ρ
Quad. (MAX Stiff.)	$\frac{EA}{L}$	$\frac{EI}{L} \frac{(EI + 2GJ)}{(EI + GJ)}$	4.1ρ
Quad. (MIN Stiff.)	$\frac{EI}{L} \frac{2}{(\rho^2 + \frac{L^2}{12})}$		$0.85L$
Hexagonal	$\frac{4}{\sqrt{3}} \frac{EI}{L} \frac{12}{(52\rho^2 + L^2)}$	$\frac{4}{\sqrt{3}} \frac{EI}{L} \frac{GJ}{(EI + 3GJ)}$	$0.5L$

Analytical 'equivalent elastic properties' (membrane stiffness, bending stiffness, thickness) for the three regular tilings of the plane.

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Topology and Mechanics

Topology	Eq. Membrane Stiffness $(\frac{kN}{m})$	Eq. Bending Stiffness $(kN\,m)$	Eq. Thickness (mm)
Triangular	$2.506 \cdot 10^5$	55.06	50
Quad. (MAX Stiff.)	$3.172 \cdot 10^5$	~70	50
Quad. (MIN Stiff.)	$7.031 \cdot 10^2$		1100
Hexagonal	$7.164 \cdot 10^3$	54.94	312.5

Numerical 'equivalent elastic properties' (membrane stiffness, bending, stiffness, thickness) for the three regular tilings of the plane. The beams are considered being made of steel and having circular cross section of 50 mm of diameter ($EA = 412334$ kN, $EI = 64.427$ kNm^2).

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Statics Awareness - Intuition

The grid-shell remeshing is driven by the statics of the underlying surface.

THE INTUITION:

Making *smaller* cells over high stress regions and *stretching* them along the principal stress direction, in such a way that the global strain energy U is minimized, yields grid-shells with optimal statics.

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Statics Awareness - Pipeline

1- Find out the state of stress of the surface underlying the grid-shell. The surface is FIXED;

2- Define **local** *density* and *anisotropy* functions for sizing and shaping the cells, respectively, according to the local state of stress.

3- Remesh the surface, thus obtaining an *optimal grid-shell* (i.e. that minimizes U).

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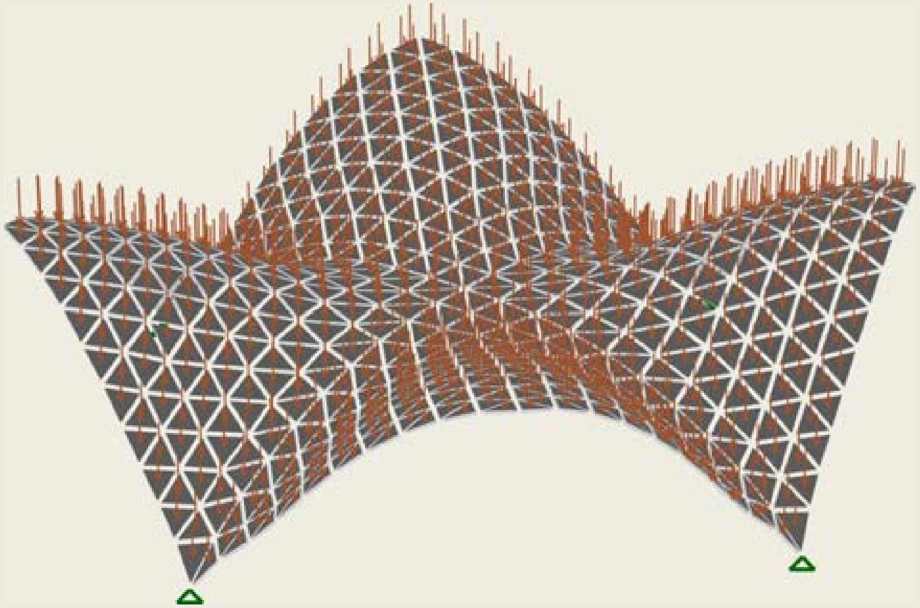
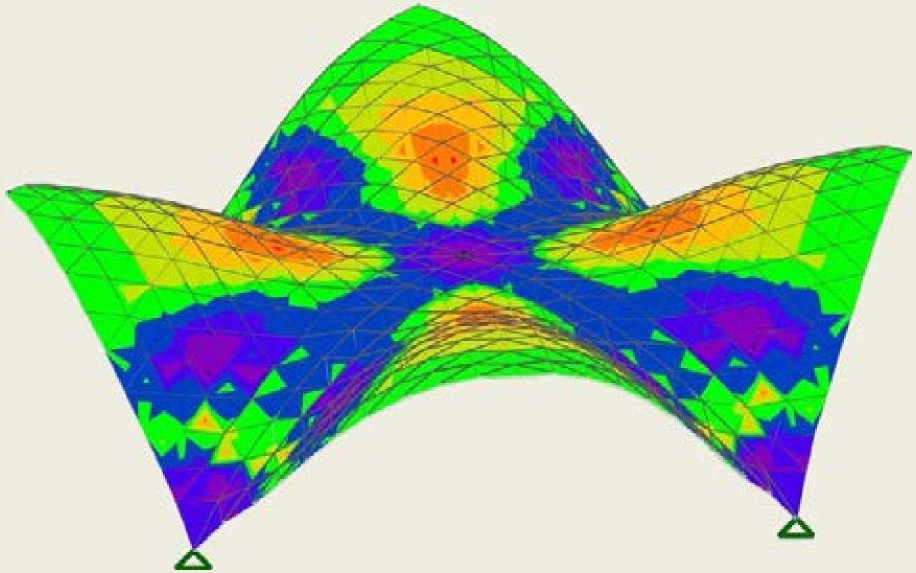
Statics Awareness

1- F.E.M. linear elastic analysis of the continuous surface underlying the grid-shell. The stress tensor is computed at each shell element.





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Statics Awareness

2 – Definition of *local transfer criteria*.

A *transition* from the continuum shell to the discrete optimal Voronoi grid-shell:
The local stress tensor $\psi_f(p)$ is related to a tailored metric tensor $\psi_e(p)$ by means of the strain energy.
ψ_e brings about a new non-Euclidean metric over the surface, that maps a unit circle on the tangent plane at p into an optimal ellipse of radiuses (a,b), aligned along the principal directions at p.

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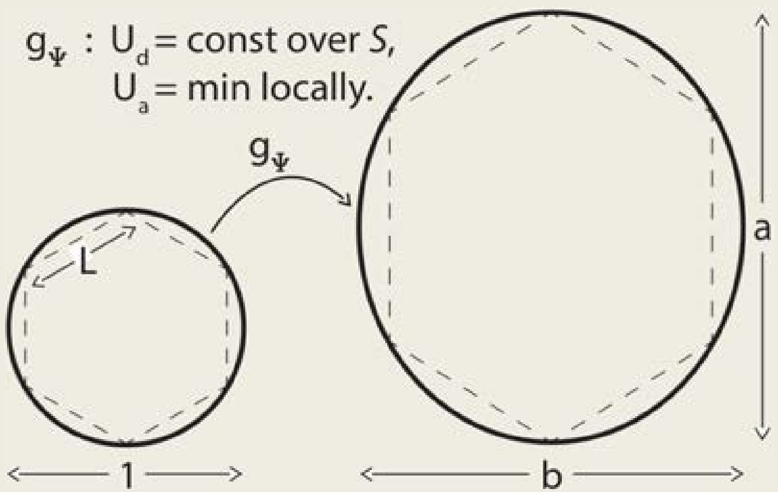
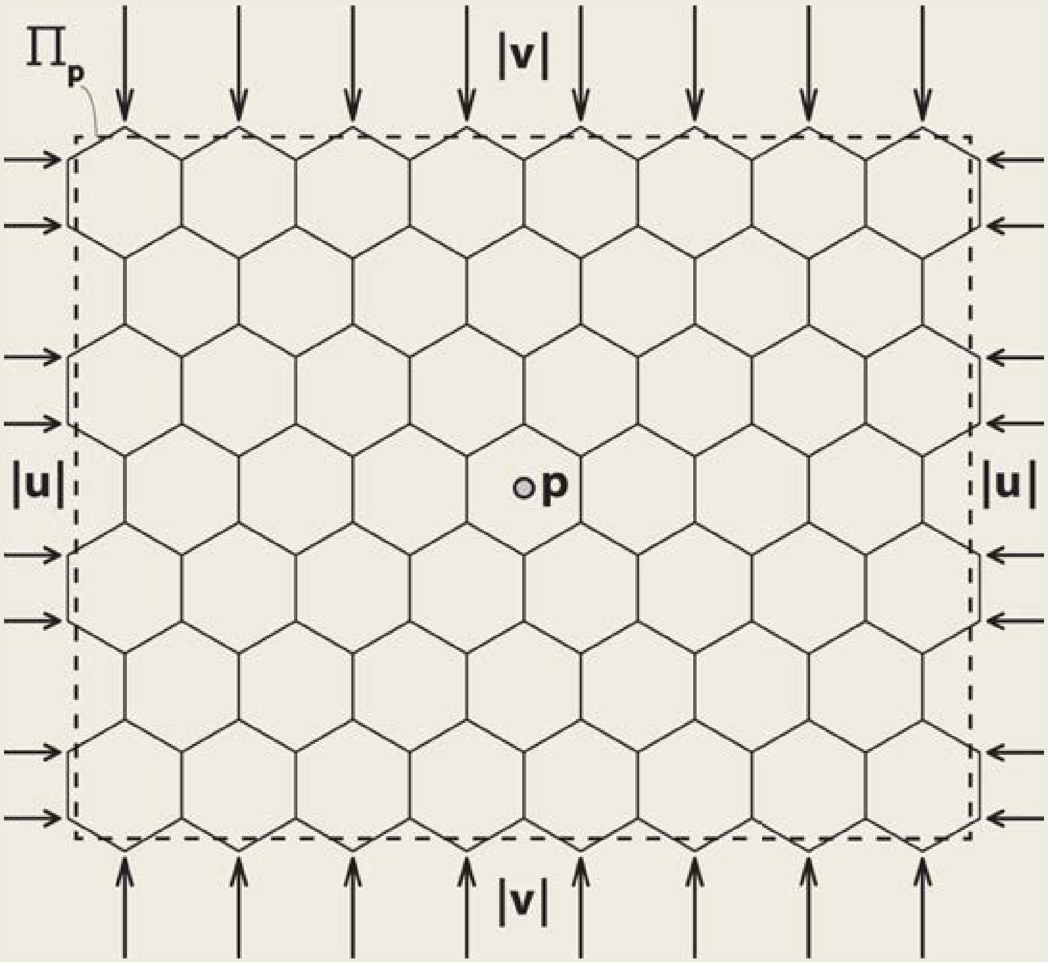
Statics Awareness

