Phase Noise

Analysis and Measurement

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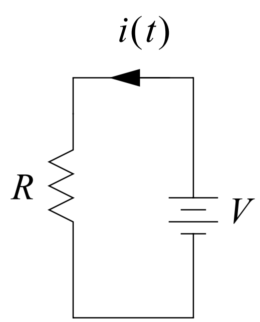
Outline

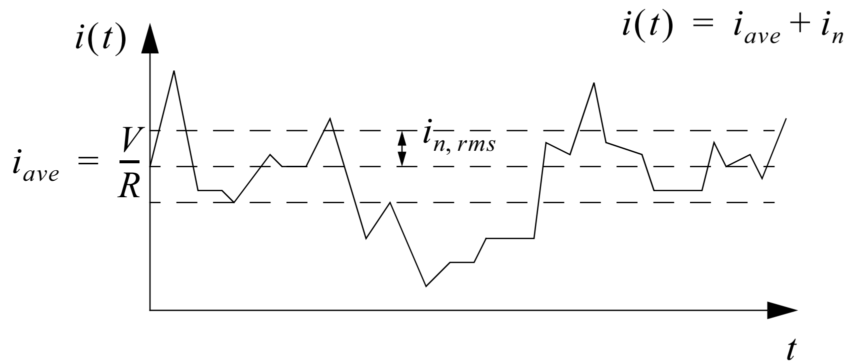
- Noise Sources
- Noise Contamination Mechanisms
- Phase Noise Spectrum
- The Importance of Phase Noise
- Hajimiri Oscillator Phase Noise Model
- Phase Noise Measurement Techniques
- Further Reading

Noise Sources

- There are three main noise sources.
  1. Thermal noise
  2. Shot Noise
  3. 1/f Noise

- Thermal noise is essentially the Brownian motion of electrons due to thermal agitations.





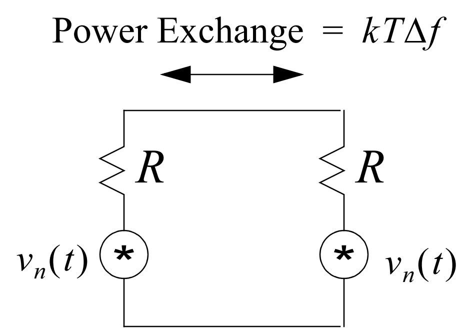
Thermal Noise

- The instantaneous current fluctuation is completely independent of its past (memory less). Hence its power spectral density is for all practical purposes white.

- It is through this thermal movements that a hot and a cold object will exchange energy till they get to the thermal equilibrium (same temperature).

- According to thermodynamics, when two equal size resistors are at thermal equilibrium (both at temperature T), the exchanged energy (or power spectral density) at any frequency is kT (k is the Boltzmann's constant $1.38 \times 10^{-23}\ J/K$ and T is the temperature in kelvins)

Thermal Noise (cont.)



Power delivered from the left resistor to the right one is:

$$\frac{|V_n(f)^2|}{4R} = kT\Delta f \Rightarrow |V_n(f)^2| = 4kTR\Delta f$$

The unit for $|V_n(f)|$ is $V/\sqrt{Hz}$ At room temperature, the noise associated with a 1kohm resistor is $4nV/\sqrt{Hz}$

Memorize this!!!

Shot Noise

- In any forward biased pn junction, the carriers (electrons or holes) should achieve a certain amount of energy to overcome the energy barrier.
- The average of the carrier current is:

$$I_{ave} = I_s(e^{V/V_T} - 1)$$

- In reality, however, the carriers go across the barrier in a random manner. The probabilistic model for this process is that of a queue which is described by the Poisson process.
- In Poisson process, the mean square is proportional to the average, i.e. $\overline{x^2} = \lambda\bar{x}$

Shot Noise (cont)

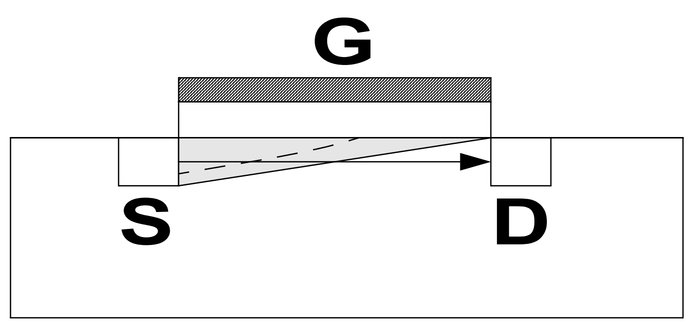
- Hence, the current across a junction can be modelled as $i(t) = I_{ave} + i_n(t)$, where $\overline{i_n^2} = \lambda I_{ave}$
- The constant of proportionality is 2q, where q is the electron charge, hence: $\overline{i_n^2} = 2qI_{ave}\ (A^2/Hz)$
- Like the thermal noise, the current noise at each instance is totally independent of its past (memory less) and as such it has a white spectrum. Hence,

$$\left|I_n^2(f)\right| = 2qI_{ave}\Delta f\ (A^2)$$

1/f Noise

- By far, the most puzzling source of noise.
- As opposed to the other two mechanisms, the 1/f noise instantaneous fluctuation does depend on its past and therefore has memory.
- It is essentially this memory that causes the spectrum of noise to be colored (not white).
- Like any other memory system, there should be a storage mechanism. But How?
- In MOSFETS, for example, some of carriers which are travelling into the channel may bend upward towards the gate due to the gate to channel vertical field.

1/f Noise (Cont.)



- Depending on the energy of these carriers, they may get trapped into the surface states between Si and SiO2 or even get trapped into the defect states within the oxide. These trapped charges will then change the threshold voltage of the transistor.
- The shift in threshold voltage will cause the current to change and this change in current is the noise.

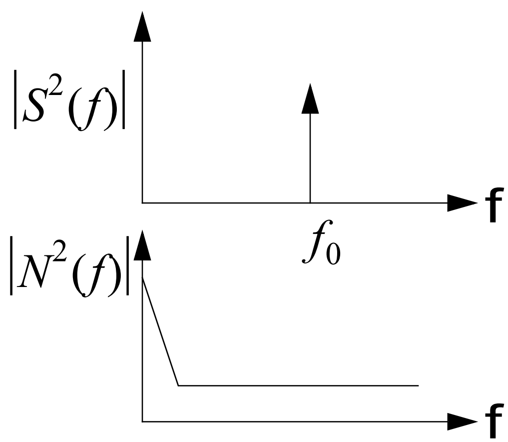
1/f Noise (Cont.)

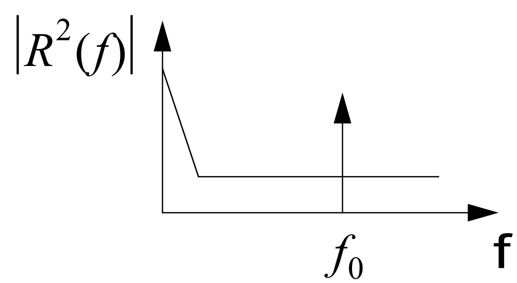
- Note that the noise current depends on the charges that have been trapped in the past (memory). The trapping phenomenon is the storage mechanism
- Crudely speaking, the storage mechanism acts like an integrator hence the input (trapped carriers energy) to output (change in current energy) transfer function is $1/f^2$.
- Since the quantum mechanical energy of the carriers is hf, one can (even more crudely) argue that the trapped carrier energy increases proportional to f.
- As such, the energy (power spectral density) of the current noise should be proportional to $f \times (1/f^2) = 1/f$

Noise Contamination Mechanisms

- In general, there are three categories for noise contamination.
    1. Additive
    2. Multiplicative
    3. Nonlinear

- In the additive case, the noise is simply added to the signal, i.e. $r(t) = s(t) + n(t)$



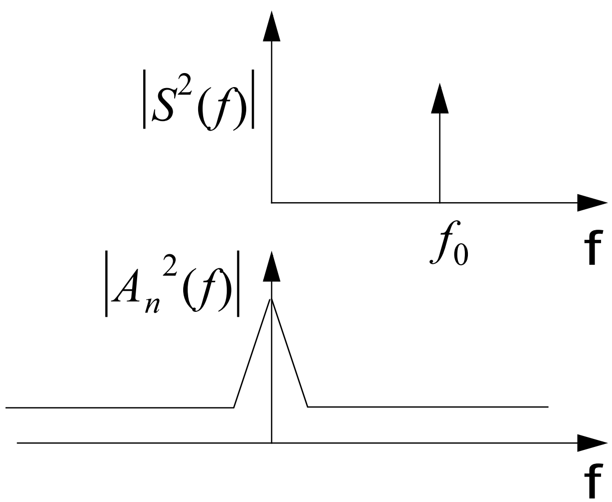


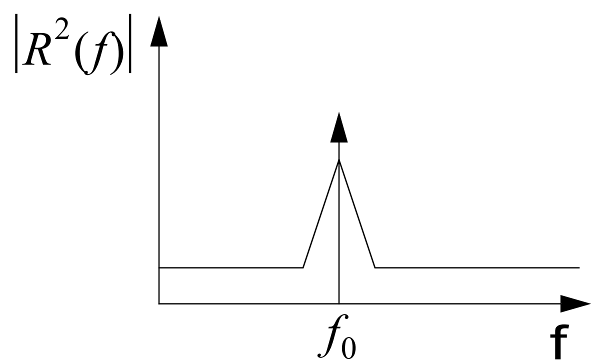
Multiplicative Noise

- Let's for the sake of simplicity assume that $s(t) = A\sin(2\pi f_0 t)$, then the multiplicative noise (also known as amplitude noise) would cause:

$$r(t) = (1 + a_n(t))A\sin(2\pi f_0 t) = A\sin(2\pi f_0 t) + a_n(t)A\sin(2\pi f_0 t)$$

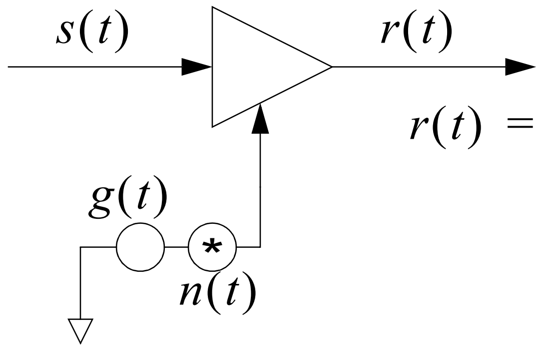
- Hence, in the frequency domain we have:





Multiplicative Noise (cont.)

- An example: A variable Gain amplifier



$$r(t) = s(t) \times (g(t) + n(t))$$

- Note that in the additive case, one can change the $s(t)$ frequency such that it lands where the noise is the lowest (i.e. away from the 1/f noise area).
- In the multiplicative case, however, this is futile since the noise spectrum will follow the signal frequency.

Nonlinear Noise

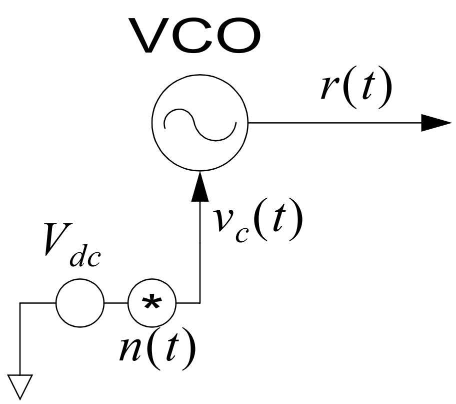
- The multiplicative noise, looks like an amplitude modulation. Can noise cause the other two modulation schemes, namely, frequency and phase modulations?
- The answer is unfortunately YES.
- Again consider the desired signal to be $s(t) = A\sin(2\pi f_0 t)$, then one has:

$$\begin{cases} \text{Frequency Noise} & r(t) = A\sin\left(2\pi f_0 t + 2\pi\int_0^t f_n(\tau)d\tau\right) \\ \text{Phase Noise} & r(t) = A\sin(2\pi f_0 t + \theta_n(t)) \end{cases}$$

- Note that in the amplitude noise case, doubling the noise $n(t)$ would double its contribution to $r(t)$ (linear)

Nonlinear Noise (cont.)