2 – Definition of *local transfer criteria.*





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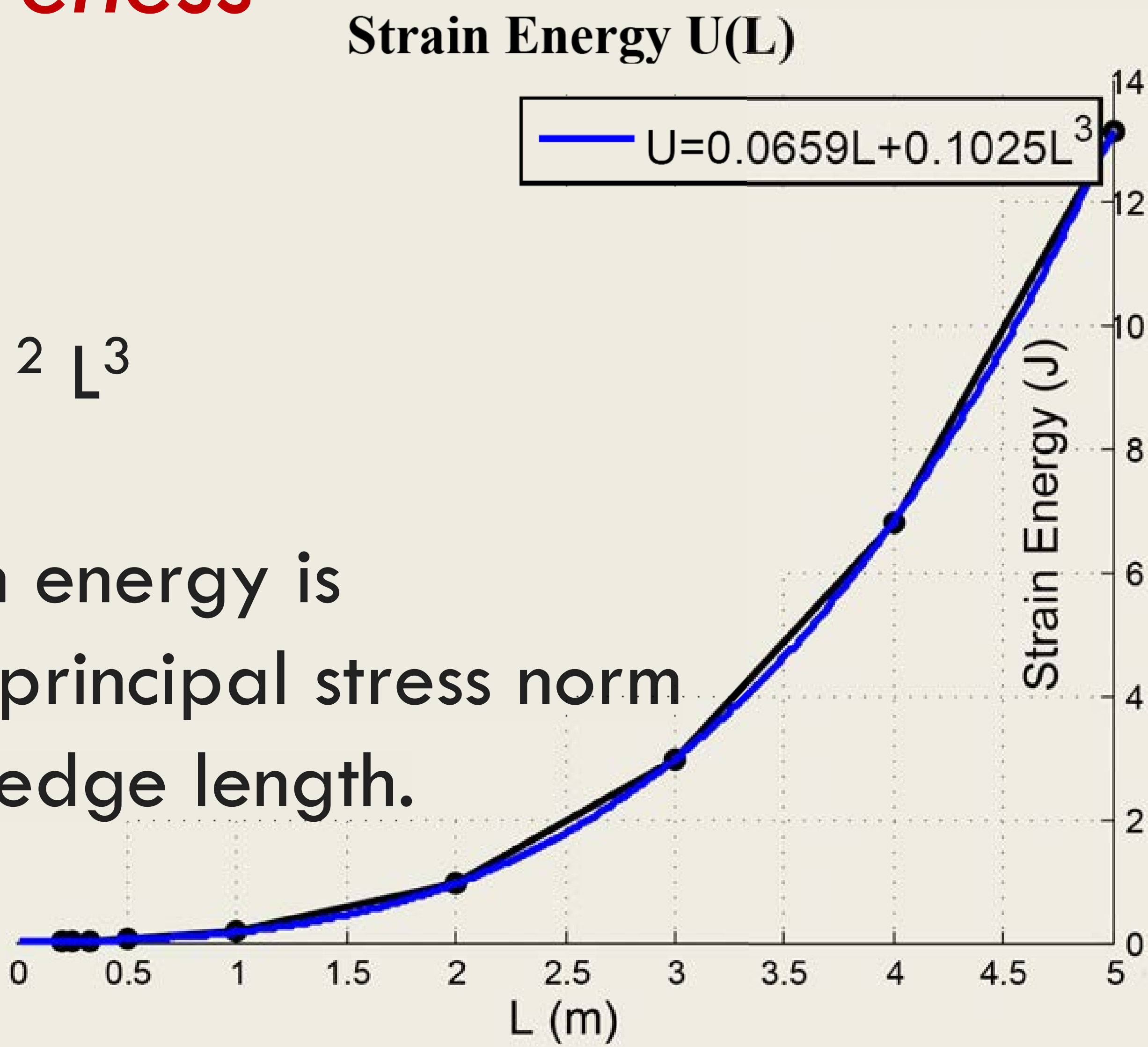
Statics Awareness

DENSITY:

$U_d(|u|, L) \propto |u|^2 L^3$

The *density* strain energy is quadratic in the principal stress norm and cubic in the edge length.



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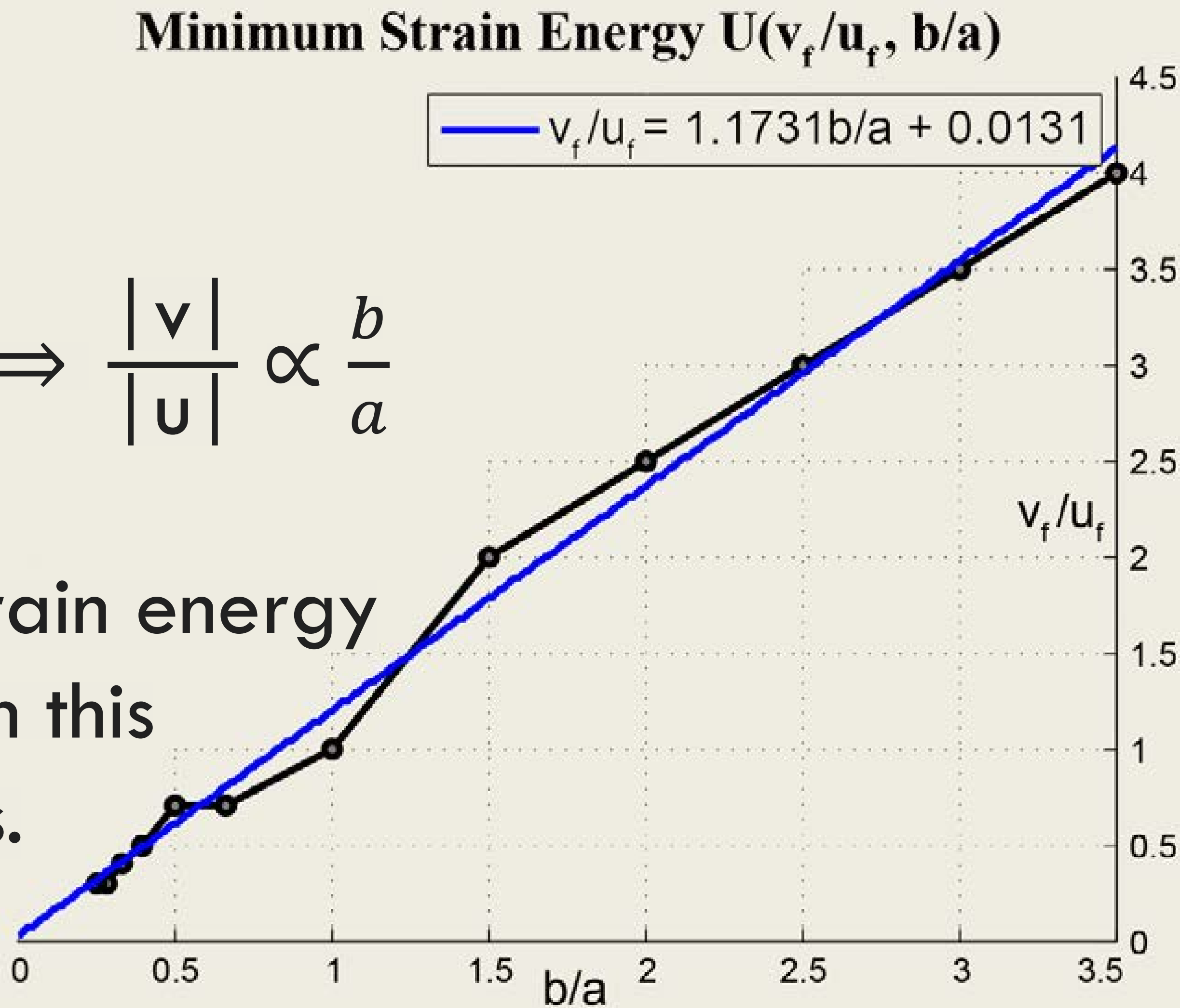
Statics Awareness

ANISOTROPY:

$$\min U_a\left(\frac{|v|}{|u|}, \frac{b}{a}\right) \Longrightarrow \frac{|v|}{|u|} \propto \frac{b}{a}$$

The *anisotropy* strain energy is minimized when this relationship holds.



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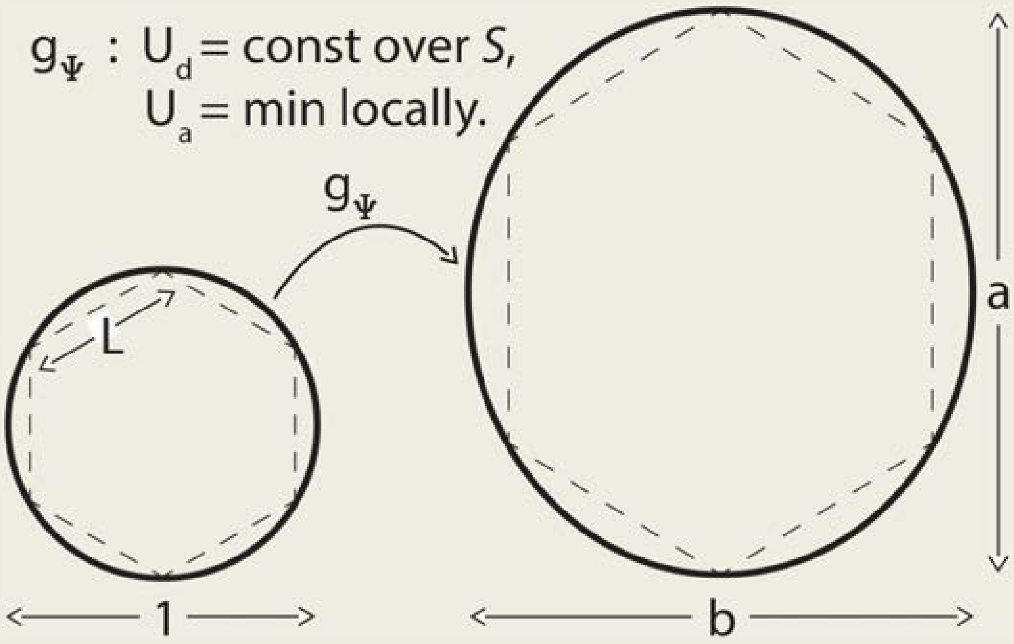
Statics Awareness