- Doubling of $f_n(t)$ (frequency noise) and $\theta_n(t)$ (phase noise), on the other hand, won't double their contributions to $r(t)$, (hence nonlinear)
- An Example: A voltage Controlled Oscillator.



$$r(t) = A\sin\left(2\pi K_{VCO}\int_0^t (V_{dc} + n(\tau))d\tau\right)$$

$$r(t) = A\sin\left(2\pi K_{VCO} V_{dc} t + 2\pi K_{VCO}\int_0^t n(\tau)d\tau\right)$$

$$r(t) = A\sin\left(2\pi f_0 t + 2\pi K_{VCO}\int_0^t n(\tau)d\tau\right)$$

$$f_0 = K_{VCO} V_{dc}$$

$$\theta_n(t) = 2\pi K_{VCO}\int_0^t n(\tau)d\tau$$

$$\left|\theta^2{}_n(f)\right| = K^2{}_{VCO} \times \left|N^2(f)\right| / f^2$$

Nonlinear Noise (cont.)

- Note that $f_n(t)$ and $\theta_n(t)$ are related to one another:

$\theta_n(t) = 2\pi\int_0^t f_n(\tau)d\tau$ or in freq. domain $|\theta^2{}_n(f)| = |F^2{}_n(f)|/f^2$

- But what about the relationship between $|R^2(f)|$ and $|\theta^2{}_n(f)|$?

- Though in general the relationship between the $r(t)$ and $\theta_n(t)$ is a nonlinear one, in practice due to the small nature of the noise (at least for large frequency offsets), one can apply some approximations.

Phase Noise Spectrum

- Let's assume that $\theta_n(t) = \theta_p \sin(2\pi f_m t)$, then we have:

$$r(t) = A\sin(2\pi f_0 t + \theta_p \sin(2\pi f_m t))$$

- Using some trigonometric identities, one derives:

$$r(t) = A\sin(2\pi f_0 t)\cos(\theta_p \sin(2\pi f_m t)) + A\cos(2\pi f_0 t)\sin(\theta_p \sin(2\pi f_m t))$$

- Assuming that $\theta_p \ll 1$, then $\cos(\theta_p \sin(2\pi f_m t)) \approx 1$ and $\sin(\theta_p \sin(2\pi f_m t)) \approx \theta_p \sin(2\pi f_m t)$, hence:

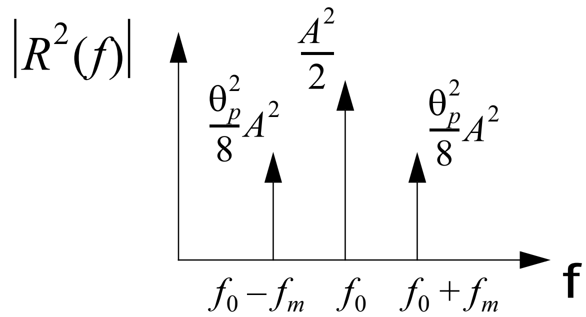
$$r(t) \approx A\sin(2\pi f_0 t) + \theta_p \sin(2\pi f_m t) A\cos(2\pi f_0 t)$$

- Which in turn is:

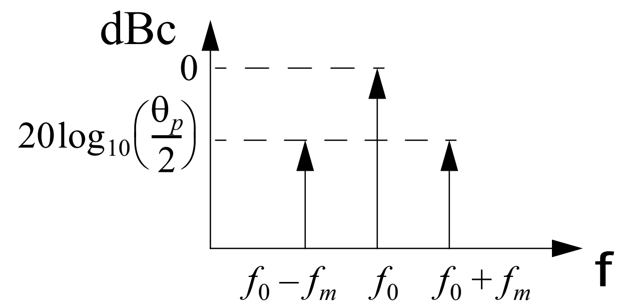
$$r(t) \approx A\sin(2\pi f_0 t) + \frac{\theta_p}{2} A\sin(2\pi(f_0 + f_m)t) - \frac{\theta_p}{2} A\sin(2\pi(f_0 - f_m)t)$$

Phase Noise Spectrum (cont.)

- So in Frequency domain, the spectrum will be:



or



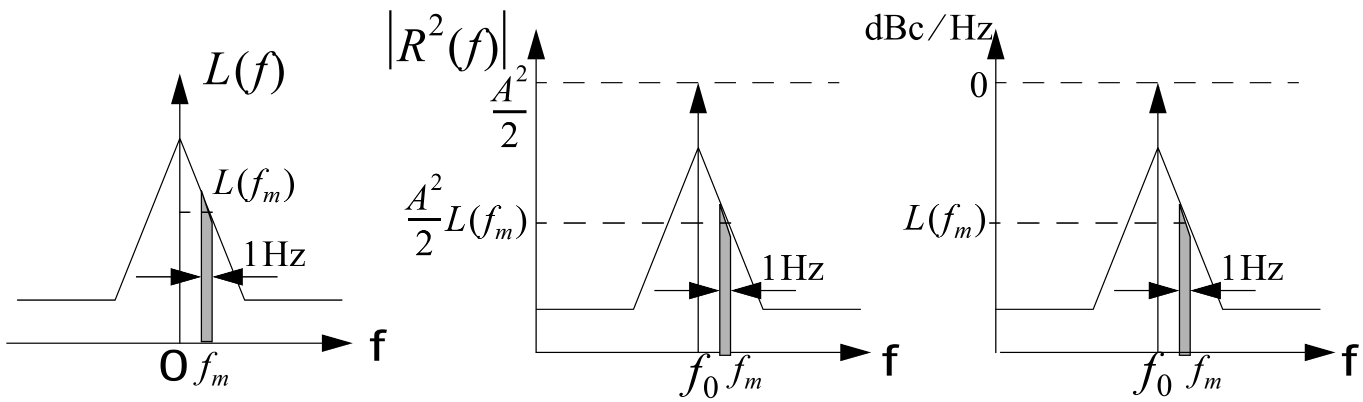
- So a periodic $\theta_n(t)$ with a frequency f_m will cause two side tones ($f_0 \pm f_m$) to appear at the $r(t)$ spectrum.
- These side tones which are due to periodic phase noise are referred to as "Spurs" and their normalized power to the carrier power is measured in dBc (dB with respect to carrier power).

Phase Noise Spectrum (cont.)