NEW METRIC: $g_\psi = W^{-T} W^{-1}$

$$D = (|u|\,|v|)^{2/3}$$

$$A = \frac{|u|}{|v|}$$

$$W = \begin{bmatrix} \sqrt{DA} & 0 \\ 0 & \sqrt{\frac{D}{A}} \end{bmatrix}$$



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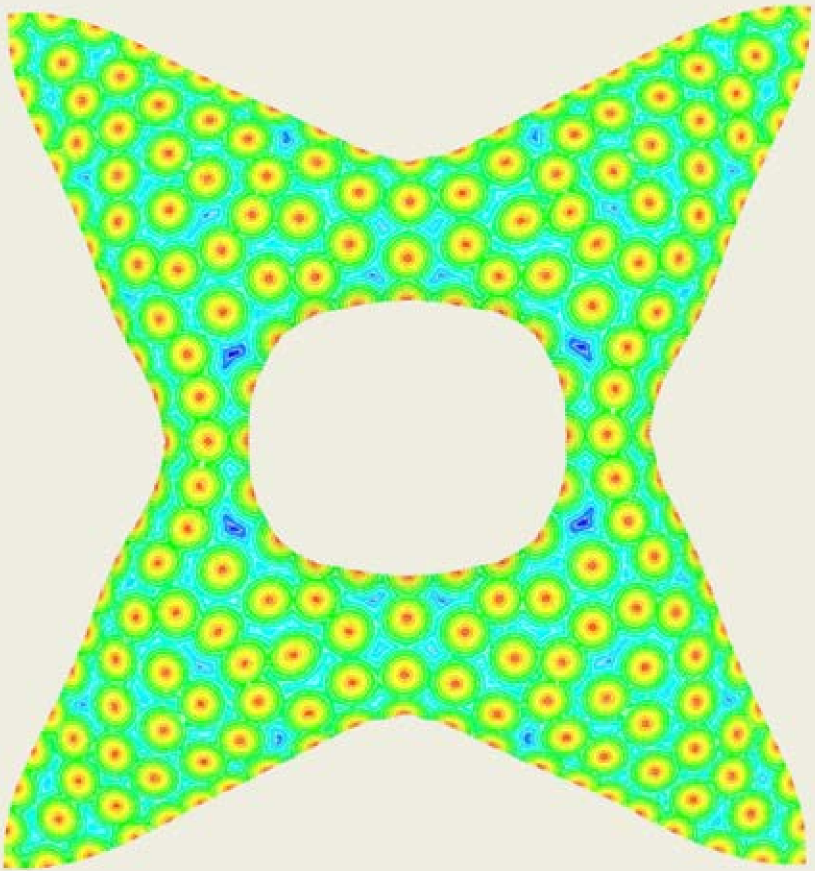
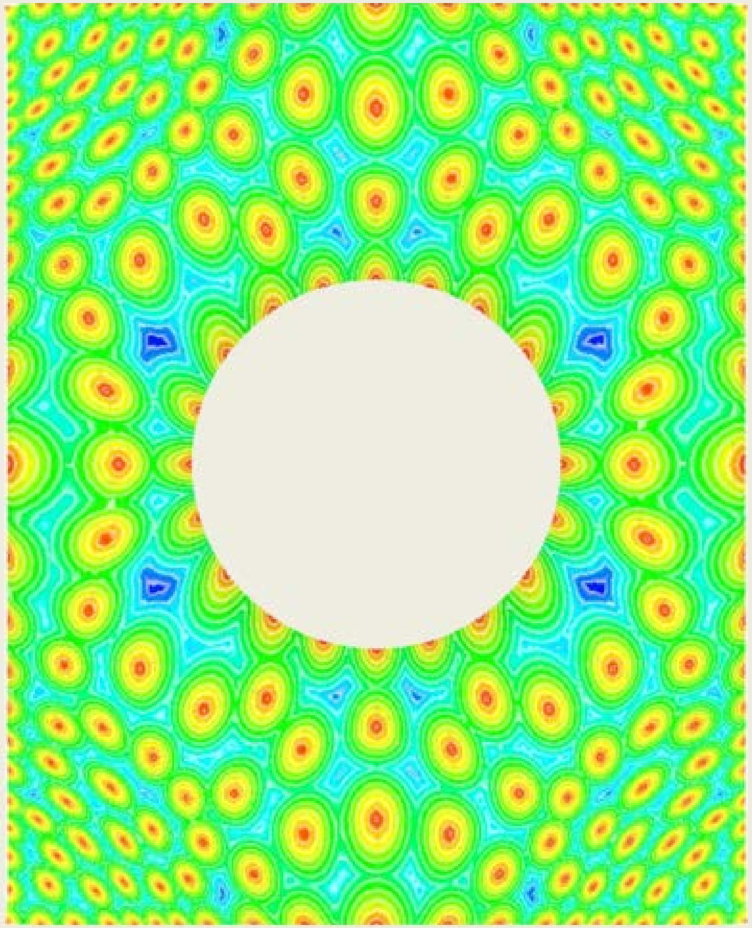
Statics Awareness

3 – Surface Remeshing.





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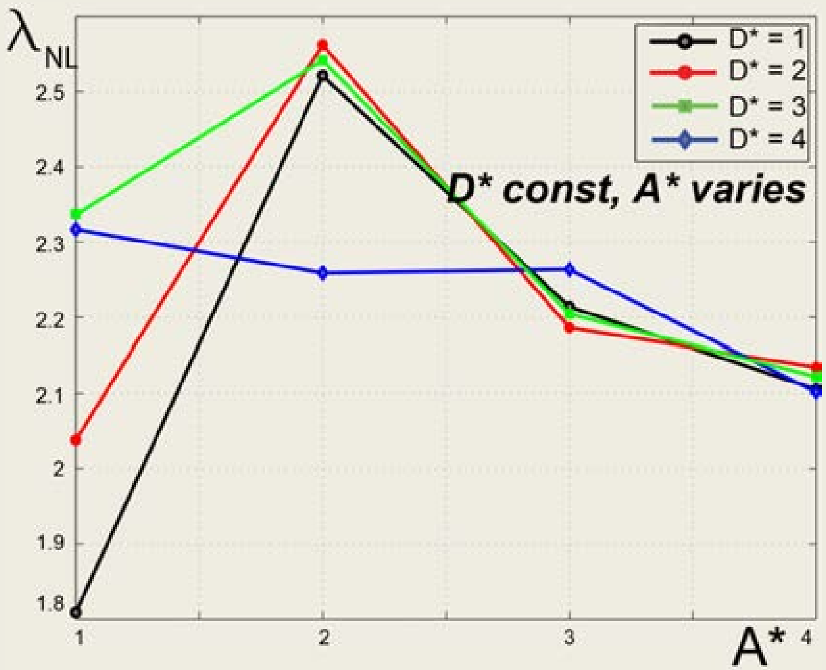
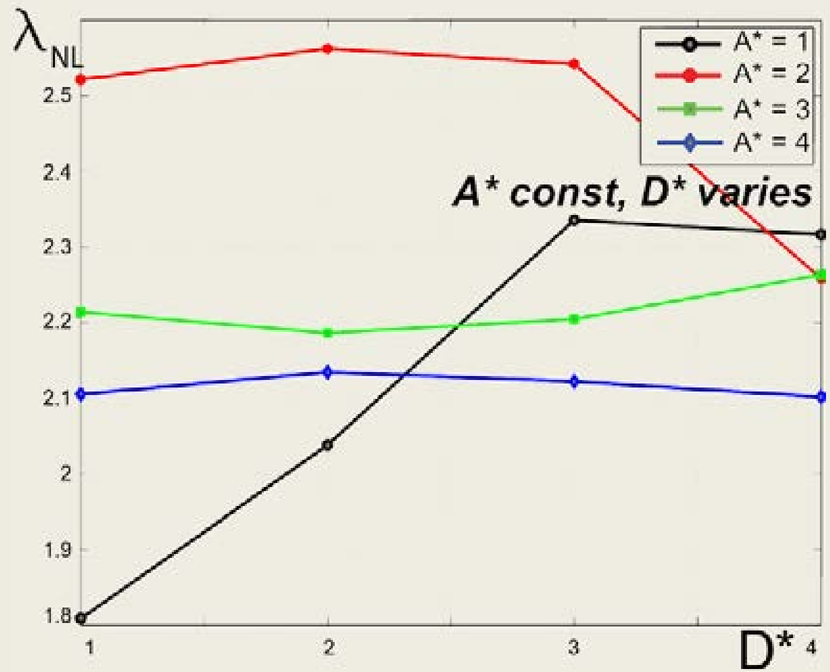
Statics Awareness

Parameters D, A can be manually tuned (i.e. they can be limited and rescaled within a user-defined scale, usually ranging from 1 to 4).





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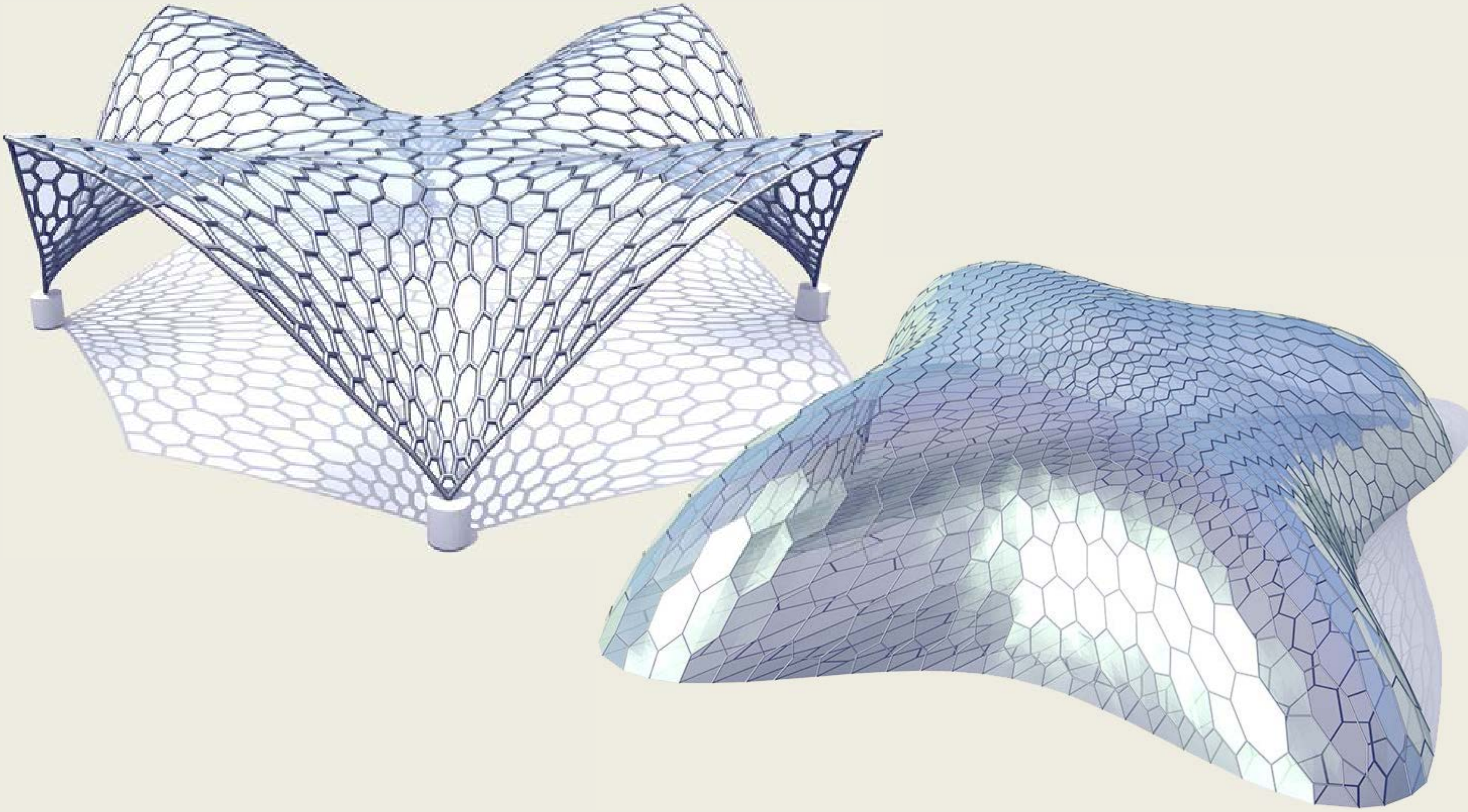

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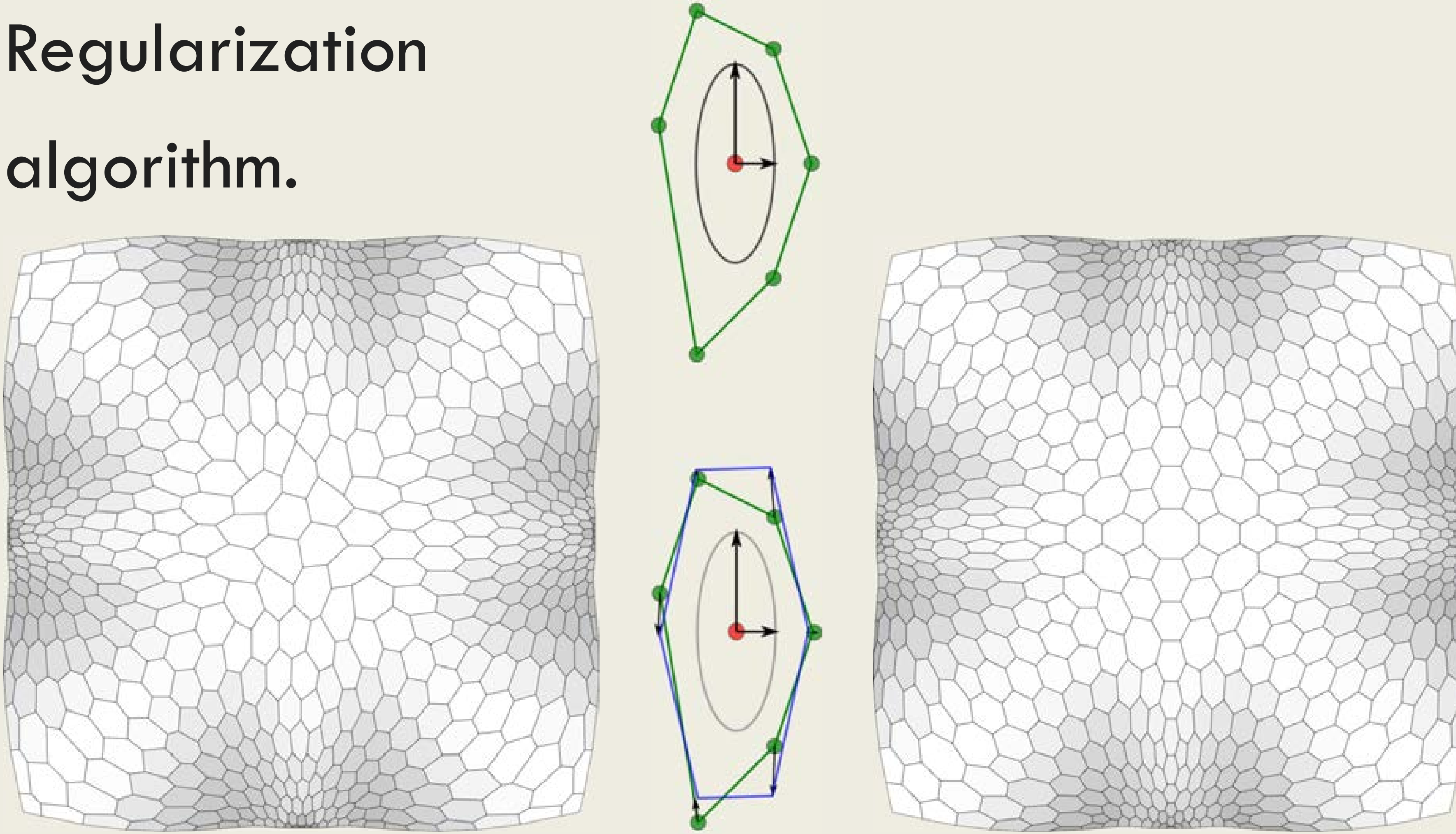
Mesh Quality Improvement

Regularization algorithm.



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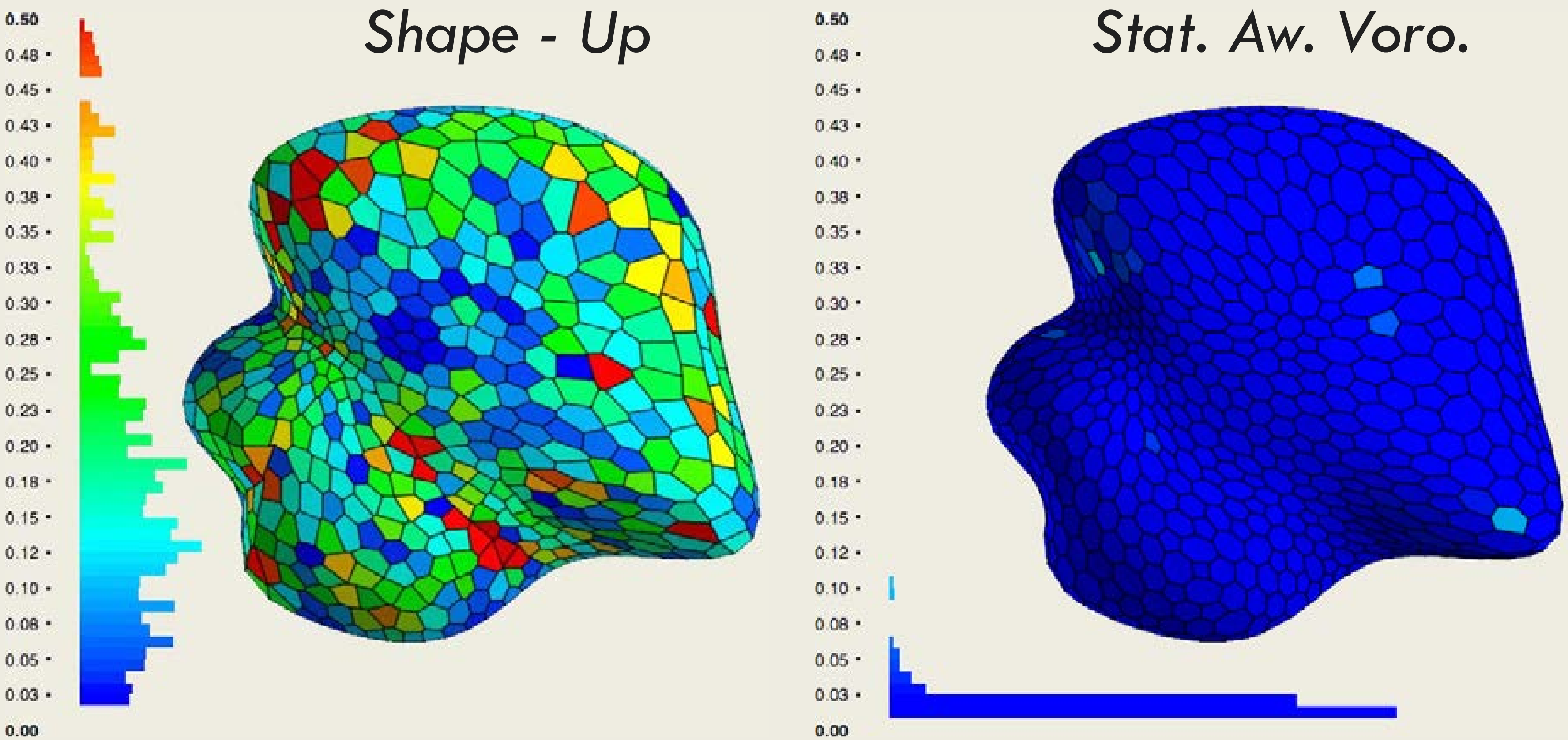
Mesh Quality Improvement