- We can generalize this analysis to non periodic $\theta_n(t)$ so long as $\theta_n(t) \ll 1$.
- The result is:

$$r(t) \approx A\sin(2\pi f_0 t) + \theta_n(t) A\cos(2\pi f_0 t)$$

- Let's denote the double sided PSD of $\theta_n(t)$ as $L(f)$. Then one has:

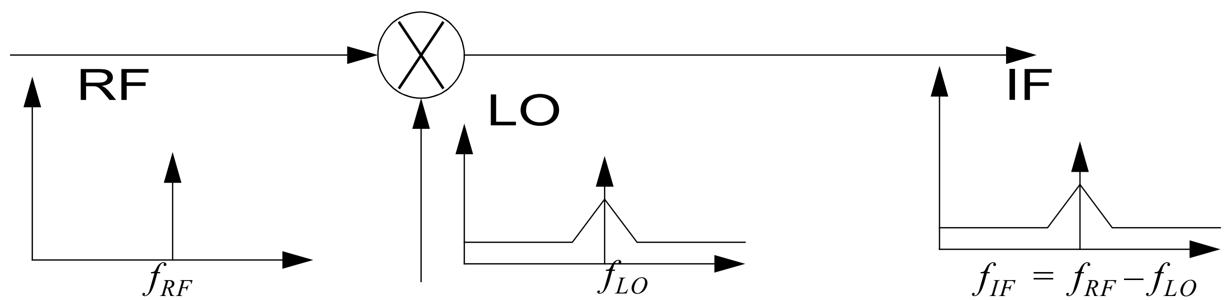


Phase Noise Spectrum (cont.)

- Note that the phase noise spectrum has been up converted to frequency f_0 which is similar to that of the amplitude (or multiplicative) noise. The small noise approximation has in effect convert the phase noise into a multiplicative noise.
- For a periodic amplitude noise ($a_n(t)$), however, the two spurs are in phase whereas for the phase noise they are 180° out of phase.
- Hence, if both periodic phase and amplitude noise (with the same frequency f_m) are present, then the two side spurs may become asymmetric.

Importance of phase noise

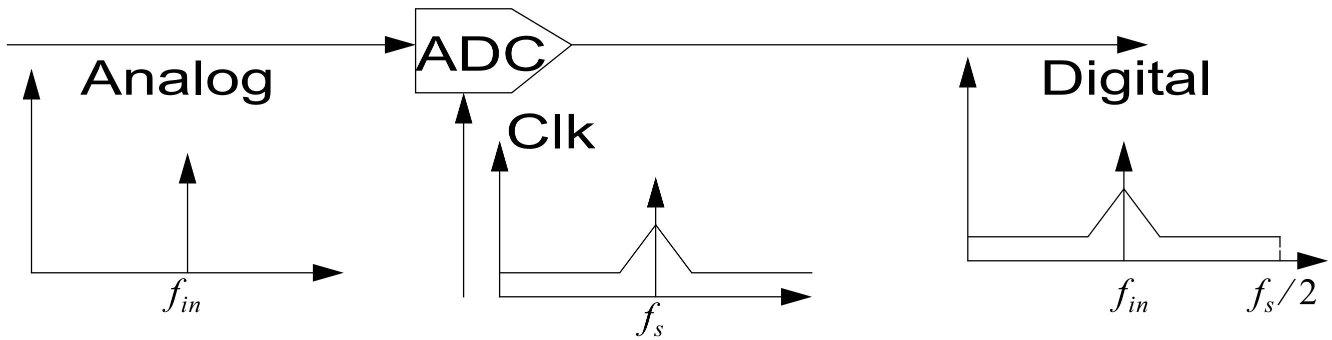
- Mixing Self Noise



- Note that the noise free RF signal, has turned into a noisy IF signal.
- The IF SNR in this case is $\mathrm{SNR}_{IF} = 2\int_{f_1}^{f_2} L(f)df$ where f_2 is single-sided bandwidth of the desired signal and f_1 is the lowest frequency of interest in the signal.
- $\varphi_{rms} = \sqrt{2\int_{f_1}^{f_2} L(f)df}$ is the RMS phase noise in radians.

Importance of phase noise (cont.)

- Sampling Jitter

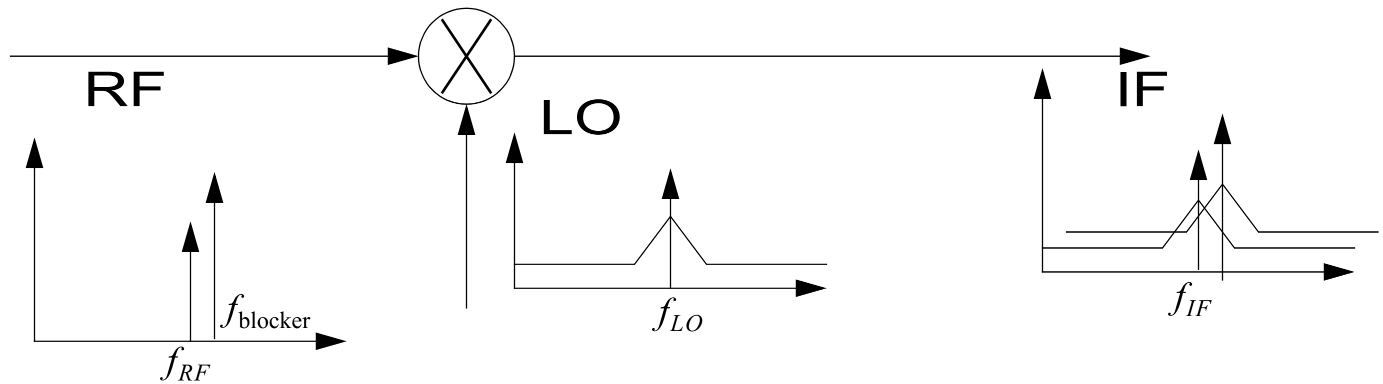


- Again a noise free analog signal will turn into a noise digital sampled signal.
- The sampled signal SNR (in addition to the quantization noise) is then:

$$SNR_{digital} = 20 \times \log_{10}(\varphi_{rms} \times f_{in}/f_s), \text{ where } \varphi_{rms} = \sqrt{2\int_{f_1}^{f_2} L(f)df}$$