Regularity – comparison of results $Reg = \sum d_i^2 / A$



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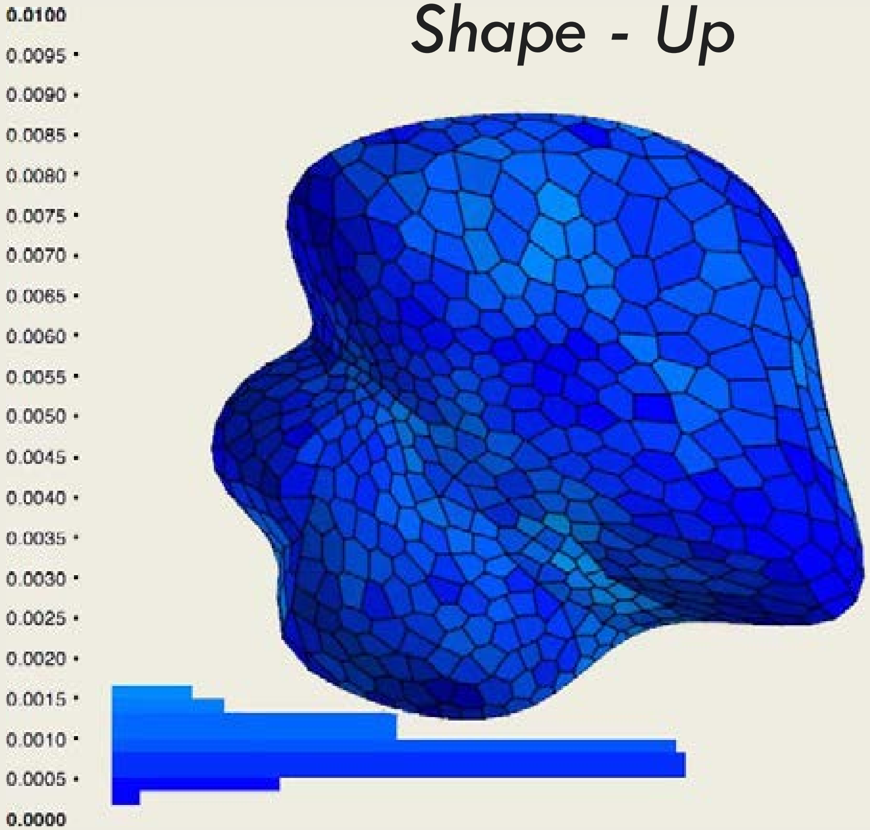
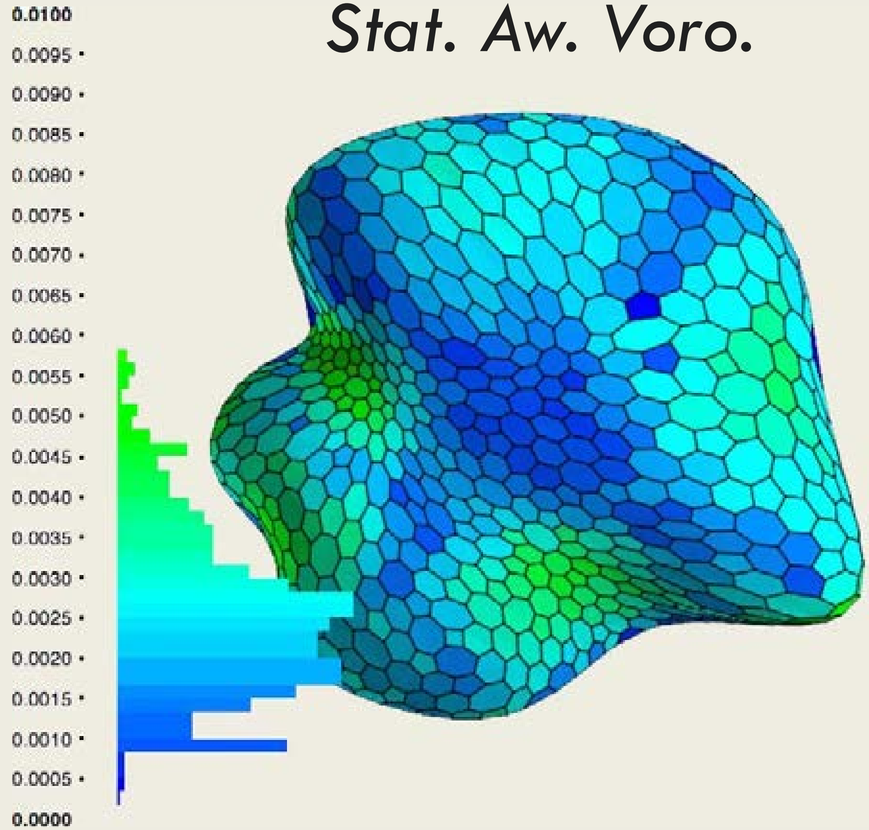
Mesh Quality Improvement

Planarity – comparison of results $Plan = 2d_{av}/P$

Shape - Up



Stat. Aw. Voro.



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Performances - *preliminary assessment*

Dataset	Model	Geometry			Beams		Structural Analyses			
		Vert.	Faces	Edges	L_{tot} (m)	(φ_e, t) (mm)	q_{es} $(\frac{kN}{m^2})$	δ_L (mm)	q_{slu} $(\frac{kN}{m^2})$	λ_{NL}
Botanic	[TSG + 14]	1121	1076	2196	1989	(70,15)	2.3	335.4	3.4	0.83
	Voro (3,3)	2352	1177	3528	2016			171.6		1.48
British	[TSG + 14]	1648	1568	3216	4286	(100,15)	2.4	27.9	3.5	2.70
	Voro (3,3)	3460	1728	5188	4114			26.7		3.23
Lilium	[VHWP12]	665	645	1300	1147	(50.10)	2.1	77.6	3.1	1.42
	Voro (3,3)	1444	723	2166	1182			37.55		2.97

Statistics on datasets and results.

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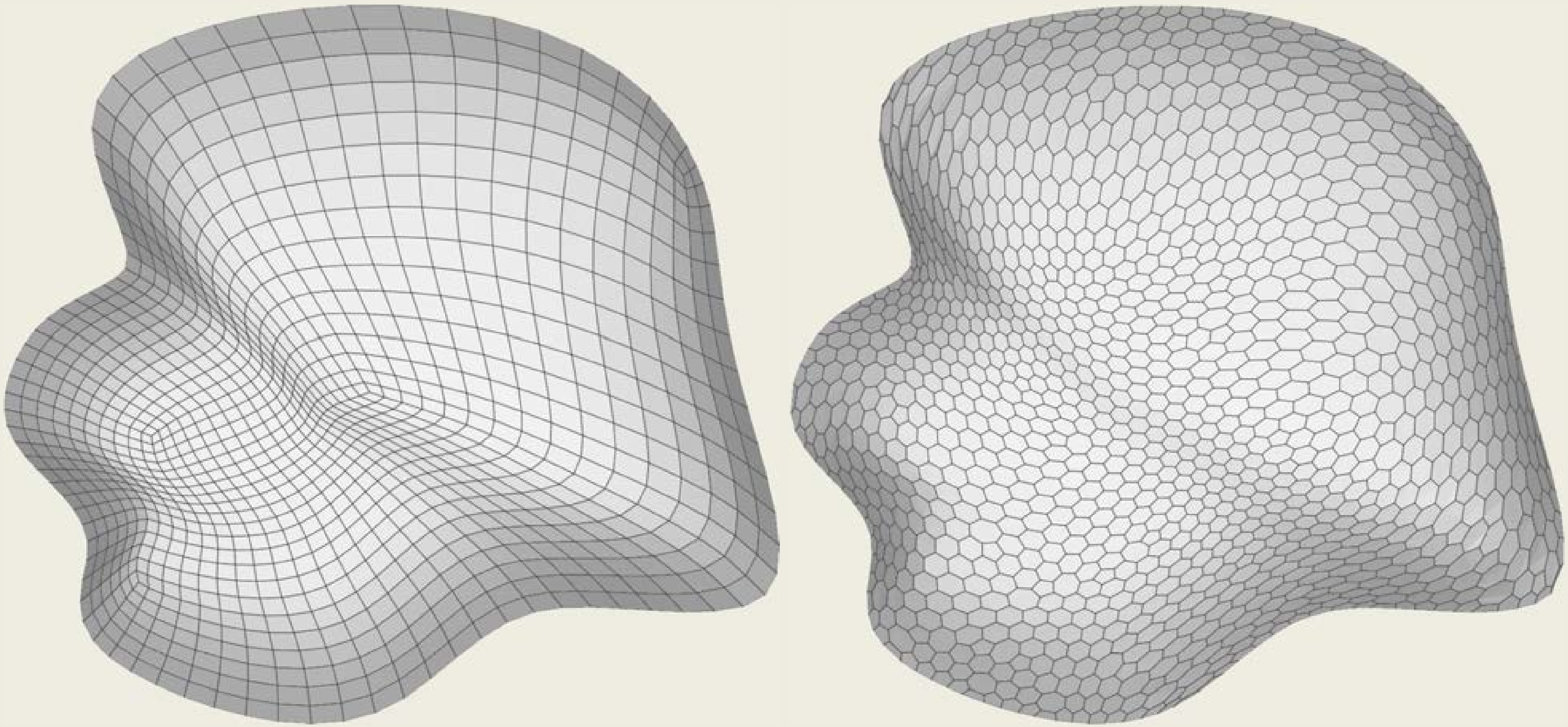
Performances - *preliminary assessment*

Botanic dataset



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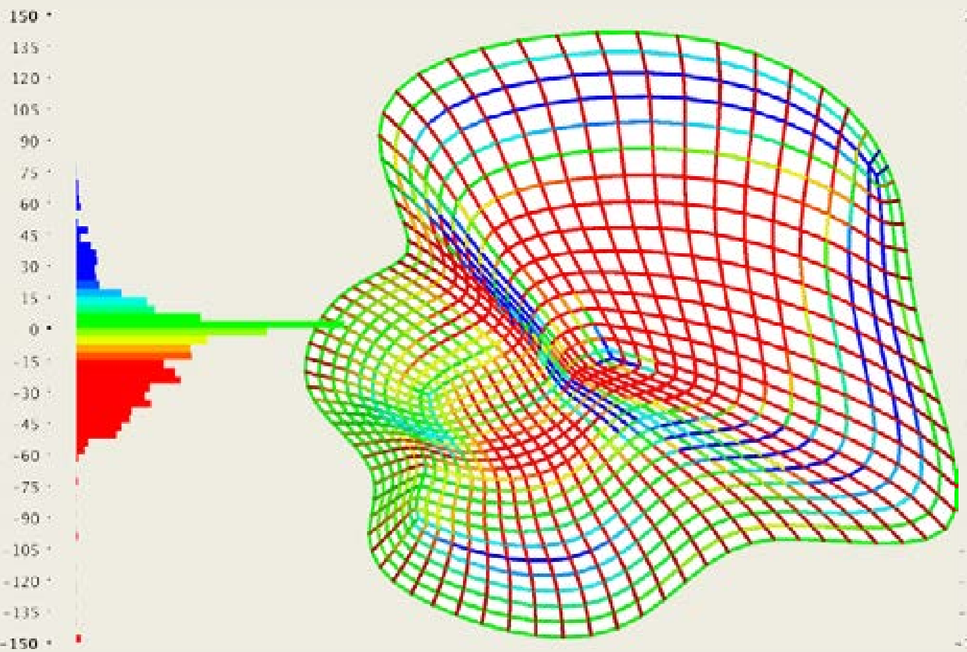
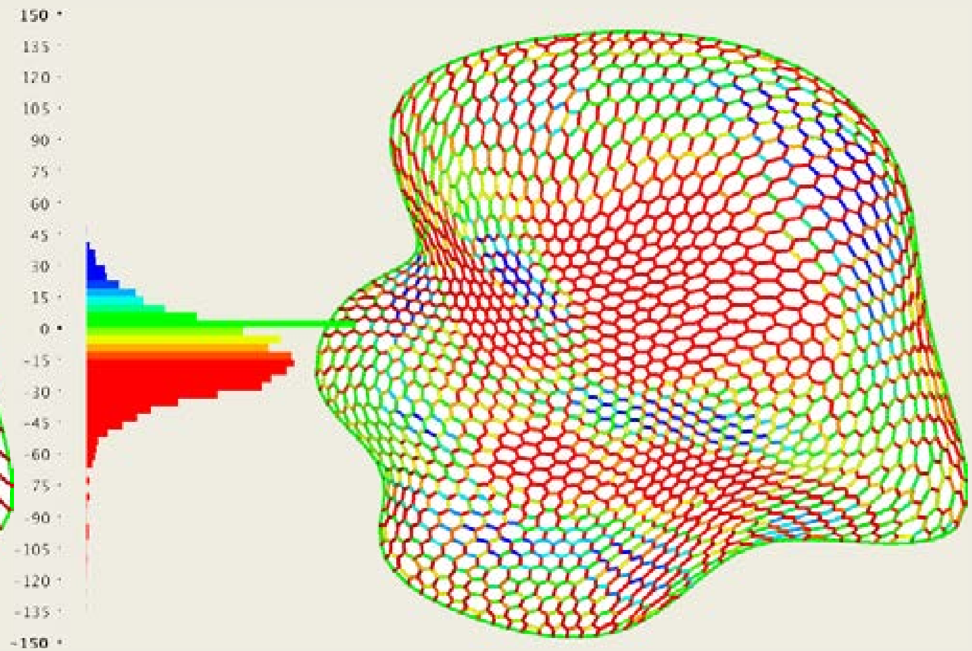
Performances - *preliminary assessment*

Botanic dataset – axial forces N distribution





$U_{tot} = 81329$ J
$\lambda = 0.83$, $\delta = 335$ mm

$U_{tot} = 32240$ J
$\lambda = 1.48$, $\delta = 172$ mm

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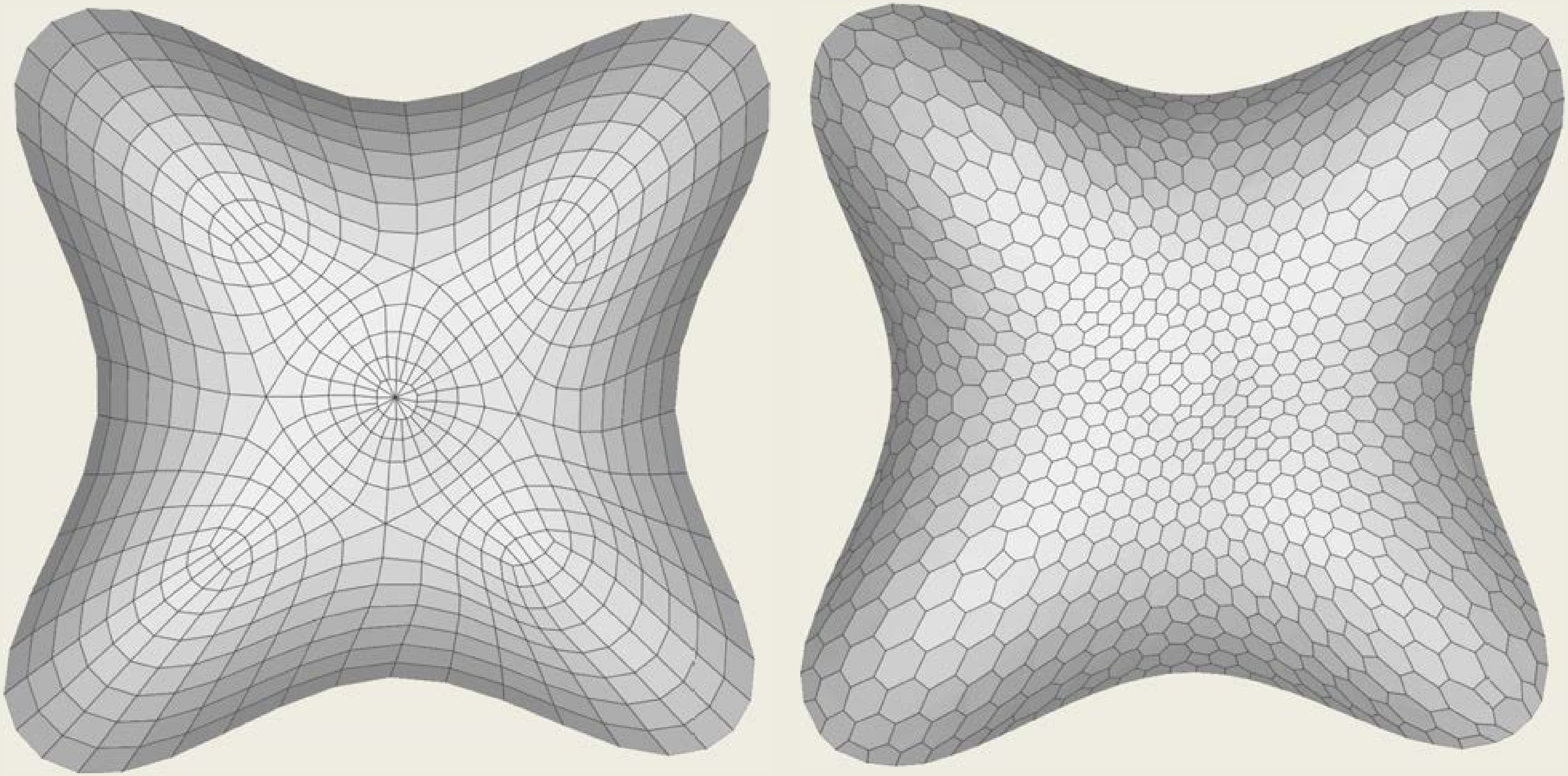
Performances - *preliminary assessment*

Lilium dataset



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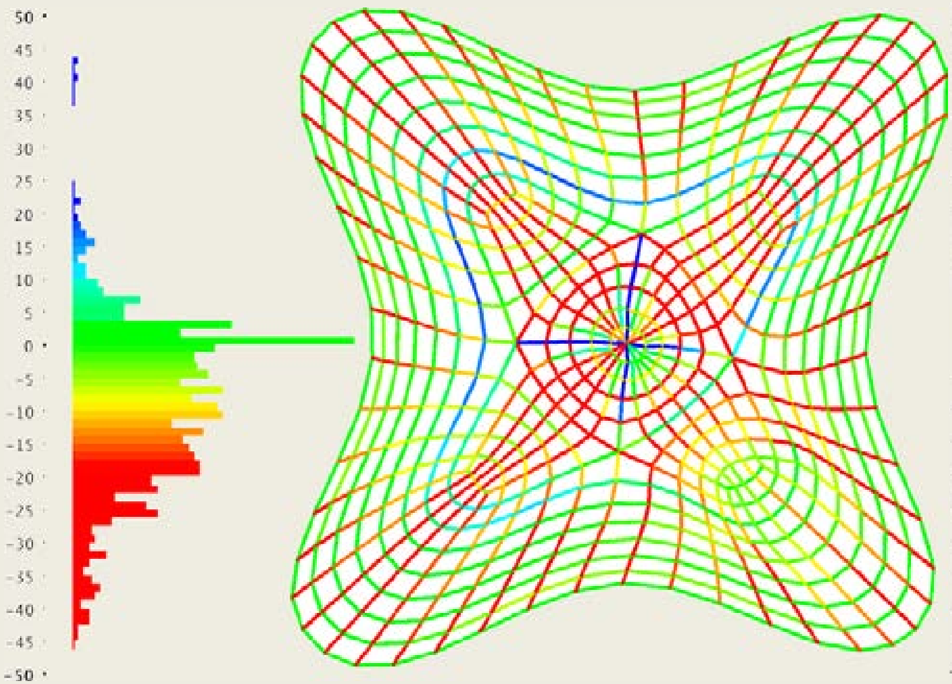
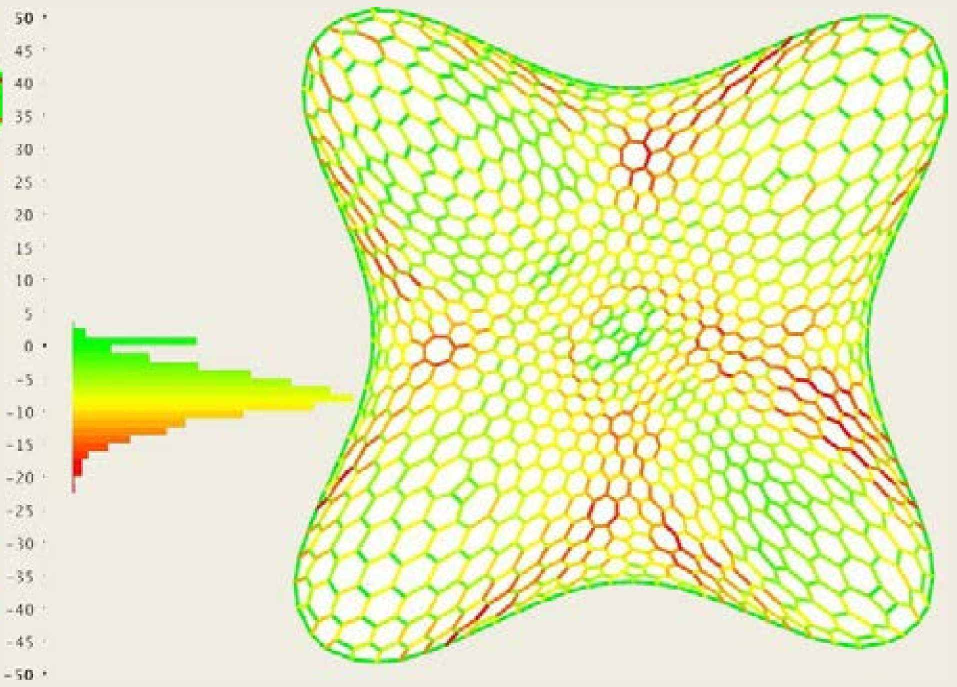
Performances - *preliminary assessment*