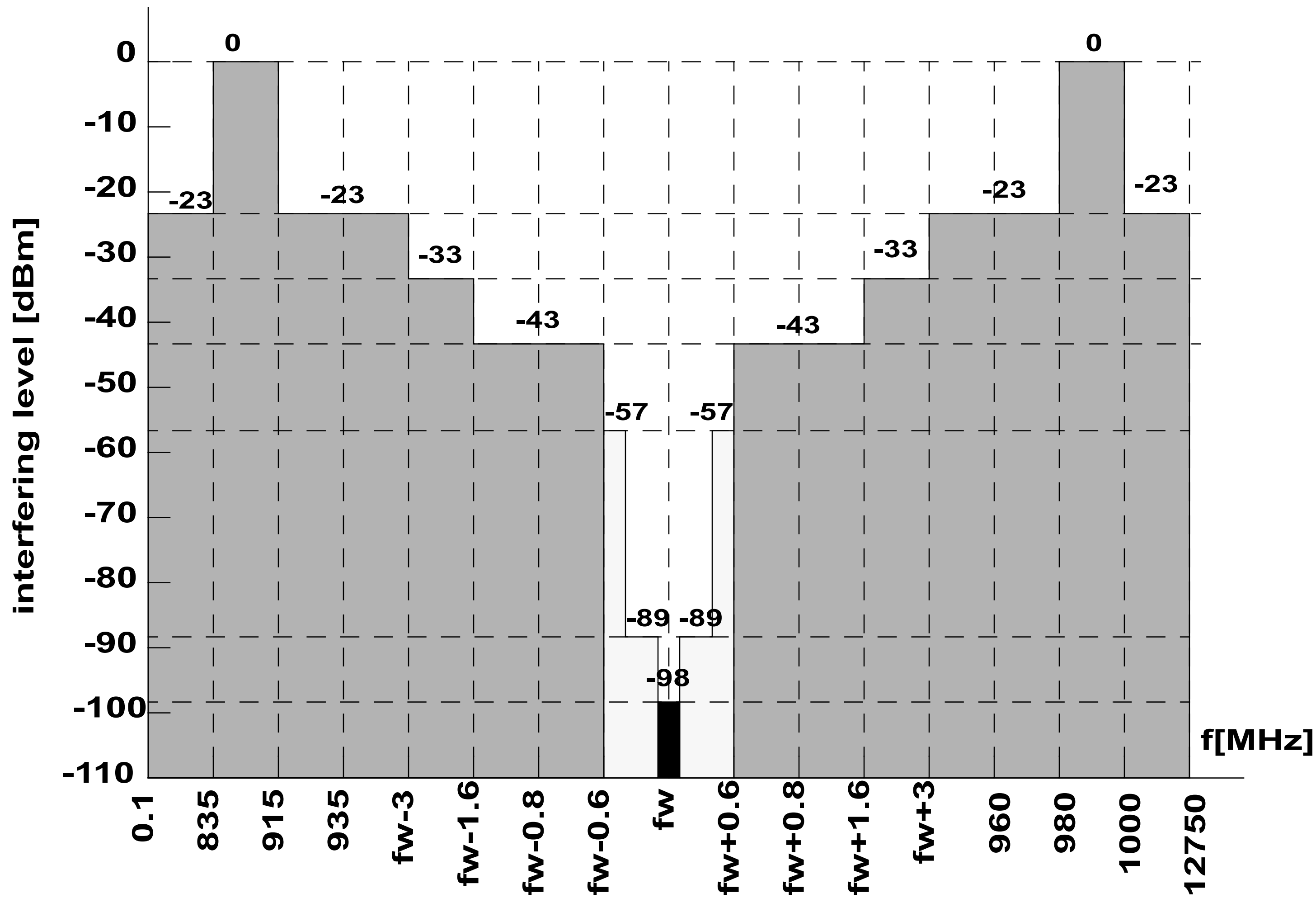
Importance of phase noise (cont.)

- Reciprocal Mixing



- The IF signal, in this case, not only will suffer from its own self noise but also from the self noise of the blocker signal.
- If the blocker signal is much bigger than the signal itself, then the reciprocal noise due to the blocker self noise would dominate the noise at IF.

The GSM Example



The GSM Example (cont.)

- Consider the -23dBm blocker which is 3MHz away from the wanted signal. This blocker will mix with the the portion of the phase noise which is 3MHz away from the LO and down convert on top of the -98dBm desired signal.
- The required SNR for GSM is 9dB. Let's allow the reciprocal noise to cause an SNR of 12dB (3dB margin)
- The GSM bandwidth is 200kHz. Hence, the phase noise requirement at 3MHz offset (integrated over 200kHz) is: $-98\text{dBm} - (-23\text{dBm} + X\text{dBc}) \geq 12\text{dB}$
- Hence, $X \leq -87\text{dBc}$ or

$$L(3\text{MHz}) = X - 10\log_{10}(200\text{kHz}) \leq -140\text{dBc/Hz}$$

Oscillator phase noise model (Hajimiri-Lee Model)

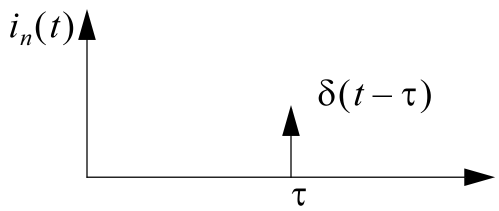
- The oscillator output can be shown as:

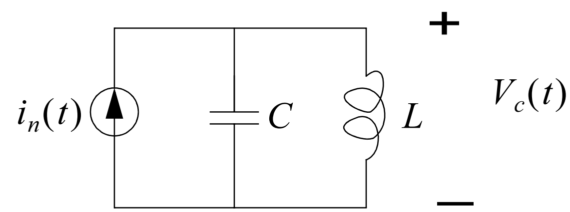
$$v(t) = (1 + a_n(t))\cos(\omega_o t + \varphi_n(t))$$

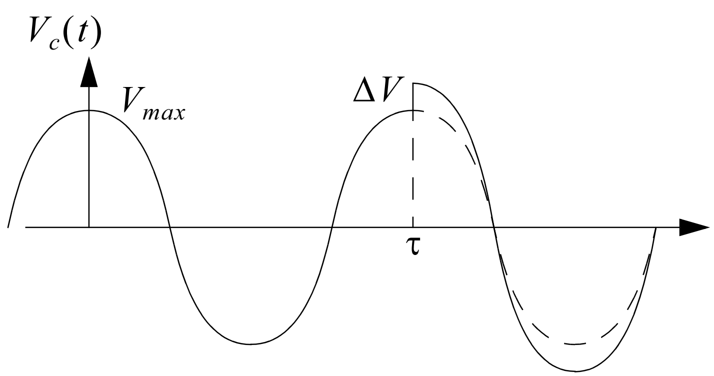
- Any amplitude shift due to the noise will get quenched due to the nonlinear amplitude control mechanism.
- However, any phase shift due to the noise will go unnoticed since oscillators have no phase feedbacks (PLL’s are an exception though).
- We will assume that the noise injections are small enough such that by doubling the noise the phase shift due to the noise will double as well. In other words, we assume that the noise-in phase-shift out transfer function is linear.