Lilium dataset – axial forces N distribution





U_{tot} = 2999 J
λ = 1.42, δ = 77.6 mm

U_{tot} = 1846 J
λ = 2.97, δ = 37.6 mm

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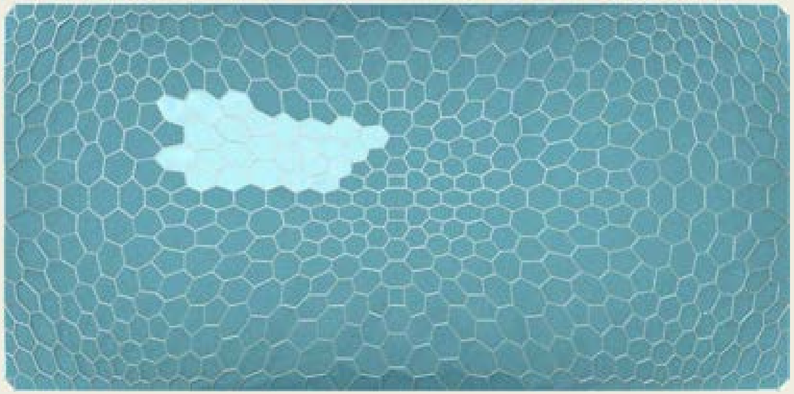
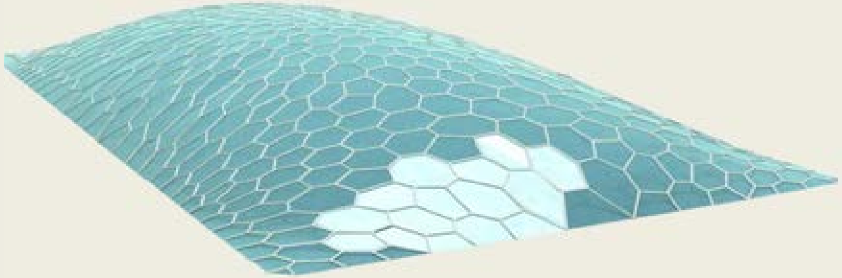
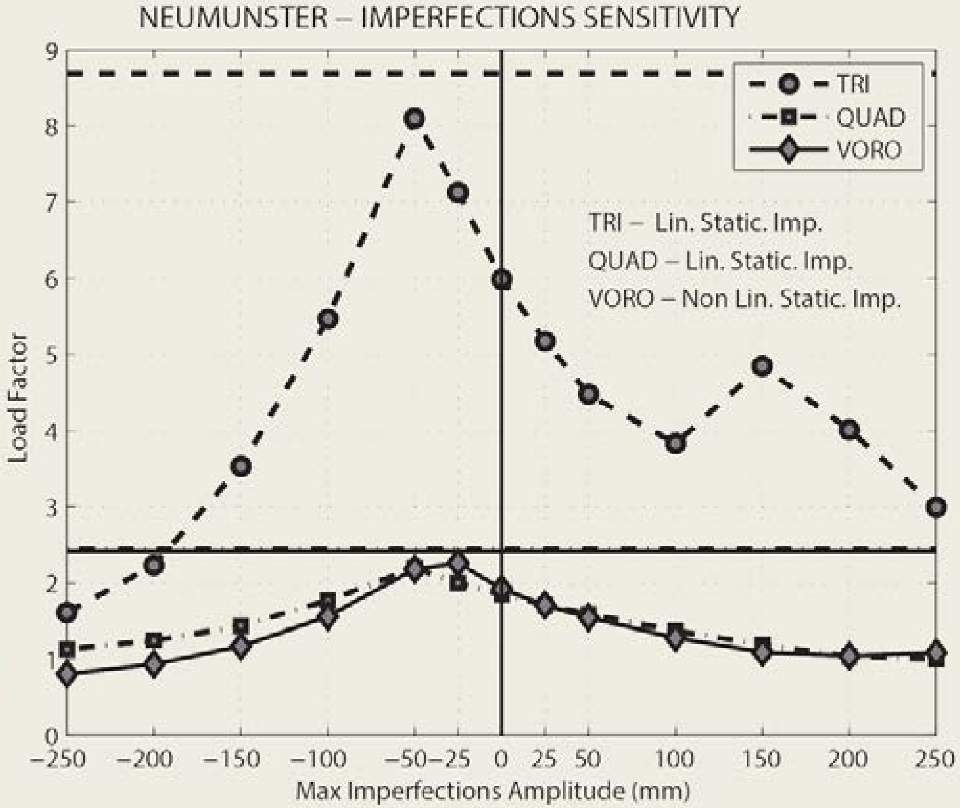
Performances - *further assessment*

Neumunster dataset







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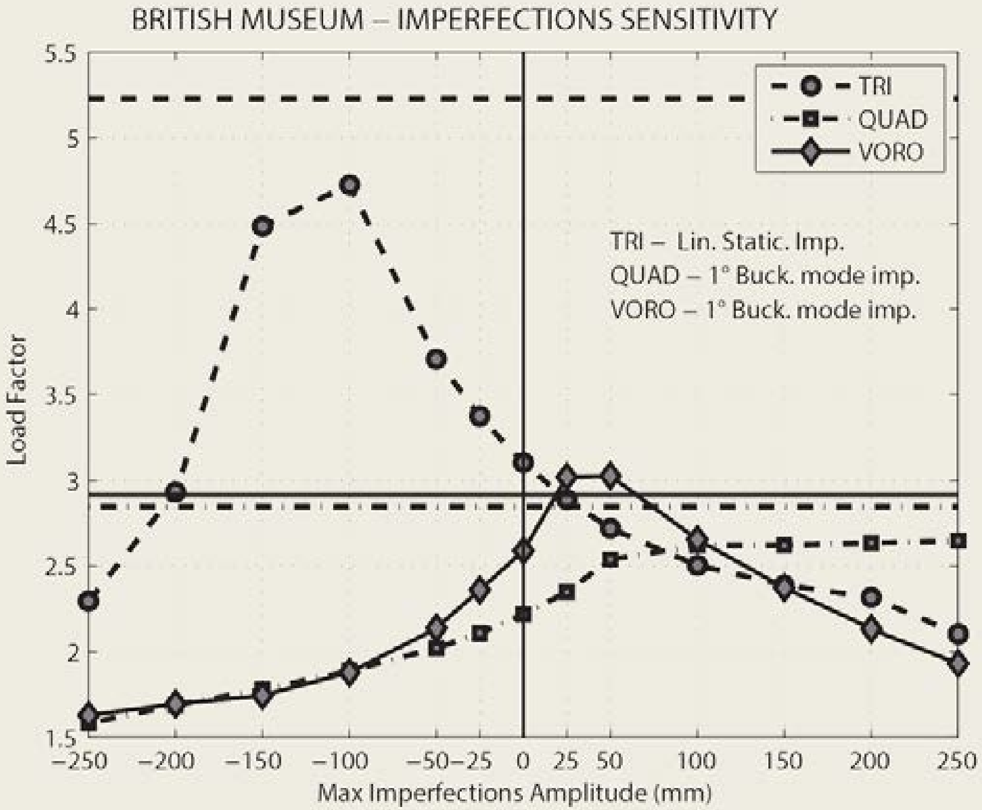
Performances - further assessment

British Museum dataset



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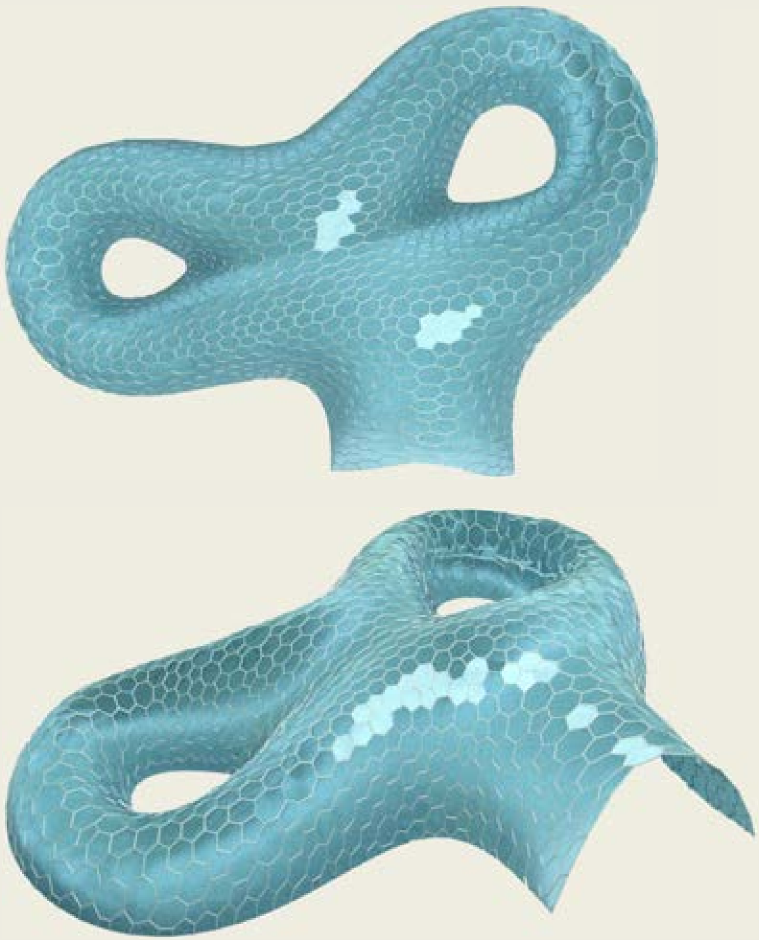
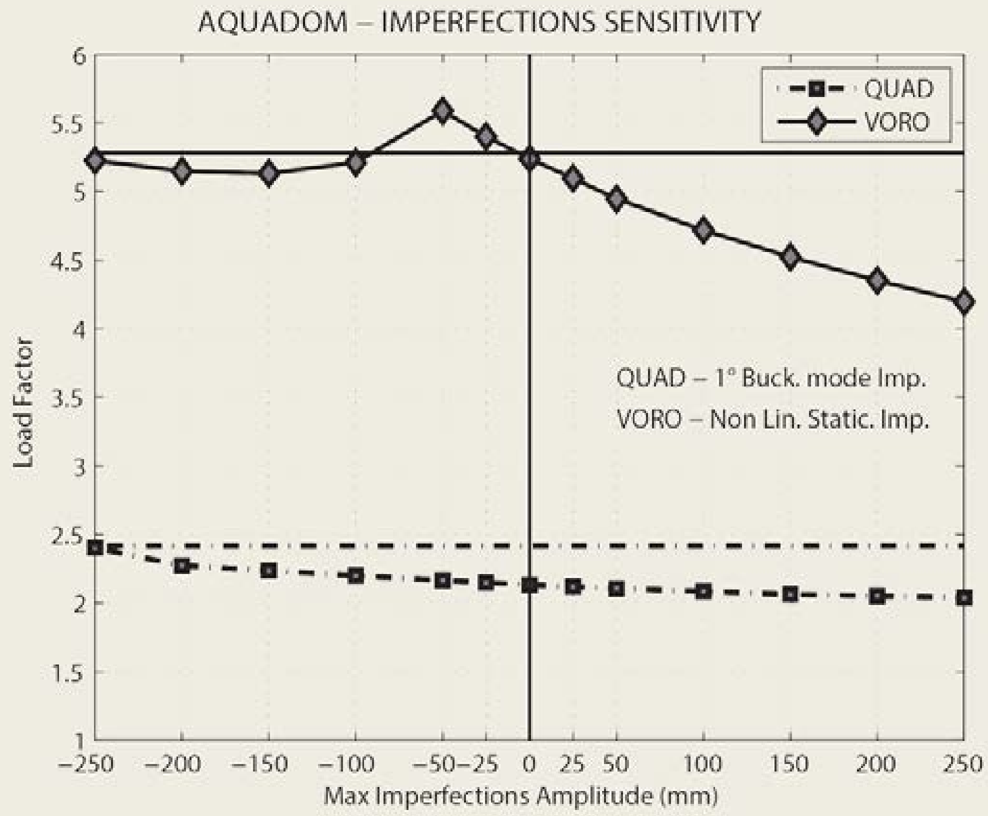
Performances - *further assessment*

Aquadom dataset





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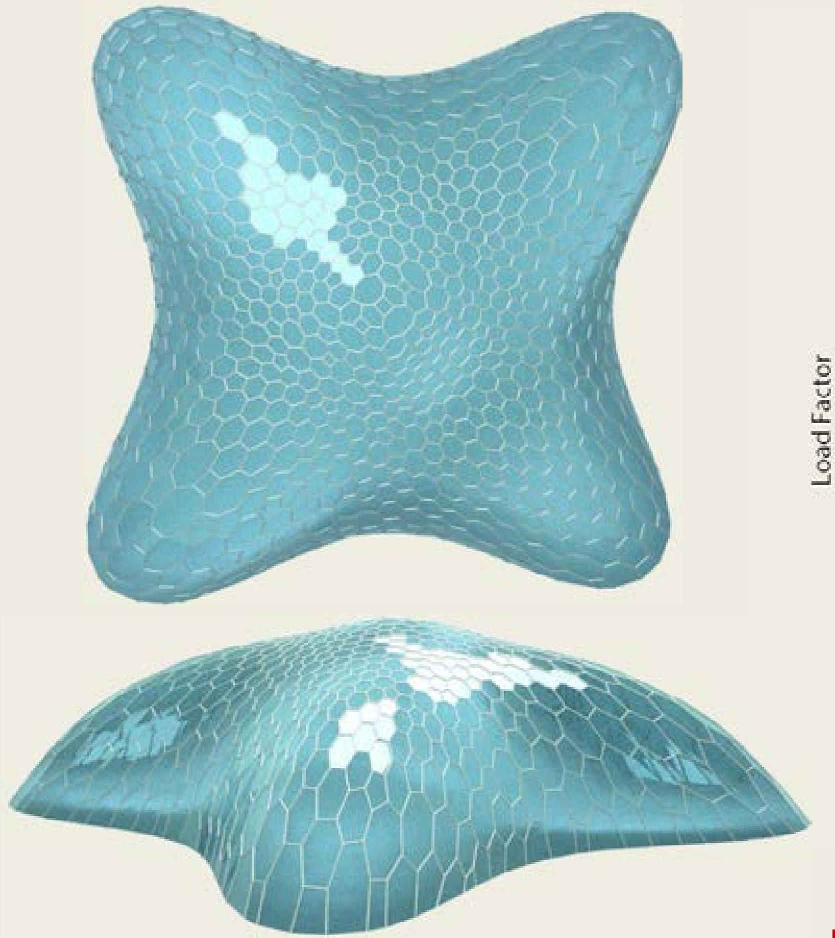
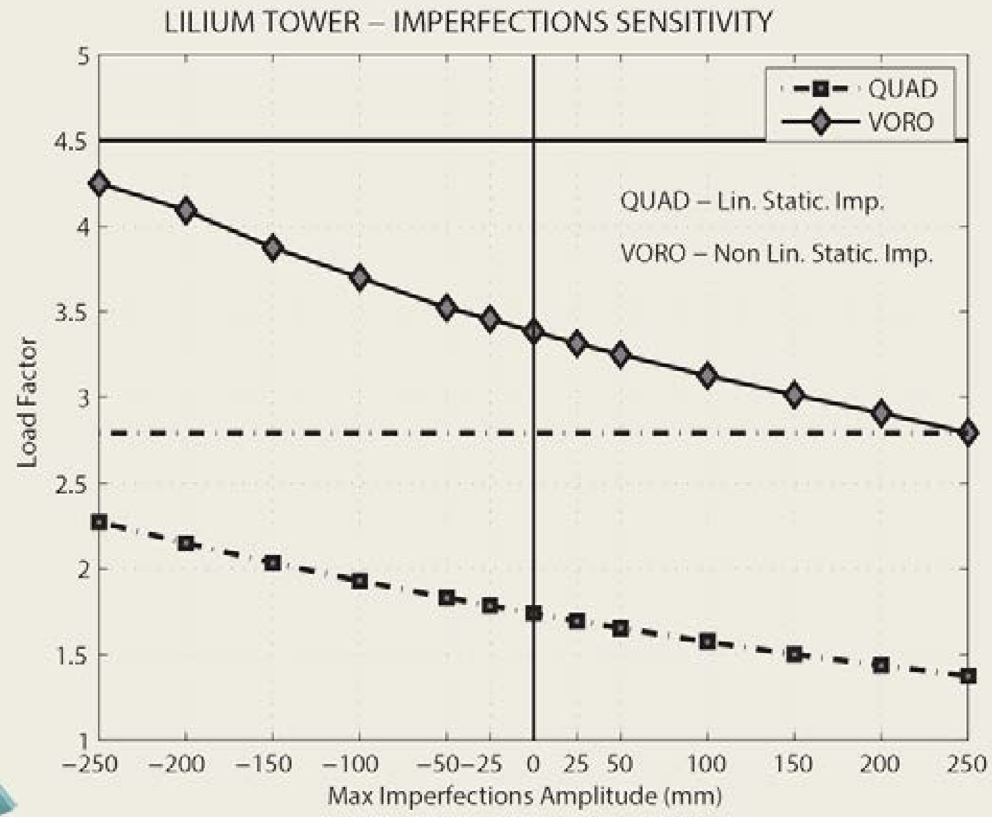
Performances - *further assessment*

Botanic dataset





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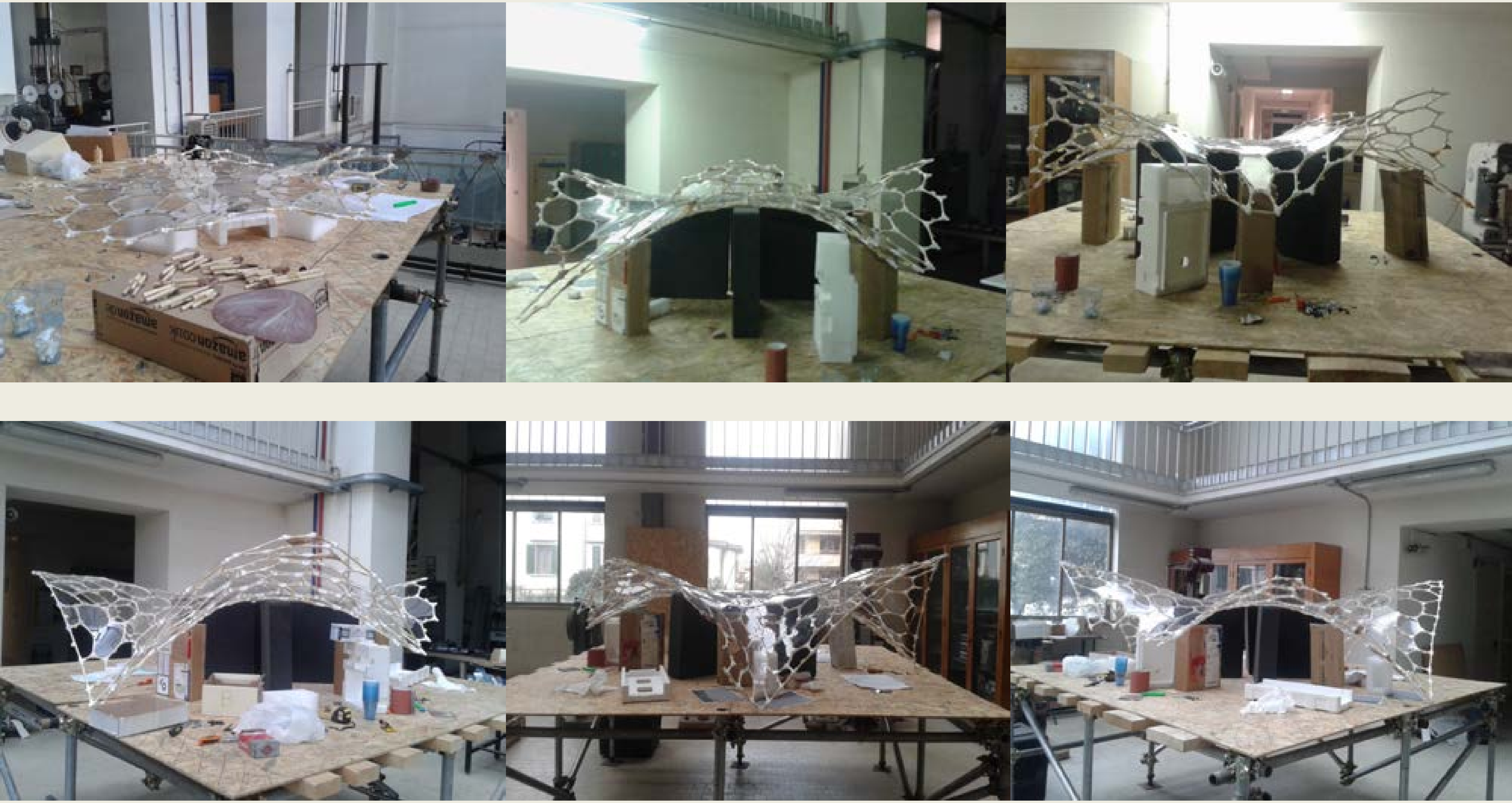
A physical mock-up



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A physical mock-up

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Conclusions

This dissertation provides:

1- A thorough insight in the mechanics of polygonal, hex-dominant grid-shells;

2- A proposal for a novel and effective kind of free-form hex-dominant grid-shells:
the ***'Statics Aware Voronoi Grid-Shells'***.

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Thank you for participating

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