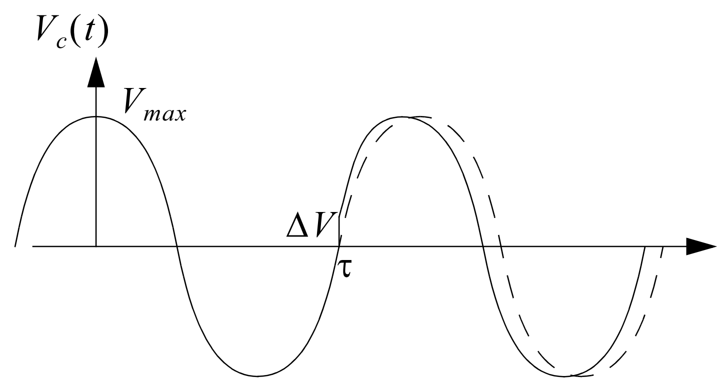
Oscillator phase noise model (cont.)

- Let's consider a simple LC tank with no loss which has a sustained oscillation. Let's subject it to an impulsive current noise.









Oscillator phase noise model (cont.)

- A current impulse will cause the V_c to change (but not I_L).
- When the impulse lines up with the peak of V_c no phase shift will occur since $I_L = 0$ at that time (note that $\varphi = \operatorname{atan}(V_c/I_L)$ and though V_c has changed, φ is still 90°. However, since energy is pumped into the tank ($i(\tau) \times V_c(\tau) > 0$) the oscillation amplitude will grow (note that in this case there is no nonlinear amplitude control).
- When the impulse lines up with the zero crossing of V_c no amplitude shift will occur since no energy is pumped into the tank ($i(\tau) \times V_c(\tau) = 0$).

Oscillator phase noise model (cont.)

- However, a permanent phase shift will occur (in this case $I_L \neq 0$, hence $\varphi = \operatorname{atan}(V_c/I_L)$ will change since V_C has changed.
- Note, that while the transfer function from $i(t)$ to $\varphi(t)$ is assumed to be linear, it is NOT time invariant.
- The phase response to the impulsive current, however, is always a step whose height depends on the time the impulse is applied. Hence,

$$h_{\varphi_n}(t, \tau) = \frac{\Gamma(\omega_0 \tau)}{q_{max}} u(t - \tau)$$

where q_{max} is the maximum charge across the cap. It is introduced to normalize Γ with respect to the oscillation amplitude.

Oscillator phase noise model (cont.)

- Noting that the convolution theorem is applicable to LTV systems as well, one can derive the phase response to any arbitrary input as:

$$\varphi_n(t) = \frac{1}{q_{max}}\int_{-\infty}^{t}\Gamma(\omega_0\tau)i_n(\tau)d\tau$$

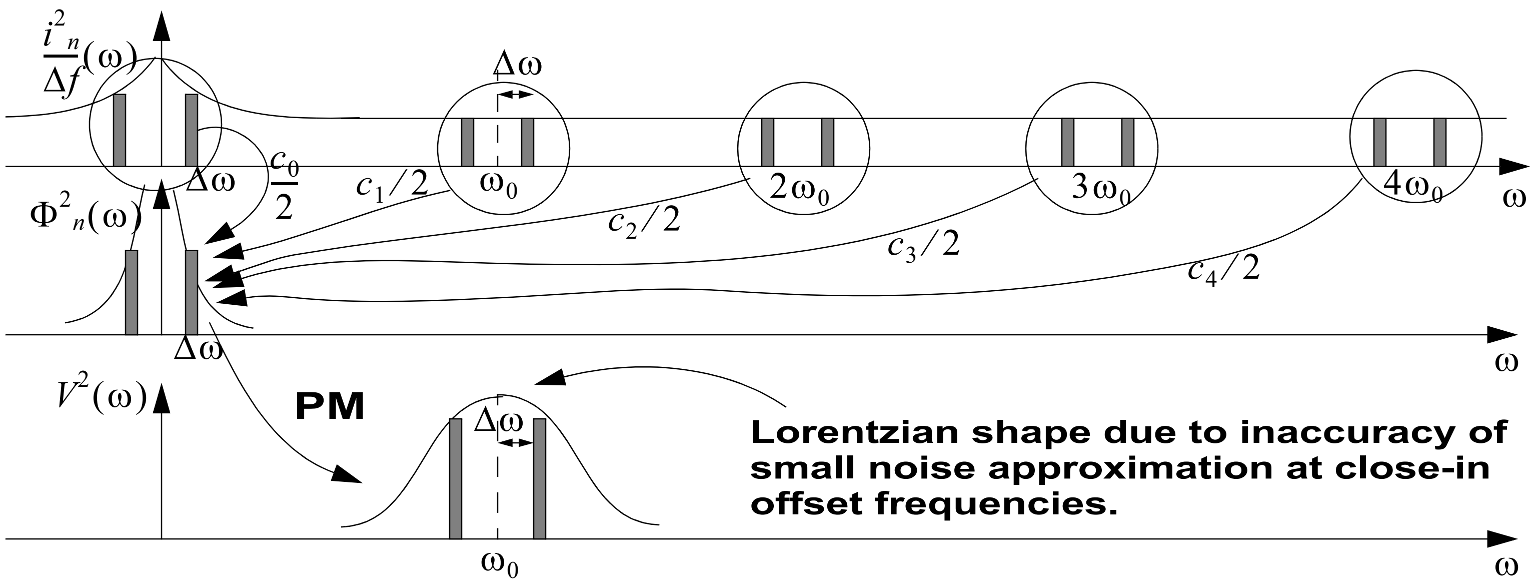
- By close inspection, one realizes that $\Gamma(\omega_0\tau)$ is periodic with the fundamental frequency of ω_0, hence

$$\Gamma(\omega_0\tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n\cos(n\omega_0\tau + \theta_n)$$ and hence

$$\varphi_n(t) = \frac{1}{q_{max}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i_n(\tau)d\tau + \sum_{n=1}^{\infty} c_n\int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau + \theta_n)d\tau\right]$$

Oscillator phase noise model (cont.)

- Hence if $i_n(t)$ has a 1/f spectrum, then the $n \geq 1$ terms can be neglected and $\varphi_n(t)$ will have a $1/f^3$ spectrum.
- On the other hand, if $i_n(t)$ has a white spectrum then $\varphi_n(t)$ will have a $1/f^2$ spectrum.

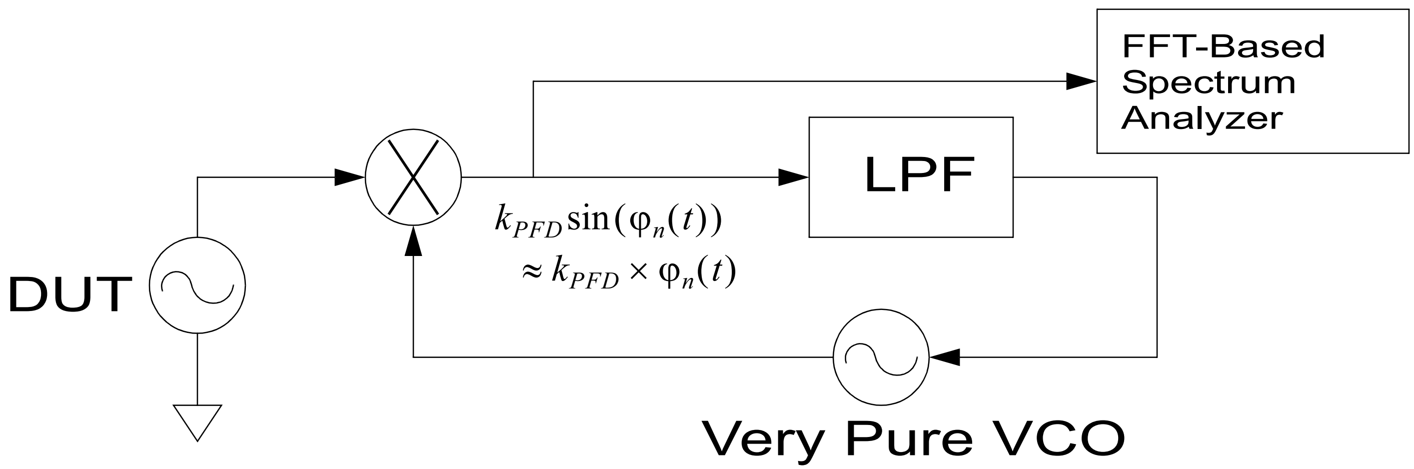


Phase Noise Measurement Methods

- Spectrum analyzer:
- While spectrum analyzer is a very popular method for measuring phase noise, it should be noted that it is measuring $V^2(\omega)$ and NOT $\Phi^2{}_n(\omega)$.
- As such it has two flaws:
  1. It is measuring the amplitude noise and additive noise on top of the phase noise.
  2. For close-in phase noise, the Lorentzian shape of $V^2(\omega)$, will report an optimistic value for phase noise.

Phase Noise Measurement Methods

- PLL-based technique



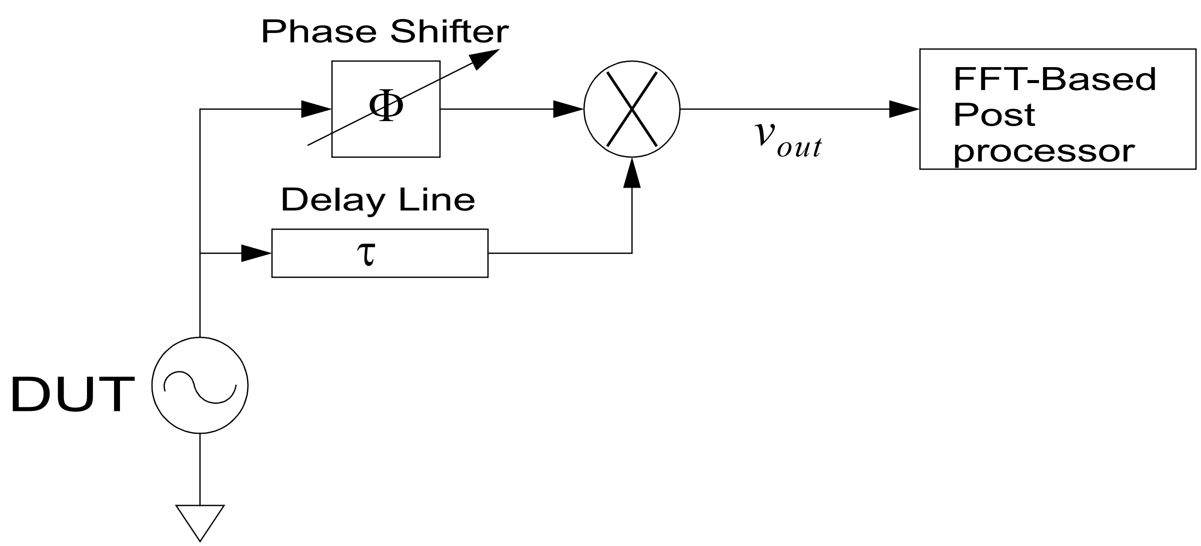
- The PLL-based method employs a narrow-band and low-phase-noise PLL.
- In this method, the multiplicative noise of DUT is completely eliminated from the measurement results.

PLL-Based Technique

- The phase detector gain (k_{PFD}) has to calibrated at the beginning. Note that the PLL will automatically force the feedback signal to be 90° with respect to the input signal which biases the PFD at its maximum sensitivity and linearity range.
- Within the bandwidth of PLL, the VCO will track any DUT phase noise and hence will cancel it. Hence, a reliable close-in phase noise can only be done for offset frequencies greater than the PLL bandwidth.
- Finally, large phase noise (for particularly close-in offset frequencies) will force the PFD to become nonlinear and hence unreliable.

Delay Line Discriminator technique:

- The PLL based technique is by far the most accurate method for phase noise measurement. It is also the most expensive in terms of equipment cost.
- An alternative is the delay line based method.



Delay Line technique (cont)

- Note that this method does not need a low-phase-noise VCO. It consists of a delay line and a phase detector.
- The delay line is essentially a linear input frequency to out phase shift converter, i.e. $\varphi_{out}(t) = 2\pi f(t)\tau$.
- The delay line will accomplish two purposes.

    1. by delaying the signal long enough, the frequency (or phase) noise of the signal and its delay can be assumed uncorrelated.
    2. the delay line will convert the frequency noise into a phase shift which is then measured by the phase detector.

Delay Line technique (cont)

- The phase shifter is used to ensure a 90° phase shift at the PFD input and hence maximum sensitivity. A calibration is required to ensure this as well as to measure k_{PFD}.

- The spectrum of PFD output v_{out} is then equal to:

$|V_{out}(f_m)| = |2\pi k_{PFD} \times \tau \times \mathrm{Sinc}(f_m\tau) \times F_n(f_m)|$, where $F_n(f_m)$ is the frequency noise measured at offset frequency f_m.

- The FFT based post processor will them compensate for the Sinc by using a Sinc-Inverse filtering. Due to the Sinc null at $f_m = 1/\tau$, the measurement result becomes questionable for $f_m \geq 1/(2\tau)$ due to measurement noise amplification of the Sinc-Inverse filter.

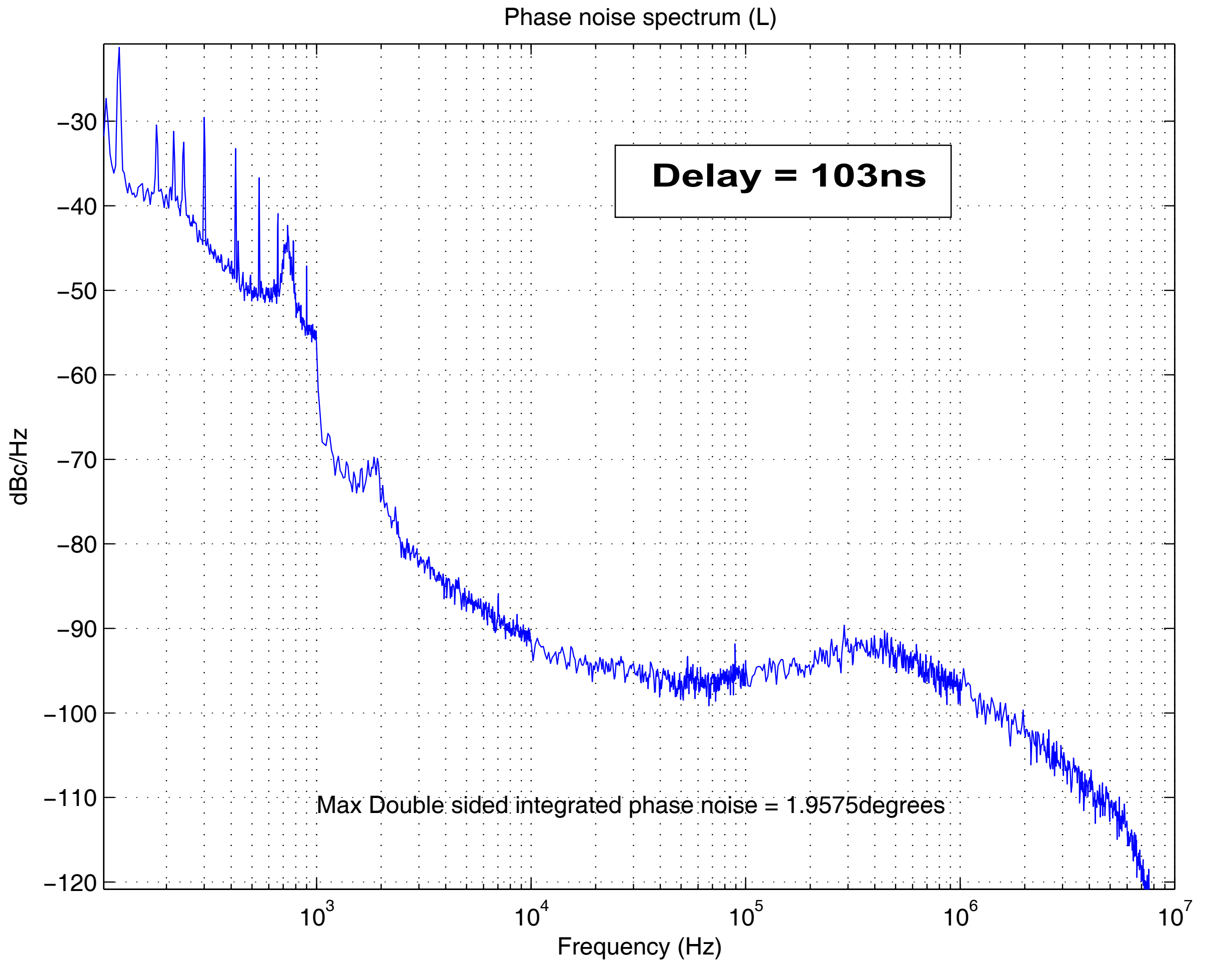
Delay Line technique (cont)

- The equipment will then calculate $F_n(f_m)$ and from there it calculates the phase noise by using the following relationship: $|\Phi_n(f_m)|^2 = |F_n(f_m)|^2 / f^2{}_m$.
- Due to this division the measurement noise will get amplified at lower offset frequencies. Hence, the equipment becomes less sensitive at lower frequency offsets.
- Hence, the main trade-off in this system is its delay of its delay line.
- The longer the delay, the larger the phase shift and hence the more robust the system is against the equipment noise, hence, better sensitivity. (Recall that $\varphi_{out}(t) = 2\pi f(t)\tau$)

Delay Line technique (cont)

- However, the longer the delay, the maximum reliable offset frequency that can be measured will decrease (recall that $\mathrm{Max}\, f_m = 1/(2\tau)$).
- Furthermore, longer delay lines are sometimes impractical or may cause too large a phase shift for the PFD to stay in its linear range.

A Measurement Example



Further Reading

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3. Ulrich L. Rohde, Microwave and Wireless Synthesizer: Theory and Design, John Wiley, 1997